DIPLOMA IN ELEMENTARY EDUCATION (D.El.Ed.)

Course-504

Learning Mathematics at Elementary Level

Block -2
Enriching Contents and Methodology

NATIONAL INSTITUTE OF OPEN SCHOOLING
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Credit points (4=3+1)

Block	Unit	Name of Unit	Theory Study Hours		Practical Study
			Content	Activity	
Block-1: Importance of Learning Mathematics at the Elementary Stage of Schooling	**U1**	How children learn mathematics	3	2	Seminar on mathematics is for all, mathematics phobia
	U2	Mathematics and Mathematics Education - Importance, Scope and Relevance	4	2	-
	U3	Goals and Vision of Mathematics Education	4	2	Taking mathematics learning beyond classroom Identification of problems in mathematics education in your class
	U4	Learner and Learning – centered methodologies	5	3	Organizing mathematics club in your school
Block 2: Enriching Contents and Methodology	**U5**	Numbers, Operations on Numbers	5	2	-
	U6	Shapes and Spatial Relationships	5	2	-
	U7	Measures and Measurements	4	2	-
	U8	Data Handling	4	3	Statistical analysis of data
	U9	Algebra as generalized Arithmetic	4	2	-
Block 3: Learner Assessment in Mathematics	**U10**	Approaches to Assessment of Learning Mathematics	3	2	Development of a lesson plans and preparation of concept maps in mathematics
	U11	Tools and Techniques of Assessment	4	3	Development of exhibits for mathematics laboratory
	U12	Follow up of Assessment of Learning Mathematics	3	2	Identification of problems and preparation of remedial measures in learning mathematics
		Tutoring	15		
		Total	63	27	30
Grand Total			**63+27+30=120 hrs.**		

Block 2

Enriching Contents and Methodology

Block Units

Unit 5 Numbers, Operations on Numbers

Unit 6 Shapes and Spatial Relationships

Unit 7 Measures and Measurements

Unit 8 Data Handling

Unit 9 Algebra as generalized Arithmetic

BLOCK INTRODUCTION

You as a leaner will study Block2: Enriching Contents and Methodology. This Block consist five units relating to content enrichment and methodology. Every unit is divided into sections and sub-sections. You have already studied about importance of learning mathematics at the elementary stage, how children learn mathematics and how mathematics teaching can be made more joyful.

UNIT-5 This unit will empower you to understand numbers and operation on Numbers. There will be acquaintance with different sets of numbers like counting numbers and whole numbers, integers and rational numbers. Understanding will also be gained about Properties of operation on numbers. An answer to question that what are common factors and common multiple and concept of HCF & LCM will be found?

UNIT-6 This unit will empower you to understand the concept of Basic Geometrical figures. You will be acquainted with Two dimension closed figure like Triangle and quadrille. The understanding of circle, congruence and Similarity, Reflection and Symmetry and Three dimensional shapes will be developed.

UNIT-7 This unit will enable you to understand the concept of Measurement and Measure. How length, area, volume weight is measured by Non standardized and standardized units. There will be acquaintance with metric system of Measurement and measurement of time.

UNIT-8 This unit will empower you to understand the data handling. You will learn different aspects like collection of data and Tabular representation of data, pictorial depiction of data with the help of Bar graph, Histogram, pie-chart. The understanding will be developed, pie-chart. The understanding will be developed about how data is analyzed through measures of central tendency and measures of variations?

UNIT-9 You will be acquainted with using symbol for numbers, Algebraic terms and Expressions Algebra holds an important place in mathematics and same as its operations like addition, subtractions etc the understanding about linear equations and how they get solved will also be developed.

CONTENTS

UNIT 5 NUMBERS AND THE OPERATION ON NUMBERS

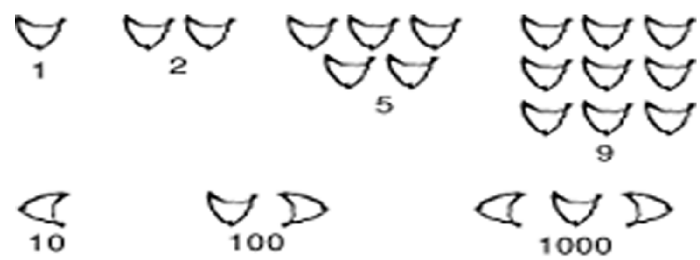
Structure

5.0 INTRODUCTION

In our day to day life we need to quantify several things that we use or come across. Members in the family, students in the class/school, money for purchasing clothes, weight of vegetables and groceries, books for the children, distance of the school from home, length and breadth of the room so on. For quantification numbers are essential. For counting objects, expressing the different quantities, measuring length, weight, volume, expressing time etc numbers are essential. Numbers have been so intimately associated with our life that we cannot think of anything without them. But the numbers that we use to-day were not invented at the beginning of civilization. Different systems of numbers were developed in different ancient civilizations. Let us have a look into some of the number systems developed in different ancient civilizations.

Mayan System:

Babylonian System :



Roman System:

I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**
1	2	3	4	5	6	7	8	9	10
XI	**XII**	**XIII**	**XIV**	**XV**	**XVI**	**XVII**	**XVIII**	**XIX**	**XX**
11	12	13	14	15	16	17	18	19	20
XXX	**XL**	**L**	**LX**	**LXX**	**LXXX**	**XC**	**C**	**CC**	**CCC**
30	40	50	60	70	80	90	100	200	300
CD	**D**	**M**							
400	500	1000							

In these systems of numbers, it was difficult to remember the numerals for different numbers. Further, it was difficult at the time of working out various operations like addition, subtraction etc.

India's Contribution: The present decimal system i.e. the numbers based on ten digits i.e.,0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 was designed basically by Indians and it was carried by Arabic people to their country and subsequently to western world. Thus, this system of numeration is named as Hindu-Arabic Numerals.

This system has a unique advantage over other system of numbers that any large number can be expressed using these 10 digits.

Place-Value in a Number: These ten digits could give rise to ten single-digit numbers. If we need a number more than 9, then we create two digit numbers like 10, 11, 12, …,25,…., 59, …,98 and 99. You are quite familiar as to how these numbers are created. There are two places in such two digit numbers, the right hand side place is the unit- place and the left hand side place is the tenth-place.

The digits in the unit place have unitary value i.e. in 26, the number in the unit place is 6 and its value is also 6. The digit in the tenth place i.e. 2 in the number 26 has the place value of two tens (i.e. 20) instead of the face value of 2. Similarly, the place value of a digit in the hundredth place of a number is 100 times the face value of the digit. You are quite familiar with such place values.

But, there is one digit which has the same place value irrespective of the place it occupies in the number. You know the number i.e. 0(zero). Zero is the unique contribution of the Hindu number system. Its place value is zero in which ever place it is posted. Then what is its importance?

Consider any three-digit number like 308. The place value of the digit 0 in the tenth place is zero. But imagine, had there be no 0 what would have happened to the numbers like 308? The number would have reduced to 38 and there would have been total confusion. Here, 0 keeps the place intact and gives proper identity to the number. Hence, 0 is called the **place holder** in the decimal system of numbers.

We shall be discussing about some very fundamental system of numbers which we use in our daily life and the properties of four basic operations on the systems of numbers in this unit.

To complete this unit, you shall require at least 10*(ten) study hours*.

5.1 LEARNING OBJECTIVES

After study of this unit you will be able to

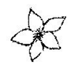
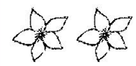
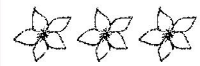
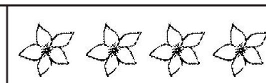
- understand the importance of numbers in the decimal system.
- recognize different sets of numbers e.g. natural numbers, integers and rational numbers.
- know the property of fundamental operations, like addition, subtraction, multiplication and division on different sets of numbers.
- determine the factors and multiples in a natural number set.

5.2 DIFFERENT SETS OF NUMBERS

5.2.1 Counting Numbers and Whole Numbers

The prime objective with which the early man wanted a system of numbers was to count the objects. Thus he created the *counting numbers* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11….. These are also called the '**Natural Numbers'**.

He associated numbers with the set of objects as follows:

Collection of objects				
Number name	one	two	three	four
Numeral	1	2	3	4

Thus, each number is uniquely associated with a set of objects in the system of counting numbers. You can observe that:

(i) 1 is the smallest counting number.

(ii) Every counting number has a successor and the successor of a number is 1 more than the number concerned i.e successor of 4 is 5 and successor of 29 is 30.

(iii) Every counting number (except 1) has a predecessor i.e. predecessor of 7 is 6 and predecessor of 60 is 59.

It follows from (ii) that there exists a counting number greater than any counting number however large.

Whole Numbers: You might have observed that 0 is not included in the set of natural numbers. It is because counting of objects always starts from 1. But when we represent numbers by numerals, we use 0 in representing numbers 10, 20. 30… 100 etc. To make the things simpler, 0 is included along with the set of the Natural Numbers to constitute the set of ‘Whole Numbers’ and is denoted by ‘**W**’.

Check your progress before proceeding further:

E1. Why the counting number 1 has no predecessor?

E2. What is the smallest whole number?

E3. What is the difference between the place value and face value of 8 when it occupies

(i) Unit’s place

(ii) Ten’s place

(iii) Hundred’s place

5.2.2 Integers

While dealing with various situations faced in the daily life, people came across some measurements with opposite characters like

Profit-Loss

Asset-Liability

Credit-Debit

Upward-Down ward

In respect of such pair of measurements with opposite characters of the above kinds, there exists a balancing position as shown in the table below.

Measurement with opposite characters	Balancing position
Profit- Loss	no Profit nor Loss
Asset- Liability	no Asset nor Liability
Credit- Debit	no Credit nor Debit
Upward-Downward	neither Up ward nor Downward

It can be seen that the balancing position in respect of each of the examples above represent the zero-level.

Thus people thought of creating numbers opposite in nature to 1, 2, 3 and so on. So we got the following pairs of number with opposite nature.

+1 and –1

+2 and –2

+3 and –3 and so on.

Zero is the balancing number in between each of the opposite pairs of numbers given above. Hence we got-

$(+1) + (-1) = 0$

$(+2) + (-2) = 0$

$(+3) + (-3) = 0$ and so on

The series of numbers which we now have are-

……………–4, –3, –2, –1, 0, +1, +2, +3, +4, ……..

This set of numbers is known as **Integers**.

+1, +2, +3, +4……………. etc are known as ***positive integers*** and –1, –2, –3, –4……….. etc. are known as ***negative integers***.

The set of integers is denoted by the symbol **Z**.

(i) There is no integer which can be said as the greatest. Any big integer you think of, there exists an integer greater than that.

(ii) There is no integer which can be said as the smallest. Any small integer you think of, there exists an integer smaller than that.

(iii) For every +p in the integer series, there exists –p in it, such that $(+p) + (-p) = 0$. +p and –p are known as opposite of each other.

(iv) Zero (0) is neither positive nor negative.

Ordering integers and their representation on a number line:

The series of integers written below go on increasing towards the right and go on decreasing towards the left.

$$\ldots\ldots\ldots\ldots\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\ldots\ldots\ldots\ldots\ldots$$

Thus $\ldots\ldots\ldots\ldots\ldots < -3 < -2 < -1 < 0 < +1 < +2 < +3 < \ldots\ldots\ldots\ldots\ldots$

We also consider that the points on a straight line can be used to represent the integers. Following is the procedure –

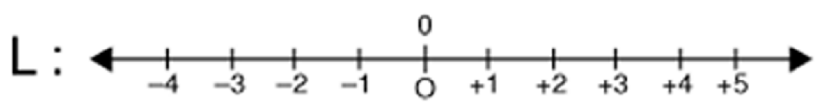
(i) Draw a straight line and name it as L (Here L denotes the entire line. It does not name any point or the line).

(ii) Mark points on it at equal intervals.

(iii) Name any of the points marked above as O and represent zero by it.

(iv) Now represent the numbers +1, +2, +3 etc. by the points towards right of 0 successively.

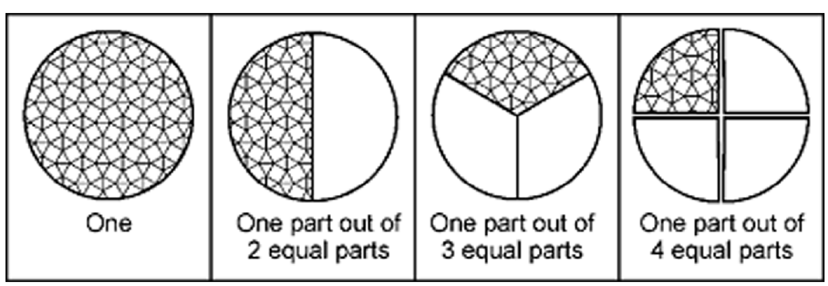
(v) Represent the numbers –1, –2, –3 etc by the points towards left of 0 successively.



Now we say the line L as a **number line**

5.2.3 Rational Numbers

Now let us look at the parts of a whole.



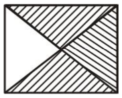
The numbers created to represent the parts of an object as shown above are as follows:

One part out of 2 equal parts of an object : $1/2$ (half, one by two)

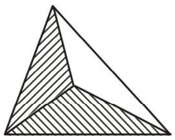
One part out of 3 equal parts of an object : $1/3$ (one third, one by three)

One part out of 4 equal parts of an object : $1/4$ (a quarter, one by four)

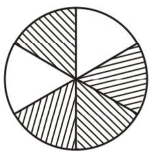
Similarly, the shadowed portion of the following figure



represents $3/4$ [3 is the numerator and 4 is the denominators.]



represents $2/3$ [2 is the numerator and 3 is the denominator]



represents $4/6$ [4 is the numerator and 6 is the denominator]

The numbers that are found to have been constructed to measure different parts of a complete object are known as **fractional numbers (or fractions)**.

There are various types of fractions:

(i) ***Proper fraction*: ** $\frac{2}{3}, \frac{3}{8}, \frac{5}{7}$ are proper fractions, where numerator < denominator.

(ii) ***Improper fraction***: $\frac{5}{3}, \frac{11}{7}, \frac{28}{5}$ are improper fractions, where numerator > denominator.

(iii) ***Mixed number* :** $2\frac{1}{3}, 3\frac{2}{7}$ etc. are mixed numbers and each of them can be changed into an improper fraction and an improper fraction can be changed into a mixed number, such as

$$2\frac{1}{3} = \frac{7}{3}, \qquad 3\frac{2}{7} = \frac{23}{7}$$

(iv) ***Unit Fraction*:** A fraction with numerator as 1 is a unit-fraction. $\frac{1}{2}, \frac{1}{5}, \frac{1}{9}$ are examples of unit fractions.

(v) ***Equivalent Fractions*:** When both the numerator and the denominator of a fraction are multiplied by a positive integer, the fraction changes in shape but does not change in value. Thus the resulting fraction is equal to the original fraction.

Fractions having same value are *equivalent fractions*. As for example

(a) $\frac{2}{3}=\frac{4}{6}=\frac{8}{12}$ Thus $\frac{2}{3},\ \frac{4}{6},\ \frac{8}{12}$ are equivalent fractions.

(b) $\frac{40}{16}=\frac{20}{8}=\frac{10}{4}=\frac{5}{2}$ Thus $\frac{40}{16},\ \frac{20}{8},\ \frac{10}{4},\ \frac{5}{2}$ are equivalent fractions.

***(vi) Like fractions*:** Fractions with equal denominators are known as like fractions.

$\frac{1}{15},\frac{2}{15},\frac{3}{15},\frac{4}{15}$ are examples of like fractions.

Rational numbers: Number written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$, are known as rational numbers. Rational numbers are denoted by the symbol Q. Obviously a fraction is a rational number.

Examples of rational numbers are :

$$\frac{2}{7},\ \frac{-3}{4},\ \frac{0}{5},\ \frac{4}{-9}$$

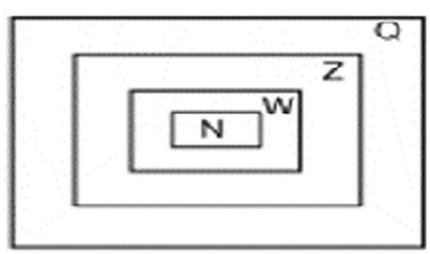
Observe the following :

$$1=\frac{3}{3},\ 2=\frac{10}{5},\ -4=\frac{-8}{2},\ 0=\frac{0}{9}$$

Thus we find that all the integers could be written in the form $\frac{p}{q}$ where ‘p’ and q” are integers and $q \neq 0$.

Thus we see that *each integer is a rational number*.

The diagram alongside shows the inter-relationship connecting the Natural numbers (N), Whole numbers (W), Integers (Z) and Rational numbers (Q).



Q includes all integers and also the non-integral numbers like $\frac{1}{2},\ \frac{2}{7},\ \frac{-3}{8}$ etc.,

Notes

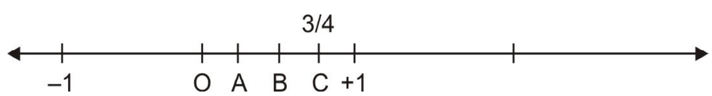
Z includes all whole numbers and also the negative integers like –1, –2, –3 ……

W includes all natural numbers and also zero.

Rational numbers on a number line: We have been already seen how to show integers on a number-line. Now we would show the rational numbers on a number line.

Let us show (i) $\frac{3}{4}$ (ii) $-\frac{2}{5}$ (iii) $2\frac{2}{3}$ on a number-line.

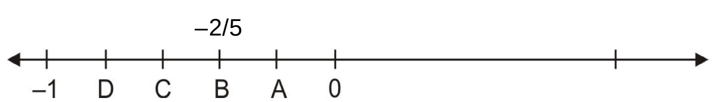
As we know, $\frac{3}{4}$ is 3 parts out of 4 equal parts of a whole.



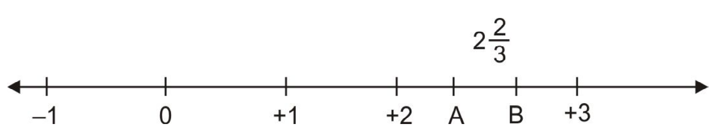
The part of the number line between 0 and + 1 is divided into 4 equal parts and A, B and C are the points of division of the whole (0 to 1) into 4 equal parts.

From 0 to C shows 3 parts out of 4 equal parts of 1 division on the number line.

$\therefore$ C represents $\frac{3}{4}$ on the number line. To show $-\frac{2}{5}$



In the number line above, B represents $-\frac{2}{5}$.



In the number line above B represents $\frac{8}{3} = 2\frac{2}{3}$.

Standard form of writing a rational number: When a rational number is written in the form $\frac{p}{q}$ where p and q have 1 as the only common factor with q > 0, the number is said to be the standard form.

Check your progress before moving to the next section:

E4. State whether the following statements are 'True' or 'False':

Notes

(a) All natural numbers are integers.

(b) All integers are whole numbers.

(c) Whole numbers are not rational numbers.

(d) Negative integers cannot be rational numbers.

(e) All rational numbers are not integers.

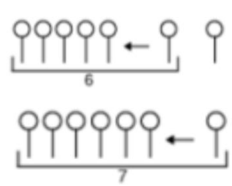
5.3 PROPERTIES OF OPERATIONS ON NUMBERS

We are quite familiar and competent in carrying out four fundamental operations-addition, subtraction, multiplication and division. In this section you will know the properties of these operations when those are performed on different sets of numbers.

5.3.1 Operations on Natural Numbers and Whole Numbers

As you have already known that the set of whole numbers differ from the set of natural numbers by inclusion of '0', the properties of four operations on the numbers of these two sets are nearly same and hence discussed together in this sub-section.

(a) Addition: When two collections of similar objects are put together, how to find the total number of objects in the new collection? Suppose 2 match sticks are to be added to 5match sticks. We can teach the children to add it in two ways. One way is to put together the two collections and then count the total number of match sticks in the mixed collection. The second way is to put one collection intact (say the collection having 5 match sticks) and add one match stick from the other at a time as shown below:



$5 + 2$

$= (5+1) +1$

$= (6+1)$

$= 7$

Some *properties of addition in natural and whole numbers*:

(i) *Closure property*: Sum of two natural/whole numbers is also a natural/ whole number.

(ii) *Commutative Property*: $p + q = q + p$ where p and q are any two natural/ whole numbers.

(iii) *Associative property*: $(p + q) + r = p + (q + r) = p + q + r$. This property provides the process for adding 3 (or more) natural/whole numbers.

(iv) *Additive Identity in Whole Numbers*: In the set of whole numbers, 4 + 0 = 0 + 4 = 4. Similarly, p + 0 = 0 + p = p (where p is any whole number). Hence, 0 is called the *additive identity* of the whole numbers.

(b) Subtraction: Subtraction means taking away. From a group of objects, we can take away some or all. For example from 5 objects we can take away less than 5 objects or all the 5 objects. From a group of objects when all are taken away, nothing is left over and since nothingness is represented by zero, p – p = 0 (where p is a whole number)

Multiplication:

Multiplication represents *repeated addition of a number with itself*. For example:

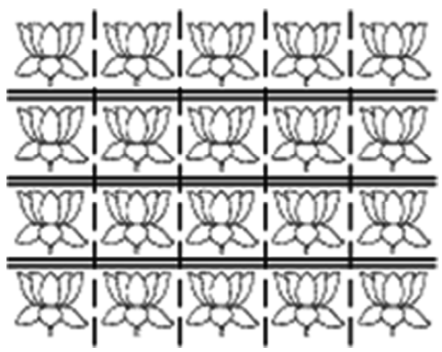
3 + 3 is represented as 3 × 2

3 + 3 + 3 is represented as 3 × 3

3 + 3 + 3 + 3 is represented as 3 × 4 and so on.

Properties of Multiplication:

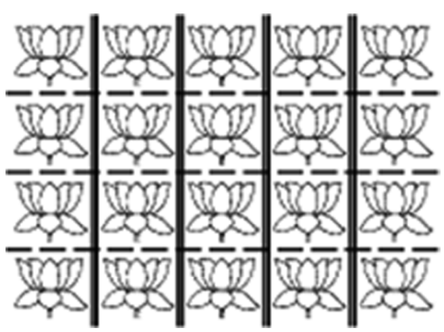
(i) *Commutative property*: Observe the tables below:



5 flowers in each row.

There are 4 rows.

Total number of flowers = 5 + 5 + 5 + 5 or 5 × 4 = 20



The above collection is shown below:

4 flowers in each column.

There are 5 columns.

Total number of flowers = 4 + 4 + 4 + 4 + 4 = 4 × 5 = 20

But both the collections show the same number of flowers.

Thus it is found that: 5 × 4 = 4 × 5

In other words, if p and q are any two natural/whole numbers, then

$$p \times q = q \times p.$$

Thus multiplication is commutative in natural and whole numbers.

(ii) *Closure property*: If p and q are natural or whole numbers then p × q is also a natural or whole number. We say natural/whole numbers are closed under multiplication.

(iii) *Associative property*: (p × q) × r = p × (q × r) (where p, q, and r are any three natural/whole numbers)

(iv) *Identity of multiplication*: The number '1' has the following special property in respect of multiplication.

p × 1= 1 × p = p (where p is a natural number)

Thus we say : The number '1' is the identity of multiplication.

(v) *Distributive property of multiplication over addition*:

p × (q + r) = p × q + p × r

We say, "multiplication distributes over addition.

Examples: 5 × (3 + 4) = 5 + 7 = 35

And 5 × 3 + 5 × 4 = 15 + 20 = 35

∴ 5 × (3 + 4) = 5 × 3 + 5 × 4.

(d) Division:

When p and q are natural numbers and p × q = r,

We say :

'r' is divisible by 'p' and 'r' is divisible by 'q'.

Each of 'p' and 'q' is a factor of 'r'.

'r' is a multiple of each of 'p' and 'q'.

We use the symbol '÷' and write r ÷ p = q and r ÷ q = p

For example : 3×5 = 15. Thus we say :

(i) 15 is *divisible* by each of 3 and 5.

Notes

(ii) Each of 3 and 5 is a *factor* of 15.

(iii) 15 is a *multiple* of each of 3 and 5.

We write: 15 ÷ 3 = 5 and 15 ÷ 5 = 3

It can be further seen that

(a) 1 × 12 = 12, (b) 2 × 6 = 12 (c) 3 × 4 = 12

Thus (a), (b) and (c) show that 1, 2, 3, 4, 6 and 12 each is a factor of 12.

What about the natural numbers 1, 2 and 3 ?

There are no two different numbers whose product is 1. Hence-

1 has only one factor and it is 1.

As 1× 2 = 2 and there is no second pair of numbers which product is 2, hence-

2 has only 2 factors and those are 1 and 2.

Similarly there are many other natural numbers each of which has only 2 factors. 2, 3, 5, 7, 11, 13….. are examples of such natural numbers each of which has 2 factors only. These are called *prime numbers.*

> *A natural number having exactly two distinct factors 1 and the number itself is called a* ***Prime Number****.*

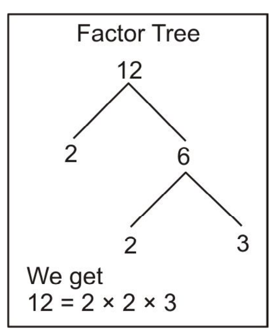
> *A natural number like 4, 6, 8, 9,…12,15… which has more than 2 factors is known as a* ***composite number****.*

> *Prime number is defined on natural number greater than one number like* $0, 1, -1, -2, \ldots, \frac{1}{2}, \frac{1}{3}$ *etc. are neither a prime number nor a composite number.*

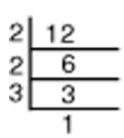
Prime factorization of composite numbers:

Writing a composite number as the product of prime numbers is known as the prime factorization of it. Such as-

12 = 2 × 2 × 3



Working process :



Thus 12 = 2 × 2 × 3

Notes

***Some terms related to primes*:**

Co-Primes (or Mutually Primes):

Two natural numbers are **co-primes** if those do not have a common factor. Following are the examples-

(i) 8 and 27 are co-primes (even if each of them is composite).

(ii) 17 and 20 are co-primes.

Twin Primes: Two prime numbers, the difference between is 2, are known as twin primes.

3 and 5, 5 and 7, 11 and 13, 17 and 19 are examples of twin primes.

Even Prime: 2 is the only prime which is even. It is also the **Smallest Prime**.

Identification of primes within a certain range :

The following is the process to find the prime numbers between 1 and 100.

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

[The Sieve of Eratosthenes. Eratosthenes was Greek Mathematician]

Procedure :

(i) Strike out all multiples of 2 greater than 2.

(ii) Strike out all multiples of 3 greater than 3.

(iii) Strike out all multiples of 5 greater than 5.

(iv) Strike out all multiples of 7 greater than 7.

Notes

All numbers (except 1) that are not struck off are prime numbers.

Why the process stops at 7 ?

Square root of 100 is 10.

Prime number just less than 10 is 7. Hence the process continues up to 7.

Thus, the prime numbers between 1 and 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Now, check your progress:

E5. What is the natural number that is neither prime nor composite?

E6. What is the additive identity in the set of whole numbers?

E7. The difference between two prime numbers is odd. If their sum is 15, what are those two prime numbers?

E8. How many pairs of twin primes occur between 10 and 30?

E9. If the divisor, quotient and the remainder in a division are 8, 12 and 5 respectively, what is the dividend?

5.3.2 Operations on Integers

A. Addition : All the properties of addition in whole numbers such as (i) Closure property (ii) Commutative property (iii) Associative property (iv) Existence of additive identity also occur in integers.

The extra property that occurs in integers is the following:-

(v) ***Existence of additive inverse***: If ‘+p’ is an integer, then there exists an integer ‘-p’ such that (+p) + (-p) = 0

‘+p’ and ‘-p’ are known as the additive inverse of each other.

Now let us discuss the operation of addition on integers as a process.

***(a) Addition of positive integers* :**

Addition of positive integers is the same as addition of natural numbers.

Such as (+5) + (+3) = +8

***(b) Addition of a positive integer and a negative integer* :**

Such as : (+5) + (–3)

We know that –

$+5 = (+1) + (+1) + (+1) + (+1) + (+1)$

Similarly, –3 can be written as-

$-3 = (-1) + (-1) + (-1)$

Now $(+5) + (-3) = (+1) + (+1) + (+1) + (+1) + (+1) + (-1) + (-1) + (-1)$

$= \{(+1) + (-1)\} + \{(+1) + (-1)\} + \{(+1) + (-1)\} + (+1) + (+1)$

$= 0 + 0 + 0 + (+2)$

$= 0 + (+2) = +2$

A shortcut process :

$(+5) + (-3) = (+2) + (+3) + (-3)$ [+5 is replaced by (+2) + (+3)]

$= (+2) + \{(+3) + (-3)\}$

$= (+2) + 0 = +2$

Another example :

$(+4) + (-7) = (+4) + (-4) + (-3)$ [-7 is replaced by (–4) + (–3)]

$= \{(+4) + (-4)\} + (-3)$

$= 0 + (-3) = -3$

(c) Addition of two negative numbers:

$(-2) + (-3) = (-1) + (-1) + (-1) + (-1) + (-1)$

$= -5$

B. Subtraction:

Subtraction in Integers is the addition of the opposite numbers (i.e. additive inverse).

Thus, *if ‘p’ and ‘q’ are two integers, then* $p - q = p + (-q)$

For example-

(i) $(+5) - (+8) = (+5) + (-8)$

(ii) $(+4) - (-3) = (+4) + (+3)$

(iii) $(-5) - (+2) = (-5) + (-2)$

(iv) $(-7) - (-3) = (-7) + (+3)$

Notes

And in course of addition, we knew the above results.

A special feature of subtraction in Integers is the following –

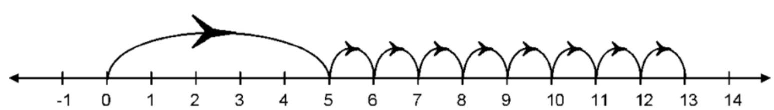
> *In Natural numbers p – q was a meaningful operation, if q < p. But in integers p – q is also meaningful when q < p, q = p or q > p as well.*

Operation of addition and subtraction on number lines :

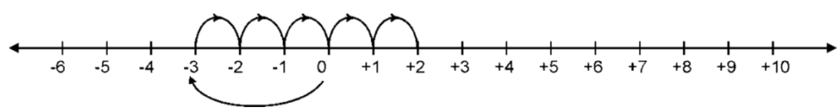
Number lines can be used to work out addition and subtraction as shown below:-

(a) ***Addition*** : (to add we move towards the right)

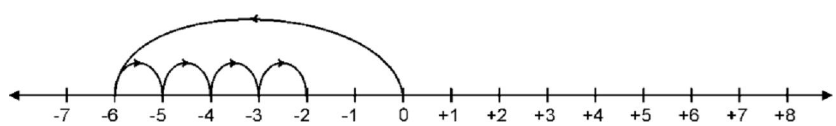
(i) (+5) + (+8) = +13



(ii) (–3) + (+5) = +2

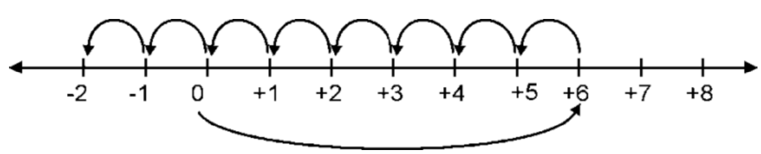


(iii) (+4) + (–6) = (–6) + (+4) = –2 [By commutative property]

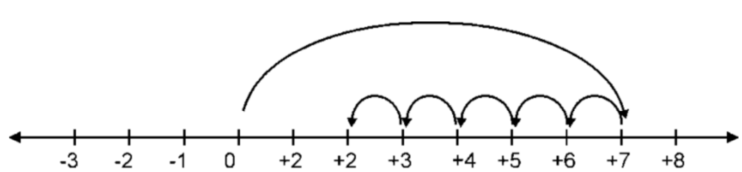


(b) ***Subtraction*** **:** (to subtract we move to the left)

(i) (+6) – (+8) = –2 [from +6 we count 8 intervals leftward]



(ii) (–5) – (–7) = –5 + 7 = 7 – 5 = +2



(c) ***Multiplication*** **:** Multiplication of two integers is the addition if one of integer is a non negative integer.

Such as :

(i) $(+5) + (+5) + (+5) + (+5) = (+5) \times 4$

Thus $(+5) \times (+4) = +20$ [as was done with natural numbers]

(ii) $(-3) + (-3) + (-3) + (-3) + (-3) = (-3) \times 5$

Thus $(-3) \times 5 = (-3) + (-3) + (-3) + (-3) + (-3) = -15$

Sign rules for multiplication of Integers:

p	**q**	**p × q**
Positive $P > 0$	Positive $q > 0$	Positive $p \times q > 0$
Negative $P < 0$	Negative $q < 0$	Positive $p \times q > 0$
Positive $P > 0$	Negative $q < 0$	Negative $p \times q < 0$
Negative $P < 0$	Positive $q > 0$	Negative $p \times q < 0$
Positive $P > 0$	Zero $q = 0$	Zero $p \times q = 0$
Negative $P < 0$	Zero $q = 0$	Zero $p \times q = 0$

Properties of multiplication in Integers

(i) Multiplication is closed.

(ii) Multiplication is associative.

(iii) Existence of multiplicative identity. The multiplicative identity in integers is 1.

(iv) Multiplication distributes over addition.

ACTIVITY - 1

Verify the above stated properties of multiplication in integers taking concrete examples.

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D. Division: We have already discussed division in natural numbers as the reverse process of multiplication. Similar is the case here.

If 'p' and 'q' are non-zero integers and $p \times q = r$, then we say

(i) $r \div p = q$

(ii) $r \div q = p$

Thus,

(i) $(+5) \times (+3) = +15$

$\therefore \quad (+15) \div (+5) = +3$

and $(+15) \div (+3) = +5$

(ii) $(+4) \times (-6) = -24$

$(-24) \div (+4) = -6$

and $(-24) \div (-6) = +4$

(iii) $(-3) \times (-5) = +15$

$\therefore \quad (+15) \div (-3) = -5$

and $\quad (+15) \div (-5) = -3$

Sign rules of division in Integers

P	**Q**	**P ÷ q**
Positive $P > 0$	Positive $q > 0$	Positive $(p \div q) > 0$
Positive $P > 0$	Negative $q < 0$	Negative $(p \div q) < 0$
Negative $P < 0$	Positive $q > 0$	Negative $P \div q < 0$
Negative $P < 0$	Negative $q < 0$	Positive $(P \div q) > 0$

Important note: Division of an integer by zero (0) is meaningless.

Answer the following to check your progress:

E10. What is the additive identity in integers?

E11. What is the additive inverse of +7?

E12. What is the integer which when multiplied by itself gives 1? How many such integers are there?

E13. Find the sum of all integers from –8 to + 8.

Notes

5.3.3. Operations on Rational Numbers

A. Addition:

(i) We add rational numbers after changing them into like fractions :

$$\frac{3}{8} + \frac{5}{12} = \frac{9}{24} + \frac{10}{24}$$

(Both the rational numbers are changed into like fractions.)

$$= \frac{9+10}{24}$$

$$= \frac{19}{24}$$

(ii) We add rational numbers by applying the following rule

$$\frac{p}{q} + \frac{r}{s} = \frac{ps+rq}{qs}$$

For example –

$$\frac{2}{3} + \frac{4}{5} = \frac{2\times5+3\times4}{3\times5} = \frac{10+12}{15} = \frac{22}{15} = 1\frac{7}{15}$$

Note : (–8) ÷ 4 = –2, 8 ÷ (–4) = –2

Hence $\frac{-8}{4} = \frac{8}{-4} = -\frac{8}{4}$; *Thus* $\frac{-p}{q} = \frac{p}{-q} = -\frac{p}{q}$

Properties of addition:

(i) Addition in rational numbers is closed.

(ii) Addition in rational numbers is commutative.

(iii) Addition is associative.

(iv) Existence of additive identity. Zero is the additive identity in Q.

(v) Existence of additive inverse. $\frac{p}{q}$ and $-\frac{p}{q}$ are additive inverse of each other.

Hence $\frac{p}{q} + \left(-\frac{p}{q}\right) = 0$

ACTIVITY - 2

Verify the above mentioned properties of addition of rational numbers with concrete examples.

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Notes

B. Subtraction:

(i) We subtract one rational number from another by changing them to like fractions.

Example:

$$\frac{5}{8}-\frac{7}{12}=\frac{15}{24}-\frac{14}{24}$$

$$=\frac{15-14}{24}=\frac{1}{24}$$

(ii) To subtract a rational number from another, we follow the rule given for addition after expressing the subtraction into addition.

Thus,

$$\frac{p}{q}-\frac{r}{s}=\frac{p}{q}+\left(\frac{-r}{s}\right)$$

$$=\frac{ps-qr}{qs}$$

As for example,

$$\frac{3}{4}-\frac{2}{5}=\frac{3}{4}+\frac{-2}{5}=\frac{3\times 5+(-2)\times 4}{4\times 5}=\frac{15-8}{20}=\frac{7}{20}$$

C. Multiplication:

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then

$$\frac{p}{q}\times\frac{r}{s}=\frac{p\times r}{q\times s}$$

Example

(i) $\frac{2}{3}\times\frac{4}{5}=\frac{2\times 4}{3\times 5}=\frac{8}{15}$

(ii) $\frac{-3}{4}\times\frac{2}{7}=\frac{-3\times 2}{4\times 7}=\frac{-6}{28}=\frac{-3}{14}$

Properties of multiplication in rational numbers:

(i) Multiplication is closed.

(ii) Multiplication is commutative.

(iii) Multiplication is associative.

(iv) Existence of multiplicative identity.(Multiplicative identity in Q is 1).

(verify these properties with concrete examples)

(v) Existence of multiplicative inverse.

$\frac{p}{q}$ and $\frac{q}{p}$ are *multiplicative inverses* of each other as

$$\frac{p}{q}\times\frac{q}{p}=1$$

$\frac{p}{q}$ and $\frac{q}{p}$ are also known as *reciprocals* of each other.

For example, multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$, multiplicative inverse of 5 is $\frac{1}{5}$.

(vi) Multiplication distributes over addition.

Such as , $\frac{p}{q}\times\left(\frac{m}{n}+\frac{k}{l}\right)=\frac{p}{q}\times\frac{m}{n}+\frac{p}{q}\times\frac{k}{l}$

Example:

$$\frac{2}{3}\left(-\frac{4}{5}+\frac{6}{7}\right)=\frac{2}{3}\times\left(-\frac{4}{5}\right)+\frac{2}{3}\times\frac{6}{7}$$

Sign-rules of multiplication are the same as in case of integers.

ACTIVITY - 3

Verify the properties (i) to (iv) of multiplication of rational numbers as stated above with concrete examples.

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D. Division:

Division of a given rational number by another is to multiply the first rational number with the multiplicative inverse of the second.

That is

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr}$$

Such as-

$$\frac{2}{3} \div \left(\frac{-4}{7}\right) = \frac{2}{3} \times \frac{7}{-4} = \frac{14}{-12} = -\frac{7}{6} = -1\frac{1}{6}$$

Note : (i) Division by 0 is meaningless.

(ii) Sign-rules of division are the same as those in case of integers.

Decimal equivalence of Rational numbers: Rational numbers with denominators equal to 10 or some power of 10 can have a different kind of representation. The following are the examples :

$$\frac{1}{10} = 0.1,\ \frac{2}{10} = 0.2,\ \frac{7}{10} = 0.7$$

$$\frac{1}{100} = 0.01,\ \frac{2}{100} = 0.02,\ \frac{14}{100} = 0.14$$

Let us look to some other rational numbers.

$$\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10} = 0.5$$

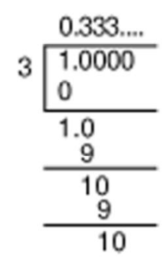
$$\frac{3}{4} = \frac{3\times25}{4\times25} = \frac{75}{100} = 0.75$$

$$\frac{5}{8} = \frac{5\times125}{8\times125} = \frac{625}{1000} = 0.625$$

$$\frac{2}{5} = \frac{2\times2}{5\times2} = \frac{4}{10} = 0.4$$

Thus it can be seen that rational numbers of which the denominators have no factor other than 2 or 5, can be represented as decimal numbers. Such decimal numbers are known as terminating decimals. What about $\frac{1}{3}, \frac{1}{6}, \frac{2}{7}$ etc.? In such cases the denominator cannot be changed into a power of 10 by multiplying it with any number whatsoever.

Hence we divide 1 by 3 to get a decimal equivalence, if any.



The division will never come to an end as at every phase of division the remainder is the same as the original dividend. So we go on getting 3 successively.

Thus we say-

$$\frac{1}{3} = 0.3333......$$

As the result never terminates and the digit 3 goes on occurring repeatedly, we say the result as a *non-terminating and recurring decimal number*.

Some other examples of the above kind are :

0.232323………

2.537373737……….

1.342342342………..

The above numbers are also written in the following symbolic form.

0.3333 …………… $= 0.\overline{3}$

0.232323 …………… $= 0.\overline{23}$

Notes

$2.5373737 \ldots\ldots\ldots\ldots = 2.5\overline{37}$

$1.342342342\ldots\ldots\ldots\ldots = 1.\overline{342}$

We may also have *non-terminating and nonrecurring decimal numbers*. Following are the examples.

0.12112111211112................

3.201001000100001...................

It can be seen that neither a single digit, nor a group of digits are found occurring repeatedly.

Expressing a non-terminating and recurring decimal number in rational form:

Example : Express in rational form :

(i) $0.\overline{4}$ (ii) $0.\overline{23}$

Solution :

(i) Let $0.\overline{4}$ be x.

$\Rightarrow$ $0.444 \ldots\ldots = x$ ...(1)

$\Rightarrow$ $0.444 \ldots\ldots \times 10 = x \times 10$ (multiplying 10 on both sides)

$\Rightarrow$ $4.444 \ldots\ldots = 10\,x$ ...(2)

From (1) and (2) we get,

$4.444\ldots - 0.444\ldots = 10x - x$,

$\Rightarrow$ $4 = 9x$,

$\therefore$ $x = \dfrac{4}{9}$

Hence $0.\overline{4} = \dfrac{4}{9}$

(ii) Let $0.\overline{23}$ be x

$\Rightarrow$ $0.232323\ldots\ldots = x$ ...(1)

$\Rightarrow$ $0.232323\ldots\ldots \times 100 = x \times 100$

$\Rightarrow \quad 23.232323\ldots\ldots = 100\,x$ ...(2)

substructing e.q. (1) from eq. (2) we get $23 = 99\,x$

$$\Rightarrow \quad x = \frac{23}{99}$$

Note : A terminating and a non-terminating and recurring decimal each represents a rational number. But a non-terminating and non-recurring decimal number does not represent a rational number.

Extension of place value system

10000 $= 10^4$	1000 $= 10^3$	100 $= 10^2$	10 $= 10^1$	1 $= 10^0$	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Thus to the right of units place, tenth place, hundredth place etc follow.

Hence, $23.715 = 2 \times 10 + 3 \times 1 + 7 \times \frac{1}{10} + 1 \times \frac{1}{100} + 5 \times \frac{1}{1000}$

Check your progress.

E14. What is the additive inverse of (a) $\frac{2}{7}$, (b) $\frac{3}{-8}$, (c) 0 ?

E15. Find the decimal equivalence of (i) $\frac{12}{25}$ (ii) $\frac{7}{8}$ (iii) $\frac{2}{7}$

E16. What is the multiplicative inverse of (a) $\frac{3}{7}$ (b) $-\frac{5}{8}$ (c) 0?

E17. Express $0.\overline{51}$ in $\frac{p}{q}$ form.

5.4 FACTORS AND MULTIPLES

While discussing the properties of multiplication and division of natural numbers, we had talked about the multiples and factors of natural numbers. In this section, we have tried to discuss about the determination of common factors and multiples and their use in calculating the highest common factor (H.C.F.) and the lowest common multiple (L.C.M.) respectively indicating their use in solving some real life problems.

5.4.1 Common Factor and the Highest Common Factor

Let us take the numbers 12 and 18.

Factors of 12 are : 1, 2, 3, 4, 6 and 12 ….. (a)

Factors of 18 are : 1, 2, 3, 6, 9 and 18 …… (b)

From (a) and (b) it can be seen that

1, 2, 3 and 6 are the factors that commonly occur in the two lists above.

Hence 12 and 18 are said to have 1, 2, 3 and 6 as their **common factors**.

The highest of the common factors is 6.

Hence 6 is known as the **highest common factor** (or **H.C.F**) of 12 and 18.

Determination of H.C.F :

Process (i) : After writing the list of factors of each of the numbers the H.C.F. can be determined (as was done for 12 and 18 above).

Process (ii) **:** ***Prime factorization*** method :

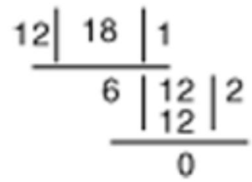
$12 = 2 \times 2 \times 3 = 2^2 \times 3^1$

$18 = 2 \times 3 \times 3 = 2^1 \times 3^2$

H.C.F is the product of the lowest powers of each of the prime factors that commonly occurs in both the numbers.

$\therefore$ H.C.F. of 12 and 18 $= 2^1 \times 3^1 = 6$

Process (iii) **:** ***Continued division*** method **:**



Step - 1 : Larger of the 2 numbers is divided by the smaller and the remainder is determined.

Step - 2 : Division of the previous division is taken as the dividend and it is divided by the remainder of the previous division.

This process continues till the remainder becomes zero.

Divisor of the last division, where the remainder is zero, becomes the required H.C.F.

Application of H.C.F. (word problems)

Let us see the example below**:**

There are 24 boys and 30 girls in a class. Separate lines of girls and boys are to be prepared with *equal number* of students in each line. What should be the greatest number of boys (or girls) with which the lines are to be made so that all girls and all boys get accommodated?

The greatest no. of children (boys or girls) in each row is the HCF of 24 and 30.

We then find the H.C.F. of 24 and 30 i.e. 6. Thus we get the answer.

5.4.2 Common Multiples and the Lowest Common Multiples

Let us take the numbers 8 and 12.

Multiple of 8 is a number which is divisible by 8.

Hence 8×1, 8×2, 8×3, 8×4 and so on are the multiples of it.

Thus the multiples of 8 are : 8, 16, 24, 32, 40, 48, 56, 64, 72…………….

(and it is a non-ending list)

Similarly, the multiples of 12 are : 12, 24, 36, 48, 60, 72………..

(a non- ending list too)

Now it can be seen that the common multiples of 8 and 12 are : 24, 48, 72………….

The list of common multiple is also non-ending.

The lowest common multiple (or L.C.M.) of 8 and 12 is 24.

Determination of L.C.M.

Process (i) After writing the lists of multiples of each of the numbers, the common multiples and the lowest common multiple (L.C.M.) can be determined as above.

Process (ii) ***Prime factorization*** method : Suppose we want to determine the L.C.M. of 12 and 18.

$12 = 2\times2\times3 = 2^2 \times 3^1$

$18 = 2\times3\times3 = 2^1 \times 3^2$

L.C.M is the product of the maximum number of each prime factor that occurs in either of the numbers.

Thus the L.C.M. = $2^2 \times 3^2 = 2\times2\times3\times3 = 36$

Relation between the L.C.M. and H.C.F of Two Numbers:

Let us observe the following examples:

Numbers	Product of the numbers	H.C.F.	L.C.M.	Product of H.C.F. and L.C.M.
12 & 18	216	6	36	216
16 & 28	448	4	112	448
25& 35	875	5	175	875

Thus we see that

Product of two positive numbers = Product of their H.C.F. and L.C.M.

E18 What is the H.C.F of two mutually prime numbers?

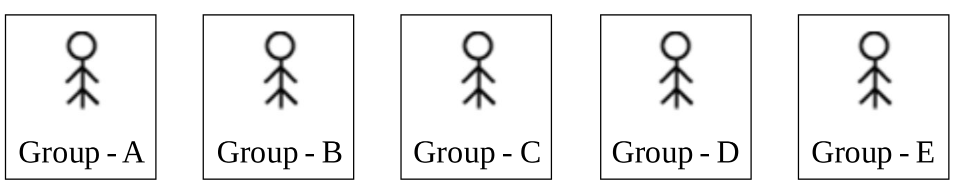
E19 H.C.F and L.C.M of two numbers are 8 and 96 respectively. If one of the two numbers is 24, what is the other number?

5.5 ARITHMETIC AND APPLICATION

A. Unitary Method

There are 20 children to be divided equally into five groups. Now we perform the work of division.

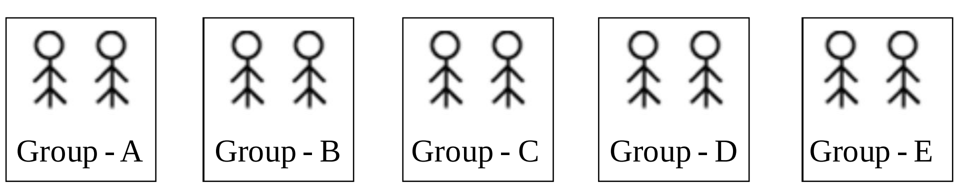
Five Places are marked for five groups and one child is made to stand in each space.



5 children have gone out of 20.

Thus 20 – 5 = 15 are left.

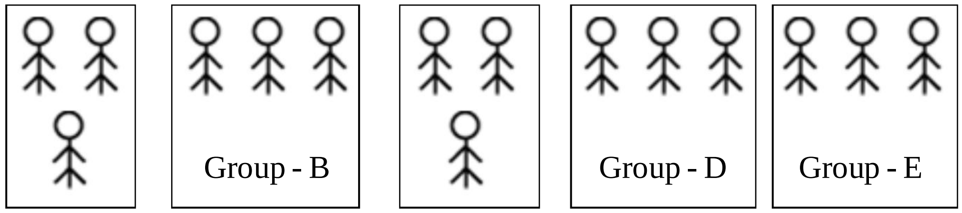
A second child is made to stand in each space.



5 more children have gone out of the 15 left earlier.
Thus 15 – 5 = 10 are left.

Notes

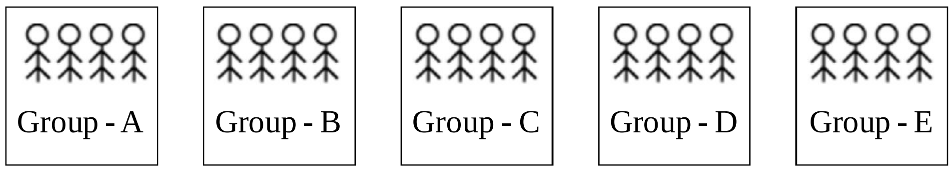
A third child is made to stand in each space.



5 more children have gone out of the 10 left earlier.

Thus 10 – 5 = 5 are left.

A fourth child is made to stand in each space.



Now 5 – 5 = 0 left.

Thus we see that there are 4 children in each group.

But we know that the number of times 5 can be taken from 20 is 20 ÷ 5.

So we say-

If 5 groups will have 20 children, then 1 group will have 20 ÷ 5 = 4 children. Of course the groups are of equal size.

Other examples –

(i) If 5 vessels (of equal size) contain 20*l* of milk, then

1 vessel would contain 20*l* ÷ 5 = 4*l*

(ii) If 5m of ribbon cost ` 20.00, then

1m of ribbon costs ` 20.00 ÷ 5 = ` 4.00

Now let us calculate for many when we know for one.

One pen costs `8.00

What is the cost for 3 pens ?

Obviously the total cost of all 3 pens = `8.00 + `8.00 + `8.00

But we know that 8 + 8 + 8 is the same as 8 × 3.

∴ We can say that 3 pens cost `8.00 × 3

Thus we came to know that-

If 1 pen costs `8.00

Then 3 pens cost `8.00 × 3 = `24.00

Other similar examples are-

Notes

(iii) If 1 packet of salt weighs 600g.

Then 4 packets of salt weigh 600g. × 4 = 2400g. = 2.400kg

(iv) If one jar of oil containts 12kg

Then 5 jars of oil contain 12kg × 5 = 60 kg

Let us analyze the examples discussed above.

In every example given above there were two variables.

Examples	1^{st} variable	2^{nd} variable
(i)	no of vessels	capacity
(ii)	length	cost
(iii)	no of packets	weight
(iv)	no of jars	capacity

In each of the examples above as many times becomes the 1^{st} variable so many times becomes the 2^{nd} variable.

Such as –

Twice is the number of vessels; twice would be their capacity;

3 times is the length; 3 times is the cost and so on.

Thus –

(i) While knowing for many and we calculate for one, we divide.

(ii) While knowing for one and we calculate for many, we multiply.

Such pairs of variables between which, as many times becomes one, so many times becomes the other, are said to have *direct variation* between them.

Variables having inverse variation in them

Let us consider the following situations:

3 workers can complete a work in 5 days.

If one worker has to work alone, how many days would he take to complete ?

Our experience shows that he can complete it in 5 + 5 + 5 = 5 × 3 days

Hence it is seen that –

If 3 worker complete a work in 5 days then,

1 worker completes the work in 5 × 3 = 15 days

Other examples of the same kind follow:-

(i) 8 persons consume a certain quantity of food in 5 days, then

1 person will consume the same food $5 \times 8 = 40$ days.

(ii) If 1 person can complete a work in 24 days, then

3 persons will complete the work in $24 \div 3 = 8$ days

(iii) If 1 person consumes a certain stock of food in 30 days, then

5 persons will consume the stock in $30 \div 5 = 6$ days

The variables in the above examples are –

No. of persons	Time required to do a work
No. of persons to consume a certain stock of food	Time they take to consume

As we have seen –

If the first variable is made twice the second becomes half.

If the first variable become one fourth, the second variable becomes 4 times.

The above pairs of variables are said to have *inverse variation* between them.

Application of unitary method in work and time.

Let us see the example below:

Example : A can complete a work in 24 days and B can complete the same work in 18 days. A and B started doing the work together. After 4 days A discontinued working. How many days would the work take to be completed?

Solution:

A can do the work in 24 days

$\therefore$ the work done by A in 1 day $= \frac{1}{24}$

B can do the work in 16 days

$\therefore$ the work done by B in 1 day $= \frac{1}{18}$

Work done by A and B together in 1 day $= \frac{1}{24} + \frac{1}{18} = \frac{3+4}{72} = \frac{7}{72}$

Work done by A and B in 4 days $= \frac{7}{72} \times 4 = \frac{7}{18}$

Work left incomplete $= 1 - \frac{7}{18} = \frac{18-7}{18} = \frac{11}{18}$

Work done by B in 1 day $= \frac{1}{18}$

$\therefore$ the time required by B to complete $\frac{11}{18}$ of the work $= \frac{11}{18} \div \frac{1}{18} = \frac{11}{18} \times \frac{18}{1} = 11$

Total time taken = 4 + 11 = 15 days

In course of calculation by unitary method, the following generalization is required to be done.

(i) Work done in unit time $= \dfrac{work\ done}{timetaken\ to\ do\ the\ work}$

(ii) Time required $= \dfrac{work\ to\ be\ done}{work\ \ done\ in\ unit\ time}$

(B) Percentage calculation

Meaning of the Term "Percentage"

'Percentage' means 'out of hundred'. But when is it used?

Let us see the situation below.

A and B are two students. A appeared in an examination with total mark 80 and he secured 64. B appeared another examination with total mark 75 and he secured 63.

Whose performance is better, 64 out of 80 or 63 out of 75? Had the total mark in both the examinations been the same, then we could compare the marks easily.

So we think of the total mark as 100.

$\therefore$ Out of the total of 80, A gets 64

$\therefore$ Out of 100, A gets $\frac{64}{80} \times 100 = 80$.

Now we say, A's marks are 80 out of 100 i.e. 80 percent or 80%.

Similarly, out of 75, B gets 63

$\therefore$ Out of 100, B gets $\frac{63}{75} \times 100 = 84$

Now we say, B's mark is 84 out of 100 i.e. 84 percent or 84%

$\therefore$ B's performance is better than A.

Thus we come to know –

> Percentage is a comparison of one number with another.
>
> While comparing, we treat the 2^{nd} number as 100.
>
> While comparing 'p' with 'q', we get $\frac{p}{q} \times 100\%$

Application of Percentage:

The following are the occasions when application of percentage is made:-

In a business, profit or loss is expressed as a percentage of the cost price.

'Profit is 12%' means profit is 12% of C.P.

In case of Loan, interest paid is expressed as a percentage of the principle. 'Interest rate is 10%' means interest in a year is 10% of the principle.

In case of increase or decrease (in production), increase or decrease is expressed as a percentage of the original.

(a) Profit and Loss

In a business –

Profit = S. P. – C. P.

Loss = C.P. – S. P.

Profit % = Profit as a percentage of C.P.

$$= \frac{Profit}{C.P.} \times 100$$

$$= \frac{S.P. - C.P}{C.P} \times 100$$

Loss % = Loss as a percentage of C.P.

$$= \frac{Loss}{C.P.} \times 100$$

$$= \frac{C.P. - S.P.}{C.P.} \times 100$$

Note : If after buying the article, some money is spent on transportation or for some other purpose, then the net cost = C.P. + Other expenditure. For calculating the profit or loss, net-cost is to be utilized instead of C.P.

For convenience of calculation of S. P. or C.P. the following rules may be used:-

$$\frac{S.P.}{100 + Profit\ \%} = \frac{C.P.}{100}$$

$$\frac{S.P.}{100 - Loss\ \%} = \frac{C.P.}{100}$$

(b) Interest Calculation

We save money in a Bank, we also borrow money form a bank.

When we save money, we get **interest** from the bank. When we borrow money from a bank, we **pay interest** to the bank.

How is the interest calculated ?

During transaction with a bank, the bank declares the rate of interest.

We decide –

(i) How much money we are to borrow (or deposit).

(ii) For what time we are to borrow (or deposit).

Supposing the money borrowed is ‘P’ and it is borrowed for ‘t’ years.

The rate of interest declared by the bank is r%.

How much of interest is to be given at the end of the loan period ?

What is the total money to be refunded back ?

Calculation :

Rate of interest is r%. That is,

on a principle of 100 ‘ in one year, interest in r ‘.

On a principle of 1 ‘ in 1 year, interest $= \frac{r}{100}$ ‘

On a principle of P ‘ in 1 year, interest $= \frac{r}{100} \times P = \frac{Pr}{100}$ ‘

Notes

On a principal of P ‘ in ‘t’ years, interest $= \dfrac{Pr}{100} \times t = \dfrac{Prt}{100}$ ‘

Thus $\boxed{\text{Interest (I)} = \dfrac{prt}{100}}$ Rule I [I.P.t.r- rule]

Total money to be refunded at the end of the loan period that is,

Amount $(A) = P + I$

$$\Rightarrow A = P + \frac{Prt}{100}$$

$$A = P\left(1 + \frac{rt}{100}\right)$$ Rule II [A.P.t.r - rule]

Application of Rule – I
P, t, r given, to find I;
P, r, I given, to find t;
P, I, t given, to find r;
I, t, r given, to find P.

Application of Rule – II
P, t, r given, to calculate A;
P, r, A given, to calculate t;
P, t, A given, to calculate r;
A, t, r given, to calculate P.

Check your progress:

E20 A text book of mathematics costs 2 rupees more than a text book of literature. If 5 literature books cost 38 rupees more than 3 mathematics books, what is the cost of each literature book ?

E21 Three persons completed half of a work in 8 days. If one of them discontinues working, how many days would they take to do the remaining half ?

E22 Gopal borrowed some money at 12% simple interest. If he had to pay back 1280 rupees after 5 years, in order to clear off the loan, how much did he borrow?

5.6 LET US SUM UP

Four types of number systems which are included in the Mathematics curriculum at the elementary school stage are:

– Natural numbers (N) : 1, 2, 3, 4, …………

– Whole numbers (W) : 0, 1, 2, 3…….

– Integers (Z) : …………. –4, –3, –2, –1, 0, +1, +2, +3, ………..

– Rational numbers (Q) : Numbers of the form $\frac{p}{q}$ where p, q are integers and $q \neq 0$ are rational numbers.

Various properties of addition in different sets of numbers are :

(i) Addition is closed in N, W, Z and Q.

(ii) Addition in N, W, Z and Q is commutative and associative.

(iii) Additive identity exists in W, Z and Q. 0 is the additive identity.

(iv) Additive inverse exists in Z and Q.

Various properties of multiplication in the four types of number sets are:

(i) Multiplication is closed in N, W, Z and Q.

(ii) Multiplication in N, W, Z and Q is commutative.

(iii) Multiplication in N, W, Z and Q is associated.

(iv) Multiplicative identity exists in N, W, Z and Q.

(v) Multiplicative inverse exists in Q.

(vi) Multiplication in N, W, Z and Q distributes over addition.

Rational number can be represented as (i) terminating decimal if the denominator has no factor other than 2 or 5; (ii) non- terminating decimal if the denominator has any factor other than 2 or 5.

Prime common factors can be used to determine H.C.F. of two or more natural number. Similarly, common multiples common multiples are used to determine the L.C.M. of two or more natural numbers.

5.7 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1 The smallest counting number is 1.

E2 0

E3 (i) 0, (ii) 72. (iii) 792.

E4 (a) True, (b) False, (c) False, (d) False, (e)True

E5 1, E6. 0, E7. 2 and 13, E8. 2, E9. 101, E10. 0,

E11 -7, E12, -1 and +1, E13. 0, E14. (a) $-\frac{2}{7}$, (b) $\frac{3}{8}$, (c) 0,

E15 (i) 0.48, (ii) 0.875, (iii) 0.285714 E16. (a) $\frac{7}{3}$ (b) $\frac{-8}{5}$, (c) does not exist

E17 $\frac{51}{99}$ E18. 1, E19. 32

E20 22 rupees, E21. 12 days, E22. 800 Rupees‘,

5.8 SUGGESTED READINGS AND REFERENCE

Mathematics text books prepared and published by

N.C.E.R.T. for Class-VI, VII and VIII.

5.9 UNIT-END EXERCISES

1. How many integers between –30 and +30 are the multiples of 3 ?

2. What is the sum of

   (i) 1 – 2 + 3 – 4 + 5 – 6 ………………. + 45

   (ii) 1 + 2 – 3 + 4 + 5 – 6 + 7 + 8 – 9 + ………………….. – 48

3. (i) Score out the composite numbers from the table belows and identify the prime numbers that occur between20 and 69.

   (iii) How marks pairs of twin prime are included among them?

20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69

Notes

4. A bought 200kg of rice at 18 ‘ per kg and sold 150 kg of them at 22‘ per kg and rest at 19‘ per kg.

   B bought 250 kg of rice at 20‘ per kg and sold them all at 23‘ per kg.

   Determine who had a greater profit ?

5. P borrowed a sum of ‘80,000.00 from a bank at 8% simple interest. How much has he to pay after3 years if he is to clear his debt completely ?

UNIT 6 SHAPES AND SPATIAL UNDERSTANDING

Structure

6.0 INTRODUCTION

Whenever we look around us, we see various objects. Some show a shape which has some regular features on it, such as a guava hanging on a tree, a lemon seen on a tree etc., and some others do not have any regular feature on them like a broken piece of stone.

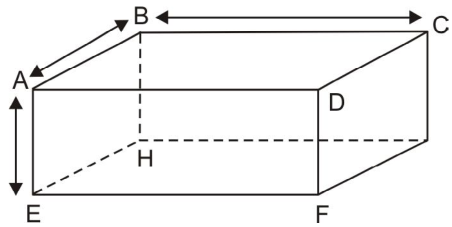


Fig. 6.1

Let us consider a piece of brick. It has extensions in 3 directions and hence it is a body of 3 dimensions. (also known as 3-D objects).

There are 6 *surfaces*, 12 *edges* and 8 *corners* or *vertices* on it.

A wall, a floor, or the top of a table represents a part of a plane. Surface of water in a container always represents a horizontal plane. On a wall or on a floor, we can draw various shapes or figures as shown below.

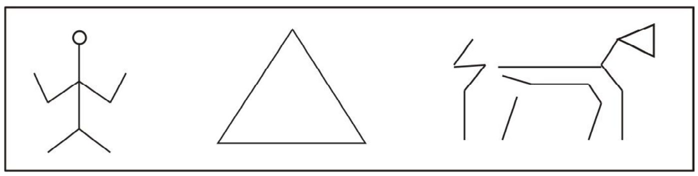


Fig. 6.2 Figures on a plane

A two dimensional plane-surface has extensions to left and to right,to top and bottom. All figures drawn on it will have two dimensions.Thus the shapes seen in the above diagram are of 2 dimensions (2.D shapes). In this unit we will be discussing of such 2-D and 3-D shapes in detail.

For completing this unit you shall need at least 10 *(Ten) hours of study.*

6.1 LEARNING OBJECTIVES

After going through this unit, you will be able to:

- recognize the basic geometric figures like point, line, ray and angle.
- identify different types of geometrical shapes drawn on a plane.
- explain the conditions of congruency, and similarity between two plane figures.
- identifies reflexive and rotational symmetry of any plane figure.
- recognize the three dimensional shapes and their properties.

6.2 BASIC GEOMETRICAL FIGURES

6.2.1 Undefined Terms

For knowing a mathematical topic, it is necessary that we come to know certain terms related to the topic. The knowledge about a term is derived from its definition. But to define a term we need other terms using which the requisite definition of the term is to be given. When we just start with the topic, we have no stock of words in respect of the topic. Hence, often it does not become possible to define the basic terms related to a topic.Such terms are known as- ***Undefined terms***.

Some of the undefined terms in Geometry are:***Point, Line, Plane (to define these terms we have to come back to the terms)***

Since no definition has been given about the above terms, to enable us to use the terms rightly, the following axioms are given:

Basic Axioms of Geometry

I: Every line (i.e. straight line) and plane is a set of points.

II: Through one point innumerable lines can be drawn.

III: Through two different points one and only one straight line can be drawn.

IV: The line drawn through two given points in a plane also lies in the plane.

V: There exists only one plane containing three points not lying in a line.

VI: If two planes intersect, their intersection is a straight line.

The above axioms provide us with the inter-relations connecting the three undefined terms.These axioms will help us in understanding and expressing geometrical facts wherein the undefined terms (point, line and plane) are used.

6.2.2 Basic Figures

Point: A dot mark given on a piece of paper using a pencil or a pen is treated as a point. We should not have any concept about its size in our mind.

Line: When we speak of a line we mean a straight line

Fig. 6.3

The figure above represent a straight line. It is drawn with the help of a straight edge. The two **arrow heads** shown at the two extremities speak of unlimited extension of the line either way.

Plane: Floor of a room, surface of a wall, a page of a book represent planes. A plane is unlimited in its extension.

Distance between two Points: If A and B are two points, then the distance between them is a unique non- negative real numbers and is denoted by the symbol AB.

Betweenness (definition) : If A, B and C are 3 distinct points, that lie in a straight line and AB + BC = AC, then B is said to lie between A and C and in symbol we write it as A - B - C or C - B - A.

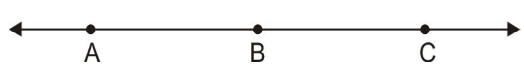


Fig. 6.4

The diagram above shows 3 points A, B and C such that A - B - C (i.e. B lies between A and C).

Line-segment (definition) : The set of two distinct points A, B and the points between them is known as the line-segment determined by A and B and it is represented by the symbol $\overline{AB}$.

A and B are known as the end points of $\overline{AB}$.

A line segment is always a part of a line and has two end points. A line segment with the end points A,B is denoted by $\overline{AB}$ and its length is AB.

Length of a line segment (definition): The distance between the end points of a line-segment is known as the length of the line segment.Thus length of $\overline{AB}$ is AB.

Ray (Definition): A portion of a line which starts at a point and goes off in a particular direction to unlimited distance. In the Fig. 6.5 the line starts at the point A and extends in the direction of B to infinity. In this figure the ray is indicated by $\overrightarrow{AB}$ and is read as "ray AB". The arrow over the two letters indicates it is a ray, and the arrow direction indicates that A is the point where the ray starts.

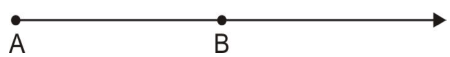


Fig. 6.5

The point A is known as the origin or vertex of $\overrightarrow{AB}$.

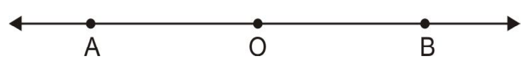
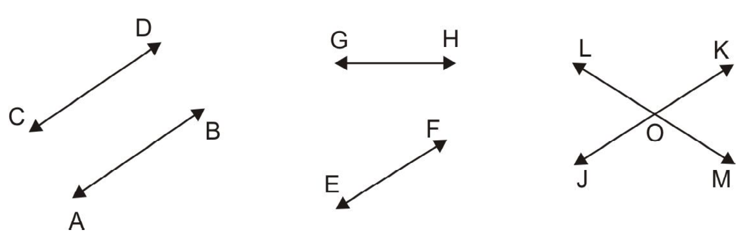


Fig. 6.6

Oppositerays:

The Fig. 6.6 shows a line $\overleftrightarrow{AB}$, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two rays both of which are parts of $\overleftrightarrow{AB}$. O is the only point common to both the rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. In such a case $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are known as two *opposite rays*. Obviously two opposite rays together form a line.**A pair of lines:**Three pairs of lines are shown in the figures below.



(I) (II) (III)

Fig. 6.7

Notes

The lines $\overleftrightarrow{AB}$ and CD shown in the Fig. 6.7 (I) are such that those, being extended, in either way, will never meet. Such lines which do not have any point common to them are known as ***parallel lines***.

Fig. 6.7 (II) shows a pair of lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ which will be found having a point common to them if $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are extended towards F and H respectively.

Fig. 6.7 (III) shows a pair of lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ which have a point O common to both of them.

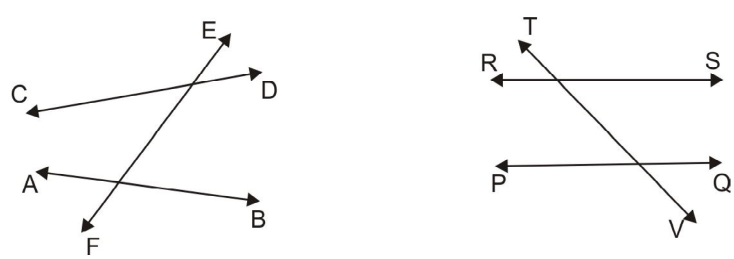
Thus the pair of lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ in fig-6.7 (II) and pair of lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$ in fig-6.7 (III) are known as ***non-parallel lines***or***intersecting lines***. The point O in Fig-6.7 (III) is known as the point of intersection of the lines $\overleftrightarrow{JK}$ and $\overleftrightarrow{LM}$. The point of intersection of the lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$, in fig. 6.8 (II), can be obtained on extending the lines $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ towards F and H respectively.

Symbolic representation of parallel lines: For a pair of parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, we write $\overleftrightarrow{AB}$ II $\overleftrightarrow{CD}$ in symbol.

Transversal of a pair of lines:

Fig. 6.8 (I) shows a pair of non-parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Fig-6.8 (II) shows a pair of parallel lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$.

$\overleftrightarrow{EF}$ is a line seen in Fig. 6.8 (I) and this line is found intersecting $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ as well. Similarly a line $\overleftrightarrow{TV}$ is seen in Fig. 6.8 (II) and it is found intersecting $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ as well.



(I) (II)

Fig. 6.8

$\overleftrightarrow{EF}$ in Fig. 6.8 (I) and $\overleftrightarrow{TV}$ in Fig. 6.8 (II) are known as transversals. $\overleftrightarrow{EF}$ intersect a pair of non-parallel lines whereas $\overleftrightarrow{TV}$ intersects a pair of parallel lines.

Characteristic of parallel lines :

In the figure along side $\overleftrightarrow{AB} \| \overleftrightarrow{CD}$. Three transversals are shown in the diagram and each of the 3 transversals makes right angle with $\overleftrightarrow{AB}$.

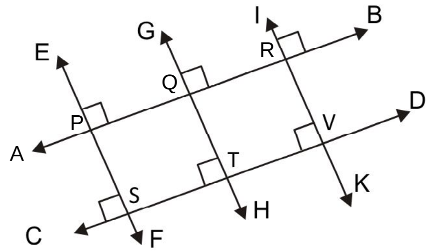


Fig. 6.9

P, Q, R are the points where the transversal cuts $\overleftrightarrow{AB}$ and S, T, V are the points where the transversal cuts $\overleftrightarrow{CD}$. $\overline{PS}$, $\overline{QT}$ and $\overline{RV}$ will have equal lengths.

Each of PS, QT and RV represents the distance between the parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. It can be verified by actual drawing that PS = QT = RV

Thus we see that the distance between two parallel lines is the same everywhere. So we say *there exists a fixed distance between two parallel lines*.

Angle(definition) : If A, B and C are 3 non-collinear points, then the figure formed by the rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is known as an angle BAC. (written as $\angle BAC$)

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are known as ***the arms*** of $\angle BAC$ and A is its ***vertex.***

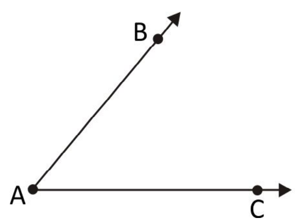


Fig. 6.10

Depending up on the situations of the points A, B and C, the angle can have different shapes (as shown below).

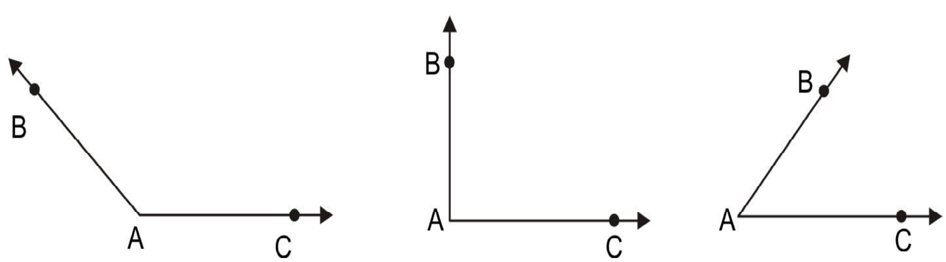


Fig. 6.11

Measure of an angle :

Protractor, provided in the geometry set (instrument box), is the instrument that helps us measuring an angle and the units of measure obtained using a protractor is degree.

In the diagram alongside,

$\angle AOB$ measures 40° and $\angle COB$ measures 135°

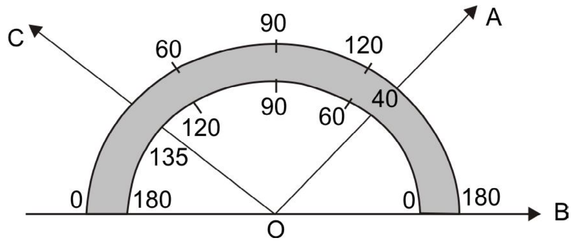


Fig. 6.12

Degree measure of $\angle ABC$ is denoted as $m\angle ABC$.

If $\angle ABC$ in Fig- 6.13 is of measure 70°, we write $m\angle ABC = 70°$

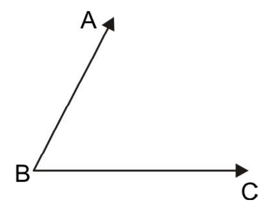


Fig. 6.13

Axiom: *With each angle a real number greater than 0 and less than 180 is associated.*

This number is known as the measure of the angle (in degrees).

Lines Perpendicular to each other:

In Fig. 6.14 (I), $m\angle AOB = 90°$. In such a situation $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are known as rays perpendicular to each other and we write $\overrightarrow{OA} \perp \overrightarrow{OB}$ or $\overrightarrow{OB} \perp \overrightarrow{OA}$

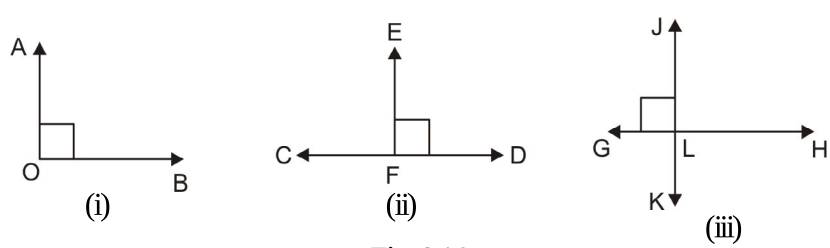


Fig. 6.14

In Fig-6.14 (II), $m\angle CFE = 90°$ and hence we write $\overrightarrow{FE} \perp \overleftrightarrow{CD}$. In Fig. 6.14 (III), $m\angle GLJ = 90°$ and we write $\overleftrightarrow{JK} \perp \overleftrightarrow{GH}$ or $\overleftrightarrow{GH} \perp \overleftrightarrow{JK}$

***Angle Bisector*:** If P is a point in the interior of $\angle BAC$ and $m\angle BAP = m\angle PAC$, then $\overrightarrow{AP}$ is known as the bisector of $\angle BAC$ (Fig.6.15).

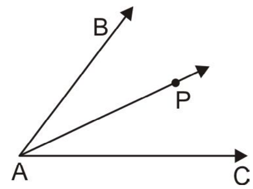


Fig. 6.15

***Classification of angles*:** From measure point of view, angles are classified into the following categories-

(i) An angle of measure less than 90° and more than 0° is known as an **acute angle**.

(ii) An angle of measure 90° is known as **a right-angle**.

(iii) An angle of measure greater than 90°and less than 180° is known as an **obtuse angle**.

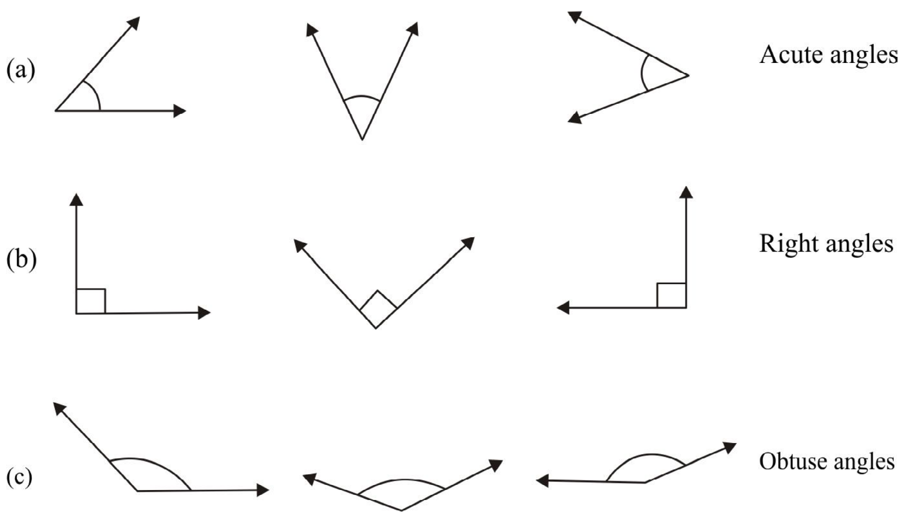


Fig. 6.16

***Pairs of angles*:**

Adjacent angles*:If two angles have a common vertex and a common arm and further their interiors have no common point, then the pair of such angles are known as adjacent angles.*

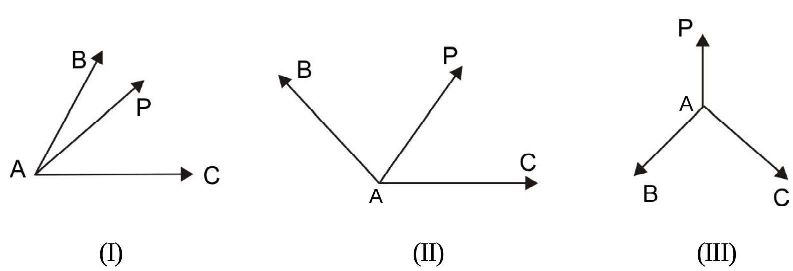
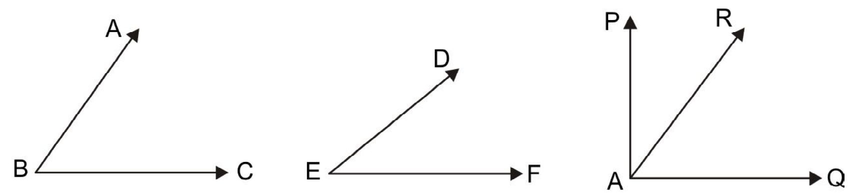


Fig. 6.17

In each of the three diagrams above, you find a pair of angles $\angle BAP$ and $\angle CAP$ such that A is their common vertex, $\overrightarrow{AP}$ is their common arm and there is no common point in their interior.

Hence $\angle BAP$ and $\angle CAP$ form a pair of adjacent angles.

***Complementary angles**: Two angles are said to be complementary if the sum of their measures is 90°*. Obviously each of them would be an acute angle.



(I) (II)

Fig. 6.18

In Fig. 6.18 (I), $m\angle ABC + m\angle DEF = 90°$.

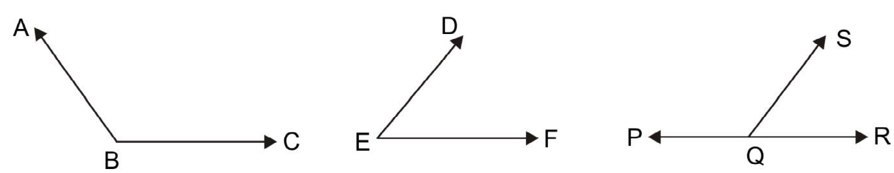
Thus $\angle ABC$ and $\angle DEF$ are complementary angles.

In Fig. 6.18(II), $m\angle PAR + m\angle RAQ = 90°$.

Thus $\angle PAR$ and $\angle RAQ$ are complementary angles.

Note : A pair of complementary angles may or may not be a pair of adjacent angles.

***Supplementary angles**:A pair of angles are said to be supplementary angles if the sum of their measures is 180°.*



(I) (II)

Fig. 6.19

In Fig. 6.19 (I) above, $m\angle ABC + m\angle DEF = 180°$

Hence $\angle ABC$ and $\angle DEF$ form a pair of supplementary angles.

In figure 6.19(II) $m\angle PQS + m\angle RQS = 180°$

Hence $\angle PQS$ and $\angle RQS$ form a pair of adjacent supplementary angles.

Note : A pair of supplementary angles may or may not be adjacent.

Vertically opposite angles:

In the figure 6.20 alongside $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect each other at O. Thus four angles are formed and $\angle AOD$ is one of the 4 angles formed by $\overrightarrow{OA}$ and $\overrightarrow{OD}$. $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are opposite rays of $\overrightarrow{OA}$ and $\overrightarrow{OD}$ respectively.

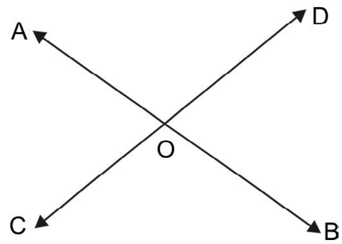


Fig. 6.20

The angle formed by $\overrightarrow{OB}$ and $\overrightarrow{OC}$ i.e. $\angle BOC$ and the angle formed by $\overrightarrow{OA}$ and $\overrightarrow{OD}$ i.e. $\angle AOD$ are known as a pair of vertically opposite angles. For similar reasons, $\angle BOD$ and $\angle AOC$ also form a pair of vertically opposite angles.

Thus two pairs of vertically opposite angles are formed when two straight lines intersect each other.

Angles formed when a transversal cuts two lines

In Fig. 6.21(i), $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two nonparallel lines and the transversal $\overleftrightarrow{PQ}$ cuts them. The angles formed at the points of intersections are marked by the numbers 1, 2, 3, 4, 5, 6, 7 and 8.

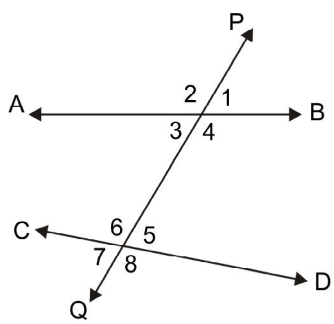


Fig. 6.21 (i)

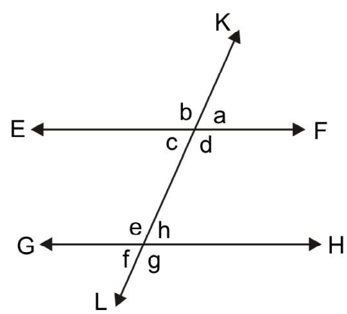


Fig. 6.21 (ii)

In Fig. 6.21(ii) $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are two parallel lines and the transversal $\overleftrightarrow{KL}$ cuts them. The angles formed at the points of intersections are marked by the letters a, b, c, d, e, f, g and h.

In Fig. 6.21 (i), angles marked as 1 and 5, 2 and 6, 4 and 8, 3 and 7 are known as pairs of ***Corresponding angles***.

Angles marked as 1 and 7, 2 and 8 are known as ***exterior alternate angles*** whereas angles marked as 3 and 5, 4 and 6 are known as ***interior alternate angles***.

In Fig. 6.21 (ii) angles marked as *a* and *h, b* and *e* , *d* and *g, c* and *f* are ***corresponding angles***.

You can measure the angles formed when a transversal cuts a pair of parallel lines and observe that

(i) *Exterior alternate angles are of equal measure and interior alternate angles are of equal measure.*

(ii) Corresponding angles are of equal measure.

Check your progress:

E1. In the diagram in Fig. 6.22, O is a point common to $\overleftrightarrow{AB}$ and $\overrightarrow{OD}$, measures of $\angle AOD$ and $\angle DOB$ in degrees are *x+30* and *x-25* respectively. Find the measures of the two angles in degrees.

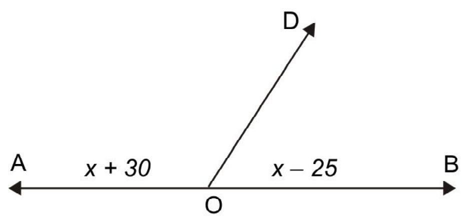


Fig. 6.22

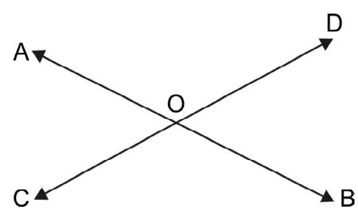


Fig. 6.23

E2. In the figure 6.23, $m\angle AOC = 72°$. Find the measure of $\angle AOD, \angle BOC$ and $\angle BOD$.

E3. See Fig. 6.24 below and name the following kinds of angles.

(i) A pair of corresponding angles.

(ii) A pair of co-interior angles.

(iii) A pair of alternate angles.

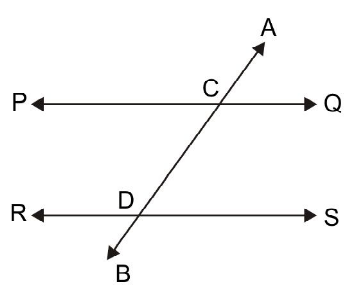


Fig. 6.24

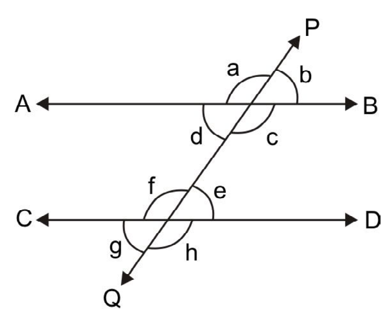


Fig. 6.25

E4. In the figure 6.25, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{PQ}$ cuts them. The measures of angles formed are indicated as a, b, c, d, e, f, g and h. If g = 35°, find the values of a, b, c, d, e, f and h.

6.3 TWO DIMENSIONAL CLOSED FIGURES

In this section we discuss the types and properties of three types of two dimensional closed figures i.e. triangles, quadrilaterals and circle.

6.3.1 Triangle

(*Definition***)** *If A, B, C are three non-collier points, then the figure formed by* $\overline{AB}$ *,* $\overline{BC}$ *and* $\overline{CA}$ *is known as triangle ABC (written as ΔABC).*

A, B and C are known as the vertices of it. $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are the sides and $\angle ABC$, $\angle BCA$ and $\angle BAC$ (written in short as $\angle B$, $\angle C$ and $\angle A$) are the angles of ΔABC.

Interior and Exterior of a Triangle **:**The region of the page enclosed within the ΔABC (Fig. 6.26) is known as the interior of the triangle.Triangle ABC and its interior taken together are known as the ***triangular region***.

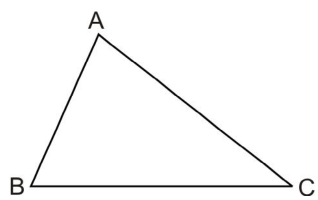


Fig. 6.26

In Fig 6.27, points P and R are in the interior of ΔABC. Point M and N are on ΔABC and points Q, S and T are in the exterior of ΔABC. The region on the page outside ΔABC. (Which contain points Q, S, T) is known as the exterior of ΔABC.

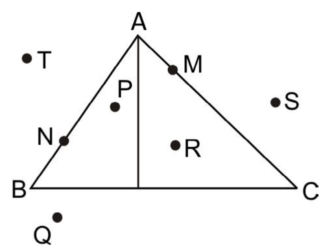


Fig : 6.27

Classification of Triangle**:**

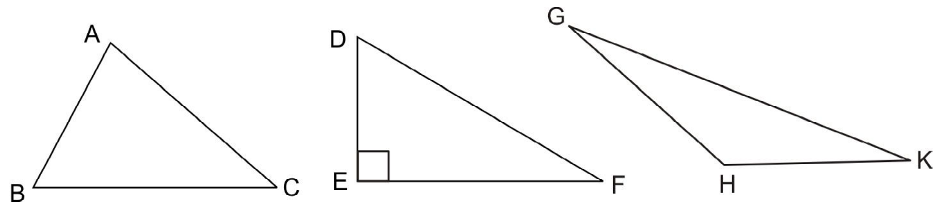
(a) Classification in respect of angle-measure :

(i) ***Acute angled triangle*****:** If all the three angles of a triangle are acute angles, then it is an *acuteangled triangle*. Such a triangle is show in Fig. 6.28 (a) below.

(ii) ***Right angled triangle*****:** A triangle in which one angle is a right angle,it is a *right angled triangle*. Such a triangle is shown in Fig. 6.28 (b) below.

Notes

(iii) ***Obtuse angled triangle*: ** A triangle in which one angle is obtuse, it is an *obtuse angled triangle*. Such a triangle is shown in Fig. 6.28(c) below.



(a) (Acute angled triangle) (b) (Right angled triangle) (c) (Obtuse angled triangle)

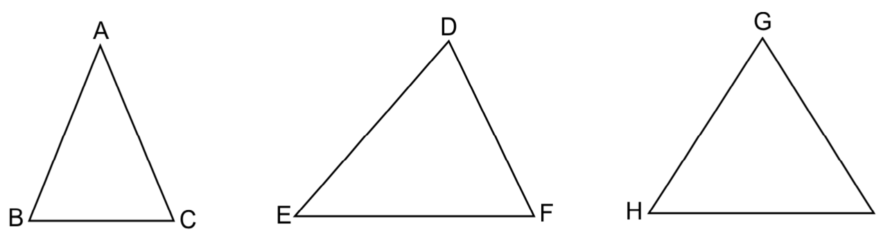
Fig. 6.28

(b) Classification in respect of the relative lengths of sides :

(i) ***Isosceles triangle* :** A triangle in which two sides are equal is an *isosceles triangle* (Fig.6.29 (I))

(ii) ***Scalene triangle* :** A triangle in which no two sides are equal is a *scalene triangle* (Fig- 6.29 (II))

(iii) ***Equilateral triangle*:** A triangle in which all the three sides are equal is an *equilateral triangle* (Fig.6.29 (III)).

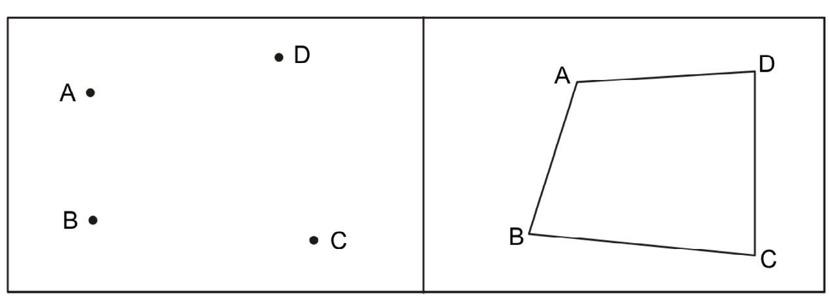


[I] (Isosceles triangle) [II] (Scalene triangle) [III] (Equilateral triangle)

Fig.6.29

6.3.2 Quadrilaterals

See the diagram below.



(i) (ii)

Fig. 6.30

In Fig. 6.30(i), there are four point A, B, C and D. No three of those four points lie along a straight line.

In Fig. 6.30(ii), traced copy of the same four point occur and the line-segment $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ are drawn.

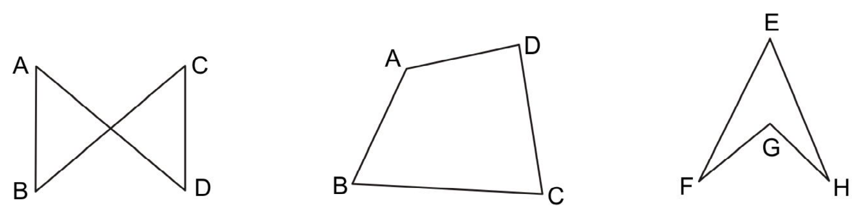
The line segments $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ are such that no two of them meet at any point other than their end-points.

Under such conditions, the figure formed by $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ is said to be a quadrilateral. The quadrilateral formed is named as ABCD.

A, B, C and D are the vertices of it; $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ are the sides of it.

Quadrilateral ABCD has four angles and those are $\angle ABC, \angle BCD, \angle CDA$ and $\angle DAB$. $\overline{AC}$ and $\overline{BD}$ are the diagonals of ABCD.

Let us see the figure 6.31 (a) A, B, C and D in a plane and no 3 of them are collinear.



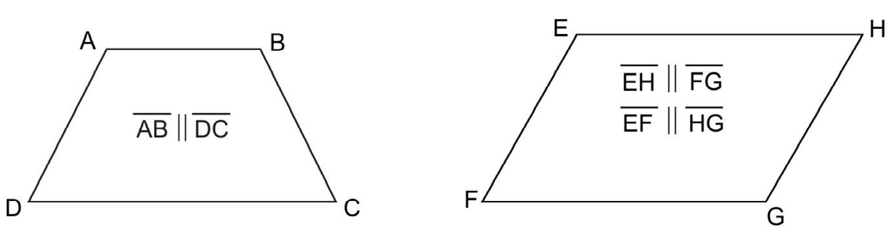
(a) (b) (c)

Fig. 6.31

But $\overline{AD}$ and $\overline{BC}$ meet at a point other than their end-points. Hence $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ do not form a quadrilateral. Fig 6.32 (b) is a convex quadrilateral and 6.32 (c) is known as a *reentrant* quadrilateral. We discuss of a convex quadrilateral only.

Categories of Quadrilaterals :

(i) ***Trapezium :*** A quadrilateral with one pair of opposite sides parallel, is known as a trapezium. Fig. 6.32 (I) below shows a *trapezium* in which $\overline{AB} \parallel \overline{DC}$.



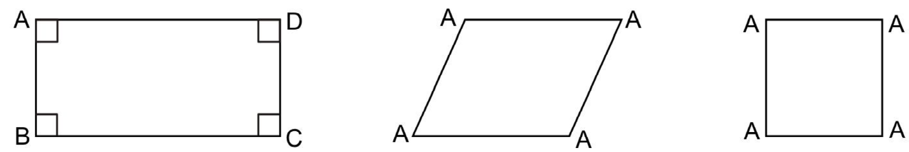
(I) (Trapezium) (II) (Parallelogram)

Fig. 6.32

(ii) Parallelogram: A quadrilateral with both pairs of opposite sides parallel, is a parallelogram.In Fig. 6.32 (II), $\overline{EF} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{EH}$. Thus EFGH is a parallelogram.

(iii) Rectangle: A quadrilateral with all 4 angles to be right angles is a rectangle.

If can also be said that a parallelogram with one angle right angle is a rectangle.



(I) [Rectangle] (II) [Rhombus] (III) [Square]

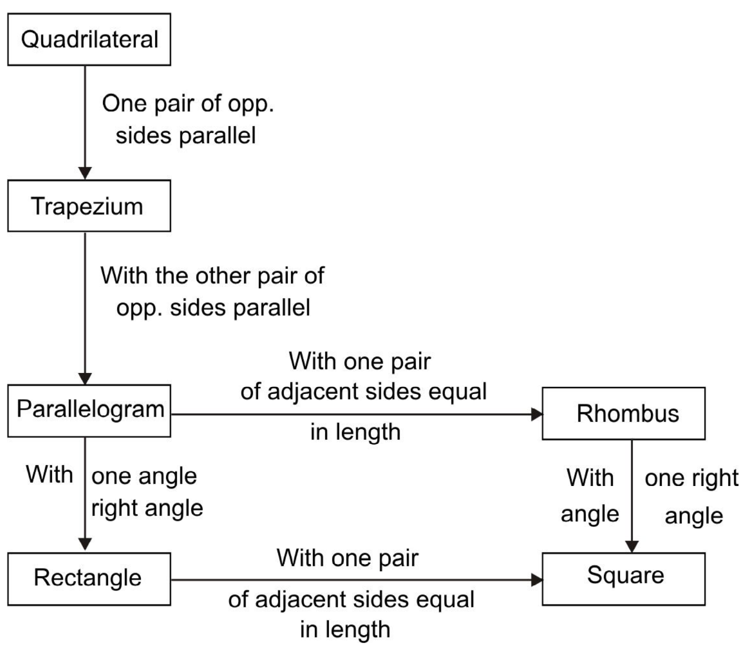
Fig. 6.33

In Fig. 6.33 above $\angle A, \angle B, \angle C, \angle D$, are right angles. Thus ABCD is a rectangle.

(iv) Rhombus: A quadrilateral with all 4 sides of equal length is a *rhombus*.

(v) Square: A quadrilateral with all sides of equal length and all angles right angles is a square.In Fig. 6.33(III) above AB = BC = CD = DA and $\angle A, \angle B, \angle C, \angle D$, are all right angles. Thus ABCD is a square.

Flow chart showing the inter relation connecting all categories of quadrilaterals:



Note : The properties of various kinds of quadrilaterals can be known through experiments.

Notes

Perimeter of a quadrilateral = sum of the lengths of all 4 sides.

Area of a quadrilateral = sum of the areas of the pair of triangles obtained by drawing a diagonal.

Perimeter of a trapezium = sum of the lengths of all 4 sides

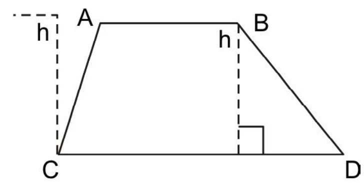
Area of the Trapezium ABCD (Fig. 6.34)

= Area of ΔABC + Area of ΔBCD

= ½ AB × h + ½ CD × h

= ½ h (AB + CD)

= ½h (sum of the lengths of the parallel sides)



Trapezium with $\overline{AB} \parallel \overline{DC}$

Fig. 6.34

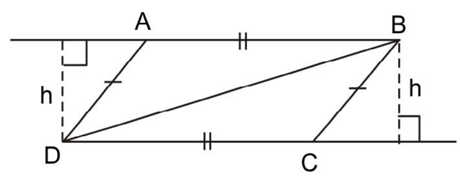
Perimeter of a parallelogram = AB + BC + CD + DA (in Fig.6.35)

= AB + BC + AB + BC

= 2AB + 2BC = 2 (AB + BC)

Area = base × height

(base is DC and height is h)



Parallelogram

Fig. 6.35

Perimeter of a rectangle = AB + BC + CD + DA (in Fig.6.36)

= $l + b + l + b = 2l + 2b = 2(l + b)$

[where l : length AB, b : breadth AD]

Area of a rectangle = $l \times b$

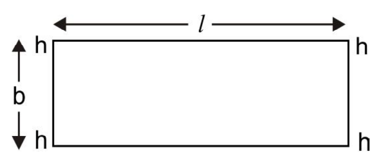


Fig. 6.36

Perimeter of a rhombus = AB + BC + CD + DA (Fig.6.37)

= AB + AB + AB + AB = 4 AB = 4 × length of a side

In a rhombus, the diagonals bisect each other at right angles.

∴ *Area of the rhombus ABCD*

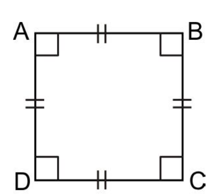
= Area of ΔABC + Area of ΔACD

= ½ AC × BO + ½ AC × DO

= ½ AC (BO + DO)

= ½ AC × BD

= ½ (product of the lengths of two diagonals)



(Square)

Fig. 6.37

Perimeter of a square

= 4 × length of a side (like a rhombus)

Area of a square

= AB × AD (like a rectangle) (Fig.6.38)

= AB × AB (as all sides are of equal length)

= AB^2 = Square of the length of a side.

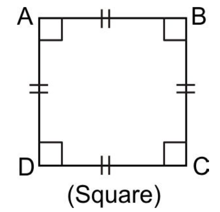


Fig. 6.38

6.3.3 Circle

Circle is a two dimensional shape made up of a curved line and it encloses a part of the plane on which it is drawn.

Definition: A circle is a collection of points in a plane such that each of them lies at a fixed distance from a fixed point in the same plane.

The fixed point is known as the ***centre*** of the circle and the fixed distance is known as the ***radius*** of it.

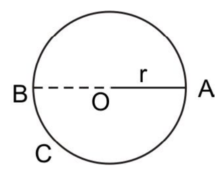


Fig. 6.39

The linesegment joining any two points of a circle is known as a ***chord*** of it. A chord that contains the centre is known as a ***diameter*** of the circle. Thus in fig 6.39, $\overline{AB}$ is a *diameter*. AB (i.e, the length of $\overline{AB}$)is the *diameter* of the circle (denoted as 'd').

$\overline{OA}$ and $\overline{OB}$ each is *a* ***radius*** of the circle.Hence $d = 2r$.

In Fig. 6.40 (a), A and C are two points on the circle ABC. These two points divide the circle into two parts and each part is known as an arc of the circle with end points A and C. $\overline{AC}$ is a chord of the circle. P and Q are two points on the circle other than A and C.P and O lie on opposite sides of the chord $\overline{AC}$ where as Q and O lie on the same side of $\overline{AC}$.

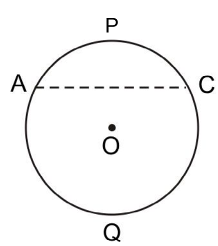


Fig. 6.40(a)

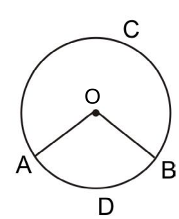


Fig. 6.40 (b)

The arc that contains P is named as arc APC and denoted as APC and the other arc that contains Q is known as arc AQC and is denoted as AQC. APC is known as a **minor arc** and AQC is known as a **major arc**. APC and AQC are known as a pair of **opposite arcs**.

In the circle ABC shown in fig 6.40(b), radii $\overline{AO}$ and $\overline{BO}$ are drawn at the end points A and B of ADB which is a ***minor arc***. Thus $\angle AOB$ is formed at the centre O. $\angle AOB$ is known as the angle subtended by ADB at the centre.

> *Measure of $\angle AOB$ is known as the degree measure of ADB. (Bigger is the arc, bigger is its degree measure. The degree measure of an arc $\overset{\frown}{ADB}$ is written as $m\,\overset{\frown}{ADB}$.*
>
> *The degree-measure of a major arc =360° — the degree major of its opposite arc.*
>
> *Such as, m ACB (in Fig. 6.40 (b)) = 360° — m ADB*

In Fig. 6.41, $\overline{AC}$ is a diameter of the circle APC As such each of the two arcs APC and AQC is known as a *semicircle*. The degree-measure of a **semicircle** is 90°.

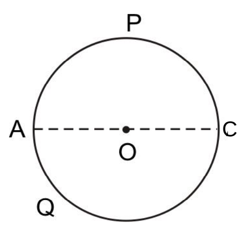


Fig. 6.41

In Fig. 6.42 (I), P and Q are two interior points of the minor arc with A and B as end points. Each of the angles $\angle APB$ and $\angle AQB$ is known as an *angle inscribed* in APB. Similarly $\angle ARB$ and $\angle ASB$ are the angles inscribed in ARB.

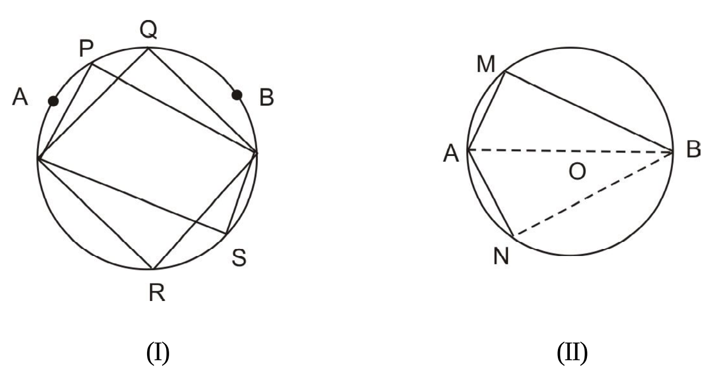


Fig. 6.42

In Fig. 6.42 (II) $\overline{AB}$ is a diameter of the circle. M and N are the interior points in the semicircles with the A and B as end points $\angle AMB$ and $\angle ANB$ each is known as an *angle in a semicircle*.

***Cyclic Quadrilateral*:**

A quadrilateral in which vertices lie on a circle (as shown in fig- 6.43 is known as a *cyclic quadrilateral*. Thus, ABCD is a cyclic quadrilateral.

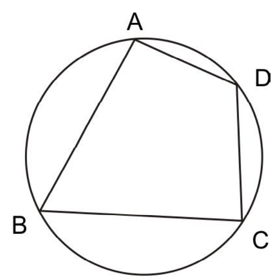


Fig. 6.43

Interior points and exterior points of a circle:

In fig 6.44, O is the centre of the circle KLM. So obviously K, L, M are points on the circle.

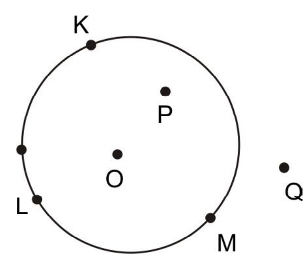


Fig. 6.44

P and Q are two points in the plane of the circle KLM such that PO < r (where r is the radius of the circle) QO > r. P is said to be a point in the *interior* of the circle KLM and Q is said to be in the *exterior* of the circle KLM.

The set of all points on a circle and the points in its interior taken together is known as the **circular region**.

***Segment of a circle*:**

In Fig. 6.45, $\overline{AB}$ is a chord of the circle ABC. $\overline{AB}$ divides the circular region into two parts and each part is known as a *segment*. The segment that does not contain the centre O, is known as the *minor segment* and segment that contains the centre O is known as the *major segment*.

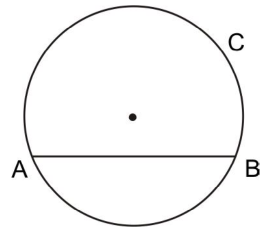


Fig. 6.45

Some properties of circle:

1. Angles inscribed in the same arc (i.e, angles in the same segment) are equal. $m\angle ABD = m\angle ACD$, $m\angle BAC = m\angle BDC$

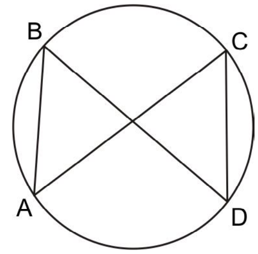


Fig. 6.46

2. Angle in a semicircle is a right angle. AB is a diameter. Thus APB is a semicircle. $m\angle APB = 90°$.

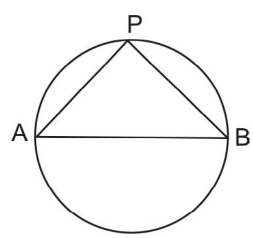


Fig. 6.47

3. Opposite angles of a cyclic quadrilateral are supplementary. ABCD is a cyclic quadrilateral as its vertices lie on the circle.

   $m\angle A + m\angle C = m\angle B + m\angle D = 180°$

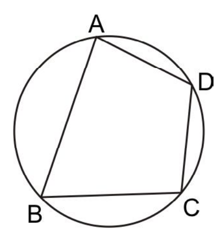


Fig. 6.48

4. The degree measure of an arc is equal to twice the measure of angle inscribed in the opposite arc.

   $m\ \widehat{APB} = 2\ \ m\angle ADB$

   $= 2m\angle ACB$

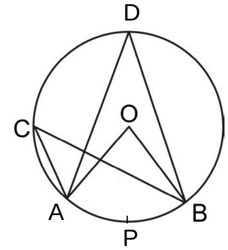


Fig. 6.49

Check your progress:

E5. Fill up the blanks

(a) A parallelogram with its two adjacent sides of equal length is a ………. .

(b) A radius of a circle is the linesegment that joins a point of the circle to the………….. of it.

(c) If the degree measure of an arc of a circle is 64^0, then the measure of an angle inscribed in the opposite arc is …….

(d) The sum of the measuresof theopposite angles of a cyclic quadrilateral is………degrees.

6.3.4 Congruence and Similarity

Congruentfigures:

Two plane- figures (figures drawn in a plane) are said to be congruent if the traced copy of one coincides with the other exactly. But the description of congruence given above involves a practical work. To make the work logical, we will give some definitions and some conditions under which figures (triangles) become congruent. The symbol that represents'is congruent to'is $\cong$.

(i) Congruence of line-segments :

If the lengths of two line segments are equal, then the line segments are said to be congruent. Thus two congruent linesegments are of equal in length.

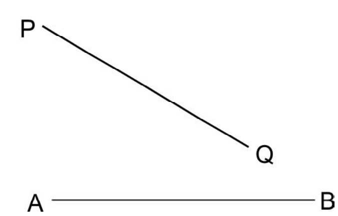


Fig. 6.50

$AB = PQ \Rightarrow \overline{AB} \cong \overline{PQ}$

and $\overline{AB} \cong \overline{PQ} \Rightarrow AB = PQ$

(ii) Congruence of angles :

Two angles are congruent if their measures are equal. Thus two congruent angles have equal measures.

$m\angle ABC = m\angle PQR \Rightarrow \angle ABC \cong \angle PQR$

and $\angle ABC \cong \angle PQR \Rightarrow m\angle ABC = m\angle PQR$

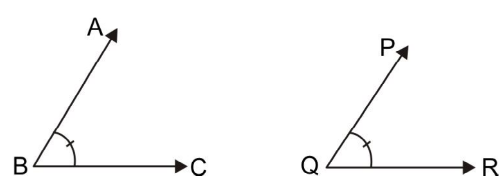


Fig. 6.51

Congruence of triangles:

Two triangles become congruent under the following conditions:

(i) Two triangles are congruent if two sides of one triangle are congruent to the corresponding sides of another and the angles included between them are congruent. This condition of congruence is known as, S-A-S postulate.

Note : Postulate is something which cannot be proved, but is accepted.

In triangles ABC and PQR, if $\overline{AB} \cong \overline{PQ}$,

$\overline{BC} \cong \overline{QR}$ and $\angle ABC \cong \angle PQR$,

then $\Delta\ ABC \cong \Delta\ PQR$

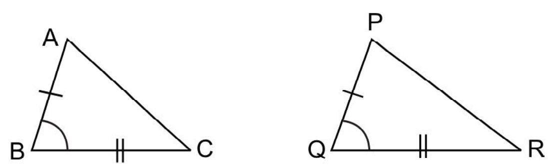


Fig. 6.52

(ii) Two triangles are congruent if three sides of one triangle are congruent to the corresponding sides of the other triangle.

This condition is known as SSS congruence.

In $\Delta\ ABC$ and $\Delta\ PQR$, if $\overline{AB} \cong \overline{PQ}$,

$\overline{BC} \cong \overline{QR}$, $\overline{CA} \cong \overline{RP}$, then $\Delta\ ABC \cong \Delta\ PQR$

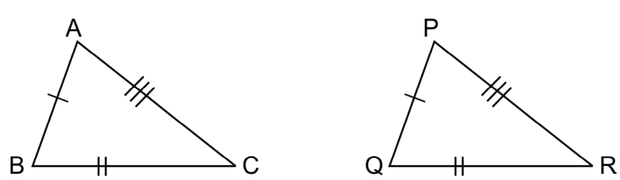


Fig. 6.53

(iii) Two triangles are congruent if two angles of one triangle are congruent to the corresponding angles of the other triangle and the sides included between them are congruent.

In Δ ABC and Δ PQR,

if $\angle B \cong \angle Q$, $\angle C \cong \angle R$, $\overline{BC} \cong \overline{QR}$

then $\Delta\ ABC \cong \Delta\ PQR$

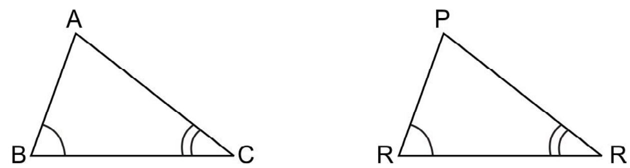


Fig. 6.54

It can be reasoned out using the above condition that if two angles of one triangle are congruent to corresponding two angles of another triangles and one pair of corresponding sides are congruent, the triangles are congruent.

This is known as "A-A-S congruence."

(iv) Two right angled triangles are congruent if the hypotenuse and one side of one triangle are respectively congruent to the hypotenuse and the corresponding side of the other.

This is known as 'RHS congruence'.

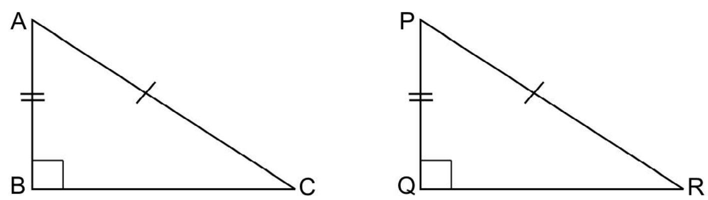


Fig. 6.55

In $\Delta\, ABC$, $\angle B$ is a right-angle and in $\Delta\, PQR$, $\angle Q$ is a right angle. If hypotenuses $\overline{AC} \cong \overline{PR}$ and sides $\overline{AB} \cong \overline{PQ}$, then $\Delta\, ABC \cong \Delta\, PQR$.

Note : Areas of two congruent triangles are equal.

Application of congruence.

When it is required to prove two sides or two angles of two triangles to be congruent, we try to prove the triangles to be congruent.

Thus congruence of triangles is an important tool that helps us solving many problems in geometry. An example is given below.

Example : In the adjoining figure, $\overline{AB} \cong \overline{CD}$,

$\angle ABC \cong \angle BCD$. Prove that $\overline{AC} \cong \overline{BD}$

Proof : In $\Delta\, ABC$ and $\Delta\, BCD$

$\overline{AB} \cong \overline{CD}$ (given)

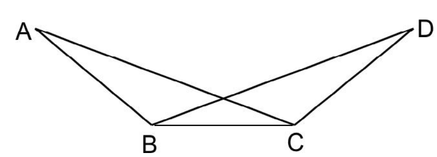


Fig. 6.56

$\overline{BC}$ is the side common to both triangles

$\angle ABC \cong \angle BCD$ (given)

$\therefore\ \Delta\ ABC \cong \Delta\ BCD$ (S-A-S congruence)

$\Rightarrow \overline{AC} \cong \overline{BD}$ [as corresponding parts of congruent triangles]

Similarity of triangles

A plane figure has two aspects in it. Those are**shape** and **size.**

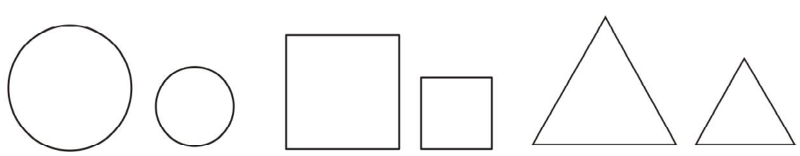
If two plane-figures have the same shape and the same size, then those become congruent, i.e. trace-copy of one coincides with the other.

Now we take two plane-figures which have the same shape. Some such examples are given below.

(i) Two photographs of the same person prepared from the same negative but with different magnification look to be exactly alike i.e., those are of the same shape, but those are different in size.

(ii) One map of India printed on the page of a book and another wall-map of India. Those are both of the same shape but of different sizes.

(iii)



(a) (b) (c)

Fig. 6.57

Two circles shown in Fig. 6.57(a) of Ex (iii) look exactly alike and hence those are similar.

Two squares are similar. Two equilateral triangles are similar.

Note: Congruent figures are similar. But similar figures may not be congruent.

6.3.5 Reflection and Symmetry

We know that reflection occurs in a plane mirror and consequently an image is formed. The concept of reflection in geometry is very much similar to the reflection in plane-mirror.

(a) Reflection in a line :

(i) *Reflection of a point in a line:* In figure 6.58(a) 'L' is a line (at times we name a line by a single letter) and P is a point.

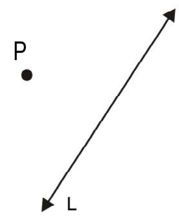


Fig. 6.58 (a)

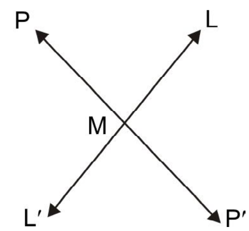


Fig. 6.58(b)

P is reflected in the line L and an image is formed. Which is the image and where is it formed?

Procedure to get the image:

See Fig. 6.58 (b) From P, $\overline{PM}$ is drawn perpendicular to L.On $\overrightarrow{PM}$, a point P' is taken such that P-M-P' and PM = MP'.Thus we get the image P' of the point P obtained under reflection in L.

Line L is known as the **mirror line**.

In Fig. 6.65., P'_1 is the image of P_1 in L.

P'_2 is the image of P_2 in L.

P'_3 is the image of P_3 in L.

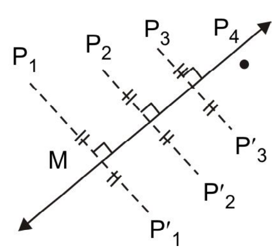


Fig. 6.59

As the point is being nearer L, the image is also being nearer to L.

P_4 is a point that lies in L. Where is the image of P_4 in L?

Distance of P_4 from L is zero as P_4 is in L.

$\therefore$ The distance of the image of P_4 from L is also zero.

Notes

Hence the image of P_4 is also to lie in L.

Thus P_4 is its own image.

Hence we say-

The image of a point lying in the mirror line is its own image.

(ii) Reflection of a line-segment :

L is the mirror line and $\overline{AB}$ is to be reflected (see fig.6.60)

The image of $\overline{AB}$ in the mirror line L is $\overline{A'B'}$ where A' and B' are the images of A and B respectively in the line L.

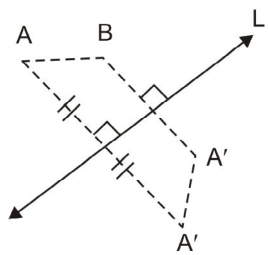


Fig. 6.60

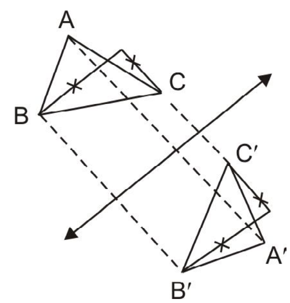
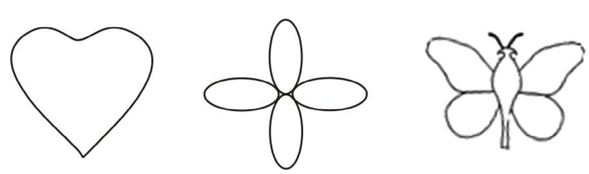


Fig. 6.61

(iii) Reflection of a triangle: As seen in fig 6.61 above, the image of ABC in the mirror-line L is $\Delta\, A'B'C'$ where A', B' and C' are the images of A, B and C respectively in the line L.

Symmetry : The shape of some objects and some designs appeal us very much. We say those objects or the designs look beautiful. Some such things are shown below.



[A betel leaf] [A design] [A butterfly]

Fig. 6.62

In each of the diagrams above, we can think of drawing a line suitably, so that-

If we take a trace-copy of the figure,

Notes

cut along the outline of the figure,

fold along the line we have drawn (or thought of drawing),

The two parts of the figure coincide with each other exactly. Then the figure is said to be a *symmetric figure* and it is *symmetric about the line* we have drawn.

The line drawn is known as the**line** (or **axis**) of **symmetry** of the figure. The figure is said to have line-symmetry.

The line of symmetry of each of the above figures is shown below:

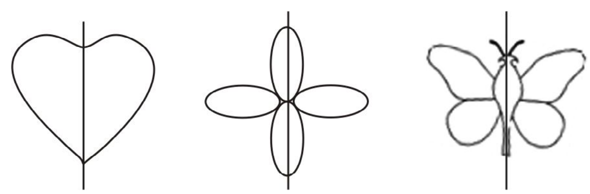


Fig. 6.63

Examples of some letters which are symmetric about a line.

A B D E

Try to write the others.

***Geometric figures with line-symmetry*:**

ABC is an isosceles triangle and $\overleftrightarrow{PQ}$ is its line of symmetry. $\overleftrightarrow{PQ}$ is the line that passes through the vertex A and the midpoint Q of the base $\overline{BC}$ (Fig. 6.64).

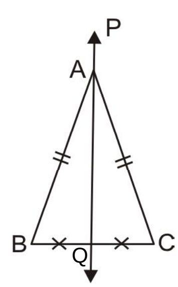


Fig. 6.64

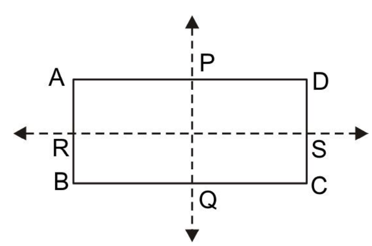


Fig. 6.65

ABCD is a rectangle and it has 2 lines of symmetry. Those are $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ where P, Q, R and S are the mid-points of sides $\overline{AD}$, $\overline{BC}$, $\overline{AB}$ and $\overline{CD}$ respectively (fig 6.65).

$\Delta\, ABC$ is an equilateral triangle. It has 3lines of symmetry and those are $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$ and $\overleftrightarrow{TV}$. Each of them joins a vertex with the mid point of the opposite side (Fig. 6.66)

Notes

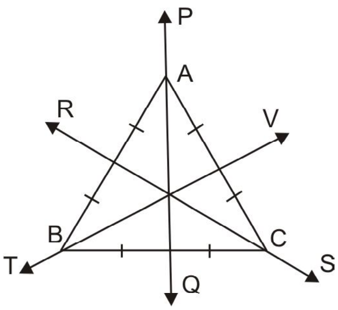


Fig. 6.66

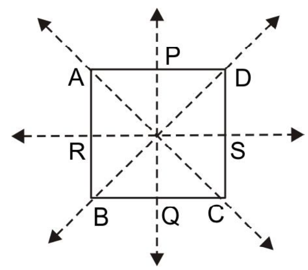


Fig. 6.67

ABCD is a square (fig. 6.67). It has 4 lines of symmetry and those are $\overleftrightarrow{AC}, \overleftrightarrow{BD}, \overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ where P, Q, R and S are the mid points of sides $\overline{AD}, \overline{BC}, \overline{AB}$ and $\overline{CD}$ respectively.

A line passing through the center of a circle is a line of symmetry for the circle.

Thus we can have geometric figures having one or more than one lines of symmetry.

Rotational symmetry :

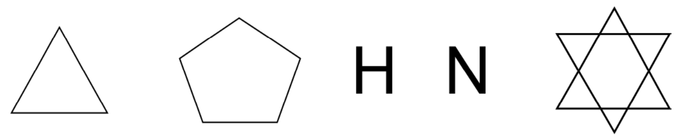
Look to the toy of a paper-made-wind-mill that rotates when shown against the wind.

Fig. 6.68

4 blades of it are named as A, B, C and D. If it is made to rotate through a right angle (in an anti clock wise manner), A will be found occupying the position now occupied by B. B, C and D will, in turn, be occupying the positions now occupied by C, D and A respectively.

Thus the wind-mill looks exactly like its first position. Hence during a complete rotation, there occur 4 positions when it looks to have the original situation. Hence we say that the wind-mill, shown in Fig6.68 has rotational symmetry of **order 4**. The point around which the wind-mill rotates is known as the **point of symmetry**.

Check if the shapes given below have rotational symmetry and if yes, state the order of symmetry in each case.



(a) (b) (c) (d) (e)

Fig. 6.69

Check your progress

E6. Fill in the blanks

(a) Two line segments are congruent if their .... are equal.

(b) Two angles are congruent if their .... are equal.

(c) If triangles are congruent then their .... angles are equal in measure.

(e) Two similar triangles have the same ....

E7. In $\Delta\, ABC$ and $\Delta\, PQR$, AB = 4cm, BC = 7cm., PQ = 6cm, QR = 10.5cm. and $m\angle B = m\angle Q$

(a) Find PR if AC = 8cm.

(b) Find the ratio of areas of $\Delta\, ABC$ and $\Delta\, PQR$.

6.4 THREE DIMENSIONAL BODIES

We have already discussed of three dimensional bodies earlier. There we said that the bodies with extensions in three directions like left to right, nearer to farther and top to bottom are known as bodies of 3 dimensions.

Bodies with extensions in three directions mutually at right angles are known as 3D bodies.

Different regular 3-D shapes:

(a) Cuboid

A box, a piece of a brick and bodies of the like has the shape which is known as a **cuboid**.

A cuboid is shown in Fig. 6.70.It has 8 vertices (A, B, C, D, E, F, G, H), 12 edges $\left(\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}, \overline{EF}, \overline{FG}, \overline{GH}, \overline{HE}, \overline{AH}, \overline{BE}, \overline{CF} \text{ and } \overline{DG}\right)$, and 6 Faces (ABCD, EFGH, ABEH, BEFC, CFGD and AHGD)

Notes

Each of BC, EF, HG and AD represents the length (ℓ) of the cuboids.nEach of AB, CD, EH and FG represents its breadth (b).Each of AH, BE, CF and DG represents its height (h).

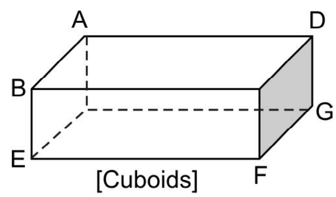


Fig. 6.70

Surface area of a cuboid = area of top and bottom surfaces + area of left and right surfaces + area of front and back surfaces= $2l \times b + 2\,b \times h + 2lh = 2(lb + bh + lh)$ square units

*Volume of a cuboid*is the measure of the space occupied by the body and it is the product of the length, breadth and height in case of a cuboid.

Thus $V = l \times b \times h$ cubic unit.

(b) Cube: A cuboid of which the length, breadth and height are equal is known as a **cube**.

Thus, all its edges are of equal length and all of its faces are squares.

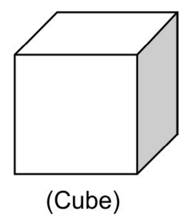


Fig. 6.71

Unit of volume measure :

The unit to measure volume of a 3-D body is the volume of a unit cube i.e, a cube each side of which is of unit length.

If the length of each side of a unit-cube is 1cm, then the volume of the unit-cube is $1cm^3$.

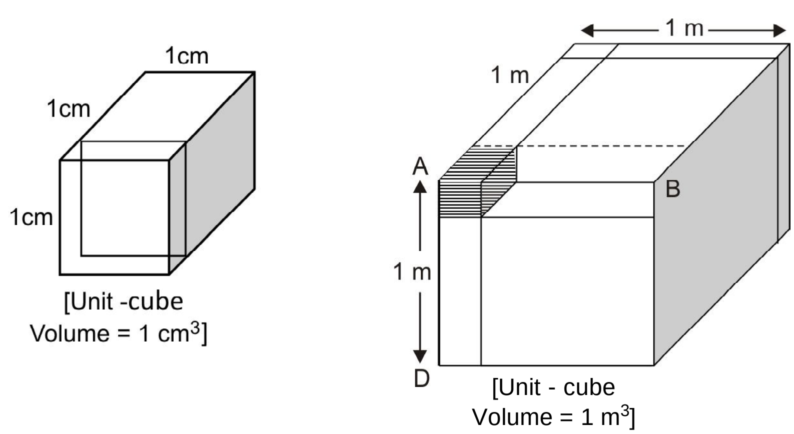


Fig. 6.72

A unit-cube, each side of which is 1cm, is known as a cm-cube.

A unit-cube, each side of which is 1m, is known as a m-cube.

The shaded part of the m-cube (Fig.6.72b) is a cm-cube i.e. each edge of it is of length 1cm. Thus if the m-cube is cut into cm-cubes we will get 100 cubes along the edge $\overline{AB}$,100 cubes along the edge $\overline{AC}$ and100 cubes along the edge $\overline{AD}$.

Hence the total number of cm-cubes available will be $100 \times 100 \times 100 = 10{,}00{,}000$.

Thus $1m^3 = 10,\ 00{,}000\ cm^3 = 10^6\ cm^3$

$1\ cm^3$, $1\ m^3$ etc. are known as cubic units.

Note :

(i) The area of cm-square and a m-square have not been calculated, those have been defined as $1cm^2$ and $1m^2$ respectively.

(ii) The volumes of a cm-cube and a m-cube have been defined as $1cm^3$ and $1m^3$ respectively.

(iii) $1cm^2$ is the unit of area-measure, but a cm-square is a square each side of which is 1 cm long.

Thus $1cm^2$ and a cm-square represent completely different concepts.

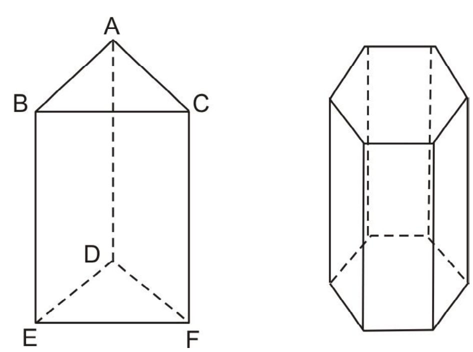
Area of a cm-square = $1cm^2$

Like wise, $1m^3$ and a m-cube are different.

Vol. of a m-cube = $1m^3$.

(b) Prisms :

3-D body shown in Fig. 6.73 (i) have the following features.



(i) (ii)

Fig. 6.73

It has 2 triangular faces (one at the top and one at the bottom). We usually name those as the *bases*.

Further it has 3 rectangular faces each of which is

perpendicular to the triangular end.

These surfaces are known as *lateral faces*.

The distance between the two triangular ends is known as the **height** of the prism (it is also said as the **length** if the prism lies horizontally).

The Fig. 6.73 (ii) presents the diagram of a prism with a hexagon as its base.

Surfacearea of a prism:

Lateral surface area = AB × h × BC + h + CA × h sq-units (where 'h' represents the height)

= h (AB + BC + CA) sq-units ('h' taken common)

= h × perimeter of the base

Area of the ends = 2 × area of each end.

Total surface area of a prism

= lateral surface area + area of the ends

= h × perimeter of the base + 2 × area of each end.

Volume of a prism = base area × height

Note : These rules apply to prisms of bases in the shape of polygons of any no. of sides.

(c) Cylinder :A cylinder is the geometrical shape that we see in a log of wood.

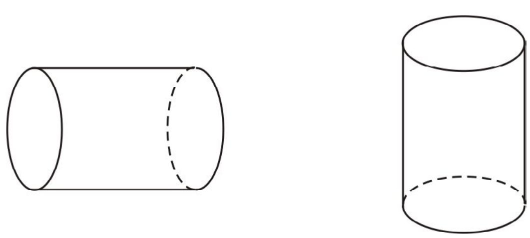


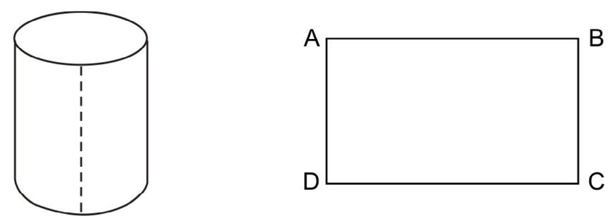
Fig. 6. 74

The cylinder has two circular ends (also known as *bases*). Except the bases, it has a curved face all around.

The distance between the two ends is known as the height (or length) of the cylinder (denoted as h).

Surface area of a cylinder :

Curved surface area : Curved surface area of a cylinder is equal to the area of a rectangular sheet of paper which can exactly cover the curved surface of it. Such a piece of paper is shown in Fig. 6.75(b).



(a) (b)

Fig. 6.75

The length (l) of the rectangular paper sheet

= circumference of the basecircle = $2\pi r$ and breadth (b) of the sheet

= height of the cylinders = h

The area of the sheet = $l \times$ b = $2\pi r \times$ h = $2\pi r$ sq. unit.

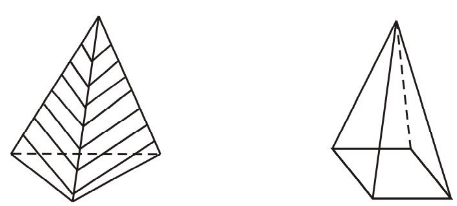
The curved surface area of a cylinder = $2\pi rh$ sq. unit.

Volume of a cylinder = base area × height = $\pi r^2 h$ cubic units

Note : A cylinder is a special form of a prism with circular base.

(d) Pyramid : Pyramids are the tombs of Pharaohs (Kings and queens) of Egypt.

The base of a pyramid is a triangle or a polygon and it ends in a point at the top. Some such shapes are shown below.



(a) (b)

Fig. 6.76

*Pyramid with triangular base*has 4 vertices6 edges and 4 faces each of which is a triangle.

*Pyramid with quadrilateral base*has 5 vertices, 8 edges and 5 faces of which slant faces are triangular and the base is a quadrilateral.

The surface area of a pyramid = slant surface area + base area

Volume of a pyramid = $\frac{1}{3}$ × base area × ht.

(e) Cone :

A joker's cap as seen in a circus, a funnel without its stem have the a shape which is known as a cone.

It has one vertex, one circular edge and 2 faces of which one is curved and the other one is flat and circular in shape.

Fig.6.78

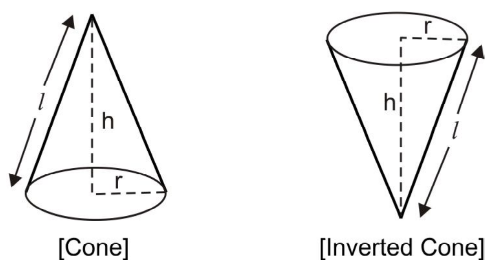


Fig. 6.77

Total surface area = Curved surface area + area of the base- circle

$$= \pi r\sqrt{h^2 + r^2} + \pi r^2 = \pi r\left(\sqrt{h^2 + r^2} + r\right)$$

[where 'r' is the radius of the circular base and 'h' is the height of the cone as shown in the diagram]

Slant height $(\ell) = \sqrt{r^2 + h^2}$

(f) Sphere: The shape of a football represents a sphere.

It has no vertex and it has no edge too.But it has one curved face.

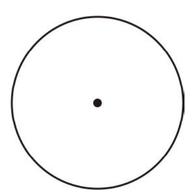


Fig. 6.78

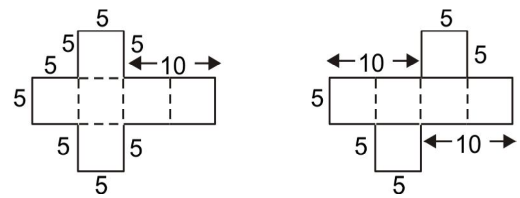
Surface area of a sphere= $4\pi r^2$

Volume of a sphere= $\frac{4}{3}\pi r^3$ (*r* is the length of theradius of the sphere and π is to be taken as $\frac{22}{7}$ or 3.14 as suitable.)

Net for building some 3-D shapes

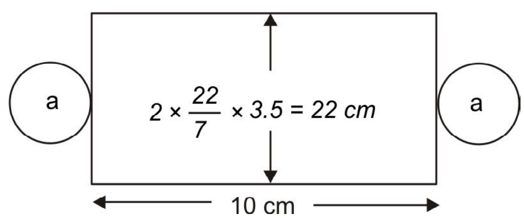
A net is an outline in 2-D which when folded results in 3-D shape. On an art paper (thick sheet of paper) an outline is so drawn that when cut along the outline, the piece of paper that comes out, can be folded suitably to give rise to a 3-D folded shape.

(i) To get a cube of edge 5cm :



The net shown above have all angles right angles and the edge-measurements are shown in the diagram.

(ii) Net to get a cylinder of base radius 3.5cm and height 10cm :



'a' and 'b' represent two circles each of radius 3.5cm and the remaining part is a rectangle with measurements as show in the diagram.

Check your progress

E9. What will be the length of each edge of the greatest cube that can be obtained by cutting a woolen cuboid of length 175cm breadth 105cm and height 63cm ?

E10**.** Determine the ratio of the volumes of the two cylindrical shapes that can be prepared using a rectangular sheet of paper of length 33cm and breadth 22cm entirely?

6.5 CONSTRUCTION USING GEOMETRICAL TOOLS

The set of geometrical instruments include a scale, a pair of dividers, a pair of compasses and a pair of setsquares (one 45°-45° setsquare and the other 60°-30° setsquare).

When we use only the straight edge of the scale, we consider it as a ruler i.e., a ruler is a rectangular plate with only straight edges.

Use of ruler and compasses:

There was a time, when mathematicians thought that many basic mathematical works can be done using ruler and compasses. However, the following geometric figures can be drawn using ruler and compass:

(i) A line segment of given length.

(ii) Perpendicular bisector of a given linesegment.

(iii) A line parallel / perpendicular to a given line,

  (a) at a given point on it.

  (b) from a given point outside it.

(iv) An angle equal in measure to a given angle.

(v) Bisector of a given angle.

(vi) Angles of measure 60°, its multiples and submultiples.

(vii) Divide a given line segment into equal parts.

(viii) Draw triangles, quadrilaterals and circles of given measure.

You must have drawn these figures in your school days. To reinforce your learning of these constructions, we have presented here the processes in brief.

(i) To construct a line segment of a given length.

(a) A st-line is drawn using a straight edge of a scale [Fig 6.79 (a)].

(b) The given length is taken as the radius in the compass. With some point on the line (A) as centre, an arc is drawn to cut the line (at B as shown in Fig.6.79 (b)]

(a) (b)

Fig. 6.79

(c) $\overline{AB}$ is the required linesegment of given length.

(ii) To draw the perpendicular bisector of a given line segment $\overline{AB}$.

Step 1: Radius equal to more than half the length of the given line segment is taken on the compass. With A as centre, two arcs are drawn on the two sides of $\overline{AB}$ [as shown in Fig. 6.80]

Step 2: With B as centre and with the same radius as taken in step-1, two more arcs are drawn to cut the arcs drawn earlier.

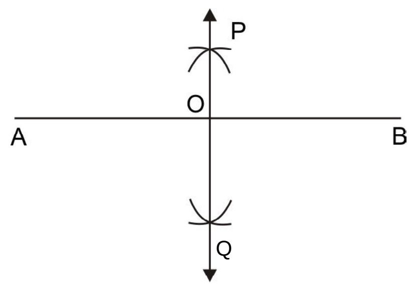


Fig. 6.80

Step 3: **A** line is drawn joining the two points of intersections of the arcs obtained in step-2. Let us name the line as $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$. O,the point of intersection of $\overleftrightarrow{PQ}$ and $\overline{AB}$, is the mid-point of $\overline{AB}$.

Examine what happens when the radius taken on the compass is exactly half or less than half of the length of $\overline{AB}$.

(iii) (a) To draw a line perpendicular to a given line at a given point on the given line :

To draw a line perpendicular to $\overleftrightarrow{AB}$ *atat a point P on it:*

Step 1: With P as centre and with a suitable radius an arc is drawn to cut the line $\overleftrightarrow{AB}$ at 2 points. Name the points as M and N.

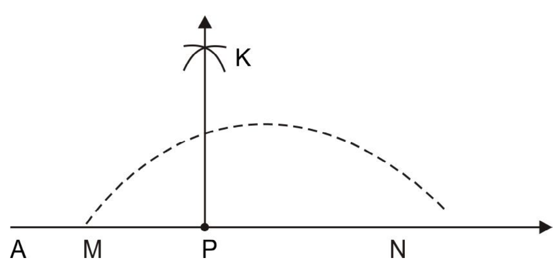


Fig. 6.81

Step 2: With a radius greater than the radius taken in step-1, two arcs are drawn successively with M and N as centres so that those arcs intersect each other on one side of the line $\overleftrightarrow{AB}$. Name the point as K.

Step 3: Draw the line $\overleftrightarrow{PK}$. $\overleftrightarrow{PK}$ is the required line such that $\overleftrightarrow{PK} \perp \overleftrightarrow{AB}$.

(iii) (b) To draw a line perpendicular to a given line at a given point outside of it :

To draw a line perpendicular to $\overleftrightarrow{AB}$ through a point P outside of it.

$\overleftrightarrow{AB}$ is a given line and P is a point outside $\overleftrightarrow{AB}$.

Step 1: With P as centre and with a suitable radius an arc is drawn to cut $\overleftrightarrow{AB}$ at 2 points. Name the points as M and N.

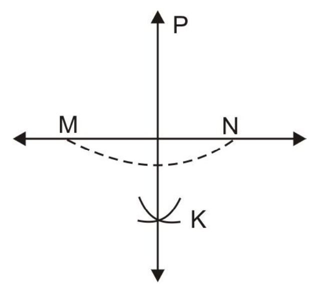


Fig. 6.82

Step 2: With M and N as centres and with a radius greater than half of MN, arcs arc drawn to cut each other on one side of $\overleftrightarrow{AB}$ (opposite to the side where P lies). Name the point of intersection as K. Draw the line $\overleftrightarrow{PK}$.

$\overleftrightarrow{PK}$ is the line perpendicular to $\overleftrightarrow{AB}$ and passes through the point P.

(iii) (c) To draw a line parallel to a given line through a given point outside it.

To draw a line parallel to $\overleftrightarrow{AB}$ through P.

$\overleftrightarrow{RS}$ is the line required to be drawn parallel to $\overleftrightarrow{AB}$.

$\overleftrightarrow{MN}$ is any line drawn through P intersecting $\overleftrightarrow{AB}$ at T.

As alternate angles $m\angle SPT = m\angle ATP$.

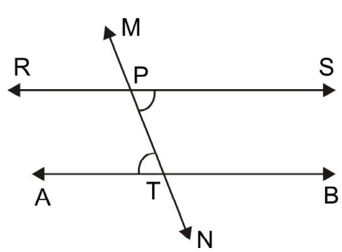


Fig. 6.83(a)

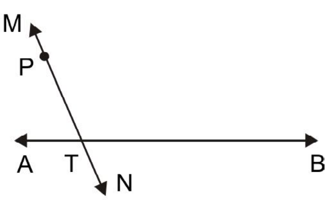


Fig. 6.83 (b)

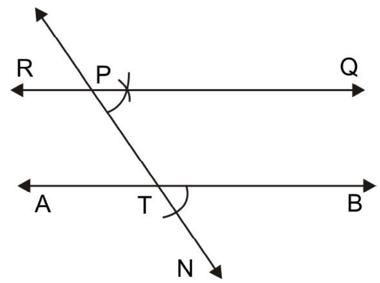


Fig. 6.83 (c)

Step 1: Any line $\overleftrightarrow{MN}$ is drawn through P which intersects $\overleftrightarrow{AB}$ at some point. The point of intersection is named as T.

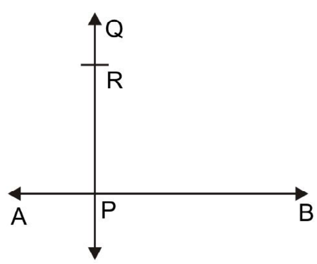
Step 2: An angle, equal in measure to $\angle BTN$ is drawn on $\overleftrightarrow{PR}$ at P. following the steps discussed in construction (iii).

Thus we get a line through P parallel to $\overleftrightarrow{AB}$. The line, named as $\overleftrightarrow{RS}$, is the required line.

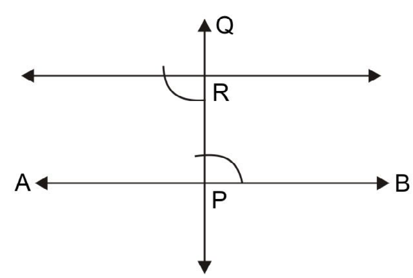
To draw a line parallel to a given line at a given distance from it.

$\overleftrightarrow{AB}$ is the given line. To draw line parallel to $\overleftrightarrow{AB}$ at a given distance (say 5 cm) from it.

Step 1: A point is taken on $\overleftrightarrow{AB}$. Name the point as P. At P a line is drawn perpendicular to $\overleftrightarrow{AB}$ following the process discussed in iv (a) above. Name it as $\overleftrightarrow{PQ}$.



(a)



(b)

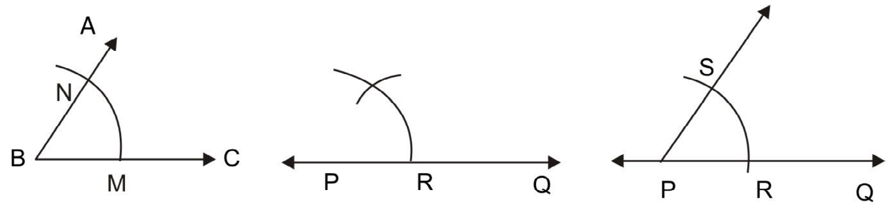
Fig. 6.84

Step 2: Taking a radius of 5 cm, draw an arc with P as centre such that it cuts $\overleftrightarrow{PQ}$. Name the point as R.

Step 3: Draw an angle equal in measure to $\angle RPB$ at R on $\overrightarrow{RP}$ following the method discussed in C (iii) such that it become alternate to $\angle RPB$. Extend the arm of the angle drawn. It gives you the required line parallel to $\overleftrightarrow{AB}$ at a given distance (5cm) from $\overleftrightarrow{AB}$.

(iv) To draw an angle equal in measure to a given angle at a given point on a given line.

$\angle ABC$ is the given angle and PQ is a given line.



(a) (b) (c)

Fig. 6.85

To draw an angle equal to $\angle ABC$ at P on another line $\overrightarrow{PQ}$:

Step-1: With some radius taken on the compass, an arc is drawn with B as centre so that the arc cuts $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Name the points of intersection as M and N respectively (Fig.6.85a).

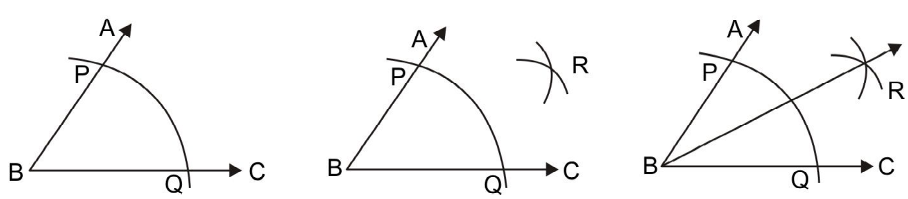
Also draw an arc of the same radius as above with P as centre so that the arc cuts $\overrightarrow{PQ}$. Name the point of intersection as R (as shown in Fig.6.85b).

Step-2: Take the distance between M and N as radius on the compass and with R as centre, draw an arc to cut the arc drawn in step-1. Name the point of intersection as S (Fig.6.85c)

Step-3: Draw the ray $\overrightarrow{PS}$. $\angle SPR$ is the required angle which is equal in measure to $\angle ABC$.

(v) To draw the bisector of a given angle:

$\angle ABC$ is the given angle. To draw the bisector of it.



(a) (b) (c)

Fig. 6.86

Step- 1: An arc is drawn with B as centre and with a suitable radius so that if cuts the arms $\overrightarrow{BA}$ and $\overrightarrow{BC}$ of $\angle ABC$. The points of intersection be named as P and Q.

Step- 2: Arcs are drawn with P and Q as centers and with radius equal to greater than half of PQ, so that the arcs intersect each other. Name the point of intersection as R.

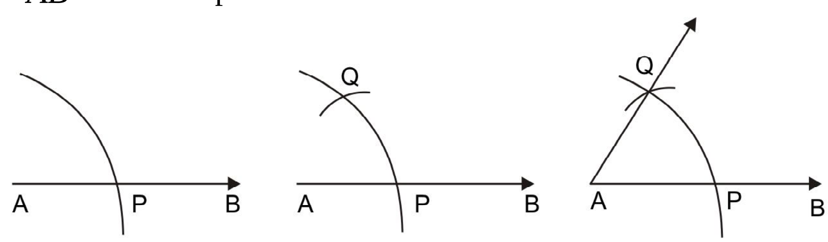
Step-3: The ray $\overrightarrow{BR}$ is drawn. $\overrightarrow{BR}$ is the bisector of $\angle ABC$. Thus $m\angle ABR = m\angle CBR$

(vi) To draws angles of specific measure (*Angles of measure 60°, its multiples and submultiples)*

(a) *To draw angle measuring 60°.*

$\overleftrightarrow{AB}$ is a given ray. To draw an angle measuring 60° at A on $\overleftrightarrow{AB}$.

Step-1: With A as centre and with a suitable radius an arc is drawn so that it cuts $\overrightarrow{AB}$. Name the point of intersection as P.



(a) (b) (c)

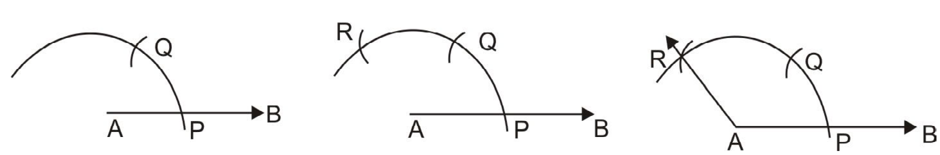
Fig. 6.87

Step-2: With P as centre and with the same radius as taken in step-1, an arc is drawn to cut the arc drawn in step-1. Name the point of intersection as Q.

Step-3: The ray $\overrightarrow{AQ}$ is drawn. $\angle QAB$ is the required angle which measures 60^0.

(a) *To draw an angle of measure 120°.*

Step-1: A ray is drawn and named as $\overrightarrow{AB}$. An arc of a suitable radius is drawn with A as centre such that it cuts $\overrightarrow{AB}$ and point of intersection is named as P.



(a) (b) (c)

Fig. 6.88

Notes

Step-2: An arc of the same radius as before is drawn with P as centre so that it cuts the arc drawn in step-1 and the point of intersection is named as Q.Another arc is drawn with Q as centre so that it cuts the arc drawn is step-1 and the point of intersection is named as R.

Step-3: Ray $\overrightarrow{AR}$ is drawn. $\angle RAB$ is the required angle and it is of measure 120°.

(b) *To draw an angle of measure 90°:*

Step-1: We draw the figure as shown in Fig. 6.89.

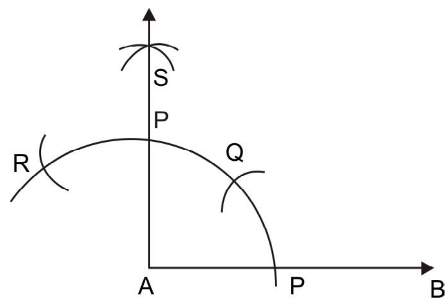


Fig. 6.89

Step-3: Draw the bisector of $\angle RAQ$. Name the bisector as $\overrightarrow{AS}$. $\angle SAB$ is the required angle which measures 90°. Angle of measure 45° can also be obtained by bisecting an angle of measure 90°.

To draw an angle of measure $22\frac{1}{2}°$ we draw an angle of measure 45° and then bisect it.

E 11 State how can you draw an angle measuring (a) 30°, (b) 15°, (c) 105°.

(vii) To divide a given line segment into any number of parts of equal length :

AB is a given line segment and it is to be divided into 3 parts of equal length.X

Step-1: With one of its ends (say A) as origin we draw a ray not coinciding with it. Thus $\overrightarrow{AX}$ is drawn.

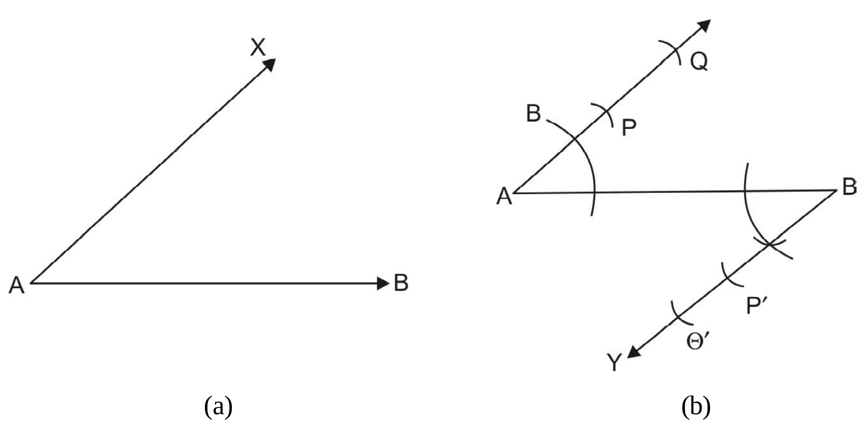


Fig. 6.90

Step-2: Ray $\overrightarrow{BY}$ is drawn at parallel to $\overrightarrow{AX}$ by drawing $\angle ABY$ equal in measure to $\angle BAX$.

Step-3: Draw an arc of a suitable radius with A as centre which cuts $\overrightarrow{AX}$. Name the point as P. Again with P as centre and with the same radius draw an arc to cut $\overrightarrow{AX}$. Name the point as Q. Thus we get 2 equal divisions AP and PQ. (i.e,. 1 less than the number equal parts into which $\overline{AB}$ is to be divided).With the same radius cut off equal division an $\overrightarrow{BY}$ and name the points of intersection as P_1 and Q_1. [as shown in Fig. 6.90(c)]

Step-4: **N**ame the points where $\overline{QP_1}$ and $\overline{PQ_1}$ cut $\overline{AB}$ as C and D respectively [as in Fig- 6.101 (c)] . Thus $\overline{AB}$ is divided at C and D into 3 equal parts.

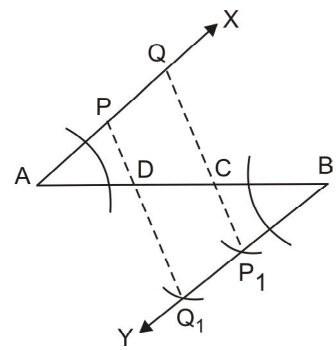


Fig. 6.90 (c)

Check your progress.

E12 Into how many parts of equal length can a line segment be divided by drawing the perpendicular bisector of the line segment?

E13 Can you use perpendicular bisector method to divide a given line segment into-

(a) 4 parts of equal length ?

(b) 8 parts of equal length ?

(c) 12 parts of equal length ?

(VIII) (a) Construction of triangles: When one acquires the understanding and skills of constructing line segments, and angles of specific measures, he/she can draw the triangles using only ruler and compass.

Minimum data required for constructing a triangle: Out of 3 sides and 3 angles contained in a triangle, the triangle can be constructed when any of the following conditions are given.

(a) lengths of three sides (S-S-S),

(b) lengths of any two sides and measure of their included angle (S-A-S),

(c) lengths of any two sides and measure of any one angle (S-S-A),

(d) measures of any two angles and the length of one side (A-S-A) or (A-A-S)

You can observe that in all the above conditions of drawing a triangle you require *at least three measures*. Through manipulation of these constructions you can draw several triangles.

ACTIVITY

State the procedure how to construct a right angled triangle when the lengths of any two of its sides are given.

(b) Construction of quadrilaterals: Once you know the construction of different triangles, it would not be difficult to construct the quadrilaterals. This is because of the fact that every quadrilateral is divided into two triangles by each of its two diagonals. Therefore, while constructing a quadrilateral first you have to complete constructing its component triangle and then complete the quadrilateral.

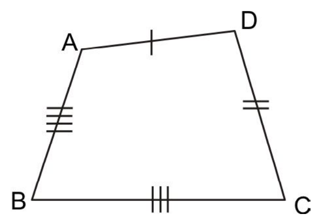


Fig. 6.91

As for example, when the lengths of the four sides of the quadrilateral ABCD and the length of AC, one of its diagonals are given, you can draw any of the two triangles, say the triangle ABC and then complete the triangle ADC . Thus, you have completed theconstruction of the required quadrilateral.

Similarly, you can undertake the construction of other variations of the quadrilaterals.

E14 Under which of the following conditions, construction of quadrilateral is possible?

(a) lengths of four sides are given,

(b) lengths of two diagonals of a rhombus are given,

(c) measures of 2 angles and lengths of 3 sides are given,

(d) lengths of two adjacent sides of a parallelogram and measure of any one of its angles are given.

6.6 LET US SUM UP

Point, line and plane are thethree undefined terms in the plane-geometry.

Line-segment, ray, angle, triangle, quadrilateral etc. are basic terms that are defined using undefined terms.

Adjacent angles, complementary angles, supplementary angles, vertically opposite angles are different examples of paired angles.

Triangles areclassified in respect of (a) the relative measure of sides: Isosceles triangle, equilateral triangle and scalene triangle, (b) the angle-measure: Acute angled, right angled and obtuse angled triangles. Sum of the three angles of a triangle is 180^0.

In case of a triangle (a) Perimeter = sum the lengths of the 3 sides, and (b) Area = ½ base × height.

Trapezium, parallelogram, rectangle, rhombus and square are the different kinds of quadrilaterals.Sum of the measures of all 4 angles of a quadrilateral is 360^0.

A circle is defined as a collection of points in a plane such that each of them lies at a fixed distance from a fixed point in the same plane. The fixed point is known as the centre of the circle and the distance between the centre and any point on the circle is the radius of the circle.

The linesegment joining any two points taken on a circle is a chord of it.Each of the two parts into which a circular region is divided by a chord, is known as a segment of the circle.

Angles inscribed in an arc of a circle are of equal measure.Angle in a semicircle is a right angle.

The angle made by the radii drawn at the end points of a minor arc is known as the angle subtended by the arc at the centre.The degree measure of a minor arc is the measures of the angle subtended by the arc at the centre.

Sum of the measures of the opposite angles of a cyclic quadrilateral is 180^0.

Line-segments of equal length are congruent while angles of equal measures arecongruent.Under different conditions (such as S-A-S, S-S-S, A-S-A and RHS) triangles can be congruent.

Congruent figures are similar but the reverse is always not true.Areas of two similar triangles are proportional to the squares of their corresponding sides.

Concept of reflection in geometry is similar to the reflection in mirror. Reflections create symmetry in figures.

Using the formulae for calculating area of plane figures, the surface areas of the 3-D shapes like cuboid, cube, cylinders, cone, prism and pyramid can be calculated.

Using only ruler compass, the following basic geometrical figures can be constructed **:**

(i) A line segment of a given length.

(ii) The perpendicular bisector of a given linesegment.

(iii) A circle of a given radius.

(iv) An angle equal in measure to a given angle at a given point and on a given ray.

(v) A line parallel/perpendicular to a given .

(vi) Bisector of a given angle.

(vii) Angles of measure 60°, its multiples and submultiples

(viii) Closed geometrical figures like triangles and quadrilateral and circles.

6.7 CHECK YOUR PROGRESS

E1. 92½°

E2. 108°, 108°, 72°

E3. (i) $\angle ACQ, \angle CDS$, (ii) $\angle \mathrm{PCD}, \angle \mathrm{CDR}$, (iii) $\angle \mathrm{QCD}, \angle \mathrm{CDR}$,

E4. $a = c = f = h = 145^0$, $e = d = b = 35^0$

E5. (a) rhombus, (b) centre, (c) 32^0, (d) 180^0

E6. (a) Length, (b) Measures, (c) Corresponding (d) shape

E7. PR = 12 cm,

E8. $\Delta\, ABC : \Delta\, PQR = 4 : 9$

E9. 63 cm

E10. The ratio of the volumes will be 3 : 2 .

E11. (a) By drawing the bisector of the angle measuring 60°,

(b) By drawing the bisector of the angle measuring 30°,

Notes

(c) By constructing $\angle ABC = 90°, \angle CBD = 120°$, then drawing BE, the bisector of $\angle ABD$, Measure of $\angle EBC = 105°$.

E12. 2 equal parts.

E13. (a) yes, (b) yes, (c) no.

E14. (a) No, (b) Yes, (c) No, (d) Yes.

6.8 SUGGESTED READINGS & REFERENCES

Mathematics Textbooks for classes V to VIII developed by NCERT.

6.9 UNIT END EXERCISE

1. Using ruler and compasses, draw the shape shown in the figure along side with the following measurements.

   AP = 3cm, BP = 4cm, BC = 7cm, PQ = 5cm, QD = 2cm, $\overline{BP} \perp \overline{AD}$ and $\overline{CQ} \perp \overline{AD}$. Complete a figure which has line symmetry with $\overleftrightarrow{AD}$ as the line of symmetry.

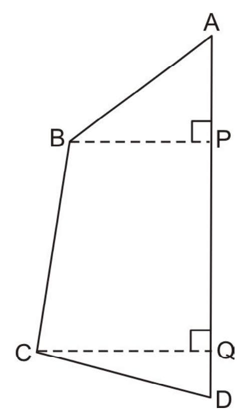


Fig. 6.92

2. In the diagram along side

   AC = BD = PF = DE = 8 cm. $\angle ABC$, $\angle BCD$, $\angle CDA$, $\angle BDE$ and $\angle BPC$ each is a right angle.

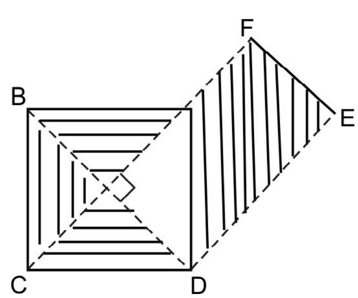


Fig. 6.93

Using setsquare and scale, draw the shape with the measurements given. Calculate and compare the areas of the shaded regions separated by $\Delta\ APD$.

Notes

3. Fill in the blanks in the table below.

Shape	**Centre of rotation**	**Order of symmetry**
Rhombus		
Equilateral triangle		
Regular hexagon		
Square		

4. Take a piece of wire of length 20cm. Bend it into shapes of rectangles of different length as indicated in the table below. Determine the area enclosed in each case and complete the table.

Rectangle of length	Breadth	Area
8cm		
7cm		
6cm		
5cm		

5. Three measures are given in each of the sets below. Which of them can be taken as the lengths of 3 sides to construct a triangle ?

   (a) 5.5cm, 6.8 cm and 7.2cm

   (b) 4.7cm, 5.3cm and 10.0cm

   (c) 8.0cm, 7.5cm and 9.2cm

   (d) 5.8cm, 12.2 cm and 6.0cm

6. To construct an angle of measure 52.5° using ruler and compasses, what should be the breakup of the given measure?

UNIT 7 MEASURES AND MEASUREMENTS

Structure

7.0 INTRODUCTION

We have already discussed in the previous unit that every physical object that we see around us has two characteristics. Those are the ***shape*** and the ***size***. Shape speaks of what the figure or object looks like. We say, "The figure looks like a circle". The term circle here speaks of the shape. Similarly when we see a sculpture made of stone in front of a temple, we say, "It is a stone-made lion". The term 'lion' here speaks of the shape of the object that we see.

But when we say that the square drawn on the page is too big, the term 'big' speaks of the size. The light house, made long back near the sea shore, is quite high. Here, 'quite high' speaks of the size of the light house.

Let us see the statement below:

"Between the two jokers playing in the circus one is too bulky and the other one is too short".

Can we say the extent of shortness of the joker?

Can we say the extent to which the joker is bulky?

We cannot say exactly.

The characteristic of an object that helps us knowing the bigness or the smallness of it is the measure of it.

Similar characteristics of different objects have a certain measure and other characteristics, not similar to the characteristics spoken of earlier have a different measure. For example, how much space a figure covers on a plane is its area measure, whereas how much water a solid body displaces when submersed in a vessel containing water, speaks of its volume-measure.

In this unit we shall be discussing about various types of measures, the units and scales of those measures and the process of measuring different aspects of physical objects and some common phenomena.

For completing the study of this unit you will need *about 7 (seven) hours*.

7.1 LEARNING OBJECTIVES

After studying this unit, you will be able to

- know different units of measurement of different aspects of objects and processes.
- using units of different measures in your daily life activities.
- make calculations relating to length, area, volume, capacity, weight, and time.

7.2 CONCEPT OF MEASUREMENT AND MEASURES

Everyone is familiar with measurement. In our daily life, we are required to measure something or other. For instance, the length of the cloth required to make a shirt, the weight of vegetables or groceries required to be purchased from the daily market, amount of water one drinks per day, the height and weight of the children in a class, the size of the classroom or our bedroom, the area of the school garden, the time you require to reach school from your home, your body temperature, etc. In all such cases we are to measure something. In all these situations, measurement means to express the attribute in terms of a quantity comparable to a unit. In other words, measurement is quantification of the size or some definite aspects of the size of an object like length, area and volume. Measurement is an application of numbers and helps children to see that mathematics is useful in everyday life, and to develop many mathematical concepts and skills.

Children in primary school need to learn about several kinds of measure. Length, area, volume, time are some important measures that are included in elementary school curriculum. But for a child of 7 to 9 years, these appear to be quite abstract. Therefore, as a teacher you need to make child familiar with these concepts.

A child, from the beginning of experiencing the world, is familiar with the 3-Dimensional (3-D) solid objects like fruits, toys, coloured blocks, cubes, rectangular solids, match box, cylindrical solids such as chalk sticks, rollers etc. By handling the 3-D objects, the child gains ideas about the 2-dimensional (2-D) figures. As for example, playing with cubes or handling match boxes or drawing the diagrams of these objects on a paper (a 2-D object), he/she recognizes the surfaces of these solids to be plane figures which can be spread on a surface of a table or on a paper. Either observing the surfaces of the 3-D objects or their representation on 2-D planes (like drawing the figures of 3-D objects on a plane like paper or wall or floor), the child experiences the features of 2-D objects. Further, observing the edges of 3-D solids and the sides of the 2-D figures, and from some common objects like thin threads and wires one gathers the experience of 1-Dimension.

Engaging children in activities like sorting different common objects into groups and drawing the figures of 3-D and 2-D objects, you can help them in strengthening their conceptions of the dimensions of the objects. From such activities, they will conceptualize that with 1-dimensional objects only one number i.e. length is associated, with 2-dimensional objects two numbers i.e. length and breadth are associated and with 3- dimensional objects length, breadth, and height (or thickness) are associated. Involving children on such activities with several variations with different shapes and sizes, developing understanding of the notions of length, area, volume and their measurement becomes easier.

The following are the measures that commonly used and the child in the elementary school can experience through different activities and from the real life experiences. In the next section, we shall be discussing these measures in detail.

1. **Distance measure:** Length, breadth, height, radius are all distance measures. Each of them represents the distance between two specific points.

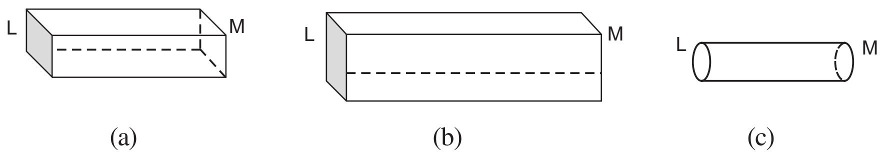
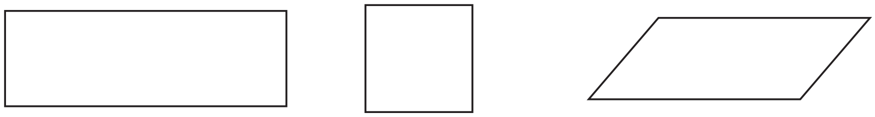


Fig. 7.1

We see three blocks of wood in the Fig.7.1. Each of them has 2 end (L, M). There is some distance between the two ends. It can be seen that the distance between the two ends of the block in (b) is more than the distance between the two ends of each of the other two.

Thus the distance between the two ends of the blocks is a common characteristic of the blocks. This is known as the **length-measure**.

2. **Area - measure**: Each 2-D shape encloses some region on a plane. The measure of enclosed region is known as the area-measure.



(a) (b) (c)

Fig. 7.2

Each of the figures above encloses a part of the page of paper. Thus each 2-D shape has an area measure.

3. **Volume–measure:** 3-D bodies occupy a portion of the space. The extent of space occupied by a 3-D body is its volume-measure. A 3-D body (not soluble in water) displaces a certain quantity of water when submersed in it. The quantity of water it displaces is known as the volume-measure.

4. **Weight – measure:** When we carry a body or lift it above the ground, in case of some, we are not to strain too much, whereas in case of some others, it strains us too much. 3-D bodies also show a force with which those are pulled towards the earth. The bigness or the smallness of the pull of the earth on 3-D bodies represents their weight-measure. The characteristic of the body that gives the feeling of heaviness speaks of the weight-measure.

5. **Time – measure:** When did an event happen during the day? How long do we take to complete a work? To answer such question we need to be acquainted with the time measure.

Measurement as comparison of two similar entities:

- Naresh was helping his students in class VI in preparing a plot in the school garden. While demarcating the boundary of the plot the group decided to carve out a rectangular plot from the school premises that the longer side would be twice in length than the shorter side. While some students were searching for a measuring device, Nitin took a stick of two cubits long and took two stick lengths on one side and took four stick lengths on the adjacent side and completed the plot.

- It was Monday and it was the turn of students of class V of the school to bring drinking water from the nearby tube well and fill up the storage container. They were given only one small bucket to fill the container. They found that the container was completely filled with 18 buckets of water.

Let us examine the above two examples of measurement. In the first example, a stick was used to decide the measure the lengths of the sides of a plot of land. In other words, the lengths of sides were compared with that of the stick and the specification that one side would be twice in length of the adjacent side was duly met. In the second example, the capacity of the container was compared with that of the given bucket. Can you use bucket as a measure of the length of the side of the plot or the stick to measure the capacity of the container?

From these examples you can realize that measurement is a process of comparison between two similar entities. The length or breath of the plot can be measured by another object known to have some definite length like a meter scale, or a stick of a definite length. Similarly the volume of the container can be measured by comparing it with another known measure of volume like a liter can or a bottle or bucket with known capacity. Since meter scale and the liter can are not similar measures, they cannot be used interchangeably.

Comparison of two similar entities can be done using any one or more than one of the five methods usually employed for the purpose. These are:

a. By observation.

b. By superimposition,

c. By indirect methods,

d. Using non-standard units, and

e. Using standard units.

The last two methods will be discussed separately in the next units. Let us see how we can use the first three methods for comparing two similar entities for measurement purposes.

You can try this activity with children in class I or II. Give them 10 coloured sticks of different lengths and ask them to arrange them in increasing order of their lengths. You will find that most of them arrange them by simply comparing them as to which one appears to be longer and which one appears to be shorter. Similarly you can show them five pebbles of different sizes and ask them to arrange them in order of their heaviness- the heaviest of them to be placed first and the lightest in the last place. You will find how quickly and how easily they can complete task correctly. These are simple examples of measuring by observation.

In a class the teacher was showing two coloured ribbons, red one in the right hand and green one in her left hand and asked children to point out the longer of the two ribbons. Some told the red ribbon is longer while some other pointed to the green one to be longer. Mere observation could not produce the correct measure. Then the teacher asked, “How can you check which one is longer?” One student proposed to superimpose one on the other. When it was done, green ribbon was found to be longer than the red one.

There are situations where observation or superimposition cannot be employed for comparison. For example, take a narrow glass and a wide glass of slightly different heights ans ask whose capacity is more. One method may be filling up one of the glasses full with water and pour it into the second glass. If completely emptying the first glass and it does not fill the second glass completely then we can conclude that the second glass has more capacity than the first one.

7.3 NON-STANDARD AND STANDARD MEASURES

Any sort of measure is always associated with a number, as numbers are the media through which bigness or smallness is expressed. The number associated with a measure comes out of a comparison. For example, to associate a number with the length of the block shown in fig.6.1 (a) we need to compare the length of the block with the length of another body. Thus a **specific length** is chosen with which the length of the body under observation is required to be compared. That specific length is known as the **unit length**. The ratio between the length of a body and the unit length is a number which expresses the length of the body.

Similarly **unit area**, **unit volume**/capacity, and **unit weight** are also used to measure area, volume/capacity and weight respectively. Each of these is taken as a ***unit of measurement*** of the specific attribute. These units can be of different forms as per the situation and requirement. This can be clearer from the following example:

A screen rod used to hang window-screen was broken and required a replacement to be purchased from the market. What should you do so that you can get the rod of right length from the market? Possible actions are that

1. The broken rod can be taken to the shop and a new rod of equal length can be purchased.
2. The length of the rod can be measured by using a stick and the new rod can be purchased by using the stick for ascertaining the required length.
3. You can measure the rod by your foot and can determine how many ‘feet’ the length of the rod is and accordingly you get the rod of equal length.
4. You can cut a thread equal in length of the rod and can procure a new rod of the length of the thread.
5. You can use a meter scale to determine the required length of the rod.

The solutions provided in 2, 3, and 4 are situation or person specific and are subject to vary from person to person who uses these measures. These are examples of ***non-standard units*** of measurement the examples of which will be discussed for different attributes in the next sections. In contrast to these measures, a meter scale is a standard measure used in almost all countries throughout the world. Any where the length of a meter is fixed and does not depend on the person or situation or time. Hence, it is an example of a ***standard unit*** of measuring length.

Standard units are comparatively easier to use as it is simple and is understood easily by everybody throughout the world. The standard units, although do not have any strong logical basis, have evolved through common acceptance and scientific refinements over the ages. Thus these units have been nearly perfected for accurate measurements. Sub-units (e.g., centimeter and millimeter are sub-units of meter) and compound units (e.g., kilometer is a compound unit of meter) are well defined in most of the standard units which are usually not available with non-standard units.

Non-standard units of measurements have been evolved to meet some immediate or some local needs. If you are preparing a sweet dish, you do not always go for the measures of rice, sugar and milk by the cooking manuals. By experience you can have handful of rice, five spoonful sugar and two glasses of milk and yet the dish would be as tasteful as the dish prepared by following the accurate measures of those ingredients. These non-standard measures may work well for you, but may not be as useful for another person who might have different measures fulfilling his/her requirements.

In a locality/community, some commonly agreed units are used to measure length, weight, area and volume since a long time. These are standardized units within that locality/community or culture. You will find such units in every culture. However, such units are confined to one culture and may not be intelligible in another culture.

ACTIVITY - 1

Prepare a list of non-standard units of measurement use in your locality, the equivalence of each with the respective standard units and the advantages and limitations of those non-standard units for the people of your locality.

....

....

....

Check Your Progress

E1 State any three differences between standard and non-standard units of measurement.

E2 What are the needs for the standard units of measurement?

Notes

ACTIVITY - 2

Use string to measure the distance around your wrist and neck. How many wrists make a neck? Compare your findings with those of your class mates.

....

....

....

ACTIVITY - 3

Draw several different rectangles on a squared paper. State a rule for finding the area of a rectangle given its length and width.

....

....

....

In spite of the advantages of the standard units, at the beginning stage of learning to measure, the non-standard units need to be used. Because of the familiarity with these types of units, the children would know the different ways of comparing and gradually become aware of the need of standard units.

There are mainly two categories of non-standard unit. One category of units varies according to person like the hand or cubit. You can try such units for measuring the length of any object in the classroom (say the length of the table). You can ask the children to measure it by their hand with cubits and fingers as the unit of measurement. Record the length of the table so measured by different children in a tabular form.

Estimation and measurement: Ask the children to guess the length of any one edge of the table in terms of the measure of hand or stick before they actually measure using the particular unit of measurement. Let them realize the accuracy of their guessing or estimating the size. They can be asked to estimate the height of a plant, the capacity of a bucket in terms of the capacity of a given mug etc. How does estimation helps in measuring objects in the long run?

Estimation plays an important role in measurement. It helps in detecting and eliminating the errors committed by the child. By making estimation as a habit before measuring any attribute helps the child in choosing proper units, making mental comparisons of the attributes with the chosen unit of measurement and thus makes the process and outcome of the measurement more meaningful and accurate for the children.

Children could be asked to estimate the measures before they actually measure any particular attribute of the object of measurement because then they will understand the significance of the numbers that they obtain as the result of measurement. For example, if a child measures the length of a desk (which is actually 37cms) to be 63 cms holding the meter scale in the opposite way, then he/she can immediately realize the mistake had he/she earlier estimated its length to be around 40cms.

Check Your Progress

E3. State any two benefits of using non-standard units at the early stage of learning to measure.

E4. Give two instances when the ability to estimate the size is useful.

7.3.1 Measuring Length

As discussed earlier, learning to measure begins with measuring with familiar non-standard units. While children are engaged in measuring with non-standard units, they should be encouraged to estimate and to develop the skills of using the units properly.

Quite a large number of objects from the immediate environment of the child can be used as non-standard units of length like sticks, wires, threads, leaves, tendrils, papers etc . Most frequently the body parts are used as the non-standard units in every culture throughout the world (see the box below). The names of these measures are different as they are called in different languages.

Body parts used as non-standard units of length

***Finger*:** the width across your first finger.

***Hand*: h-** the width across your hand with your fingers together.

***Cubit:*::c** length of your arm from the end of your middle finger to your elbow.

***Pace*:** length of a step or stride.l.eide

***Fathom*:f** distance from fingertips of left hand to right hand with arms outstretched.

***Inch*:in-** distance from the tip of your thumb to the first knuckle.

(Diagram for each measure may be given for each measure 7.3)

In comparison with measurement of area and volume or capacity, the use of large number of non-standard units is much more and more frequent in the measurement of length by all irrespective of age and education. The use of non-standard units at the early stage of schooling is important for a number of reasons:

a. Objects that are familiar to children are used as non-standard units and as such they are not burdened with acquiring new names and terms while getting used to the skills of measuring objects. Further, these units because of their familiarity with children become more meaningful than the abstract units like centimeter, or meter.

b. The non-standard units are more appropriate size of a unit ,for the first practical measuring tasks that young children undertake. Centimeter may be too small and therefore too numerous for the children to measure their familiar objects like length of the desk, height of a friend.

c. The experience of measuring with non-standard units can encourage children to the idea that scales can be invented for a particular measuring task when a standard scale is not available or when the standard scale is inappropriate.

d. Through using non-standard units children become aware of the need for a standard scale. For example, when they discover that the length of the table by using their hand is around 2 cubits becomes one and half cubits when their teacher uses her hand to measure the same length. Measuring the length of various objects around using the body parts as measuring units by all children in the class may help the children to realize that these units give different results when used to measure the length of a particular object by different persons. This realization arouses the necessity for searching for a standard unit for measurement.

ACTIVITY - 4

List the materials available in the immediate environment which can be used as non-standard unit of measuring length.

....

....

....

Standard units of length: At the elementary school level, the standard units of length that are frequently used in mathematics text are meter, centimeter, millimeter and kilometer. At the initial stage, kilometer is a very long distance beyond the normal conception of the child. Therefore, at the stage of introduction of the standard units of length, children should be made familiar with meter and its sub-units centimeter and millimeter.

When and how to introduce the standard units of length to children?

Standard units like meter and centimeter should be introduced only when the children need for a standard scale for measurement. At the first stage, provide sufficient scope for children to practice with the non-standard units to develop the skills of measuring length of different objects i.e. selecting a unit of measurement, comparing the length of the body with that of the unit length and expressing the result of comparison using the number and the unit like 3 cubits, 2 sticks, 5 paces etc.

In the second stage, when the children are skillful in measuring objects with non-standard units and realize that with these units it is not always possible to determine the exact measure of an object and feel troubled to decide which one is the correct measure, at that time non-standard units like the stick with a fixed length can be used. And when they are familiar with the use of stick to measure the length of different entities with very near accuracy, as we have discussed in the earlier example, then the standard scale i.e. meter scale can be introduced. If we introduce the standard unit in this manner, it would be quite meaningful for the children.

Measuring with a meter scale: While the children are practicing to measure with a stick, you should ensure that one end of the stick coincides with one end of the object being measured. In the measurement of the length of the desk, the stick should be placed close to one of the edge (longer edge) of the desk and one end of the stick should coincide with the beginning point of the edge being measured. A mark should be given on the edge of the desk at the other end point of the stick. And after it the stick should be removed and placed coinciding the edge such that the beginning end of the stick should coincide with the mark that you have put on the edge of the desk and put a mark on the edge of the desk at the other end point of the stick. This process should continue in succession till the total length of the edge is measured by the stick. The number of sticks covering the total length of the desk is the measurement of the length of the desk. When the children are well conversant with this process, they are now ready to use the standard meter scale.

(A diagram with two pictures may be given at this place depicting the above mentioned process Fig. 7.4).

Measuring with a standard scale is nearly similar to the process just described above. Some additional care as suggested below need to be taken to enable the children to learn using the scale correctly:

- *Familiarizing children with the marking of sub-units on the scale*. They need to be clear about the division marked on the scale at equal distances. On a meter scale, to begin with, they should be able to identify the centimeter marks. After they become familiar with using centimeter in measuring length, they may be introduced to meter and millimeter.

(Diagram of a meter scale with centimeters marked on one of its edge to be given here Fig.7.5).

- *Appropriately placing the scale with the object of measurement*. Placing the scale correctly and closely alongside the object to be measured is very important.
  - Demonstrate to children to place the scale alongside the object so that the '0' mark of the scale coincide with one end/edge of the object. This is very important for the beginners. They should also be shown how disturbing such placement may change the scale reading of the other end point of the object.

(At least 2 to 3 diagrams to be given here showing the correct coincidence of the '0' point with one end/edge of the object (preferably a pencil) and incorrect placements away from '0' point. Fig.7.6).

- Allow the children to place the scale such that one end point of the object coincides with any point on the scale other than the '0' point (say 1, 2, or 3) and mark the difference in the reading of the scale, coinciding with the other end point of the object.

(appropriate diagram may be given here Fig. 7.7).

- *Estimating the length before exactly determining it.* As discussed earlier, always encourage children to estimate the length of the object being measured before going to determine the exact length of the object using the standard scale.

- *Correctly calculating the length.* The children should be led through several instances of measuring different objects with a ruler or a scale to determine the exact length of the object they are measuring.

  - At first, when the child aligns the '0' point of the scale with one end point of the object being measured, then the figure on the scale coinciding with the other end point of the object determines its length.

  - When the child aligns any point other than the '0'point (say 1, 2, or 3, etc.), then difference in readings on the scale at the two end points determines the length of the scale.

  - While using a particular unit of length for the first time, for example, the centimeter, the children should be provided with materials like wires, rods, sticks, paper strips etc whose length could be measured by complete units (i.e. 3cm, 5cm, 10cm etc.) without using any part of the unit. When they become competent in such measurements, they can be given objects where they can use the sub-units or parts of the unit (for example millimeters).

- *Developing the measuring skills.* Measuring with a standard scale requires some basic skills like alignment of the scale with the object, taking correct readings and calculating the differences of readings. None of these is a difficult skill to acquire, but children very often commit mistakes due to seer carelessness. Taking care at the early stage would help the children in developing these skills with ease.

- *Choosing appropriate ruler or scale.* Scales of different lengths and forms are available in market. Scales of 1meter, 30 centimeter and 15 centimeter lengths are commonly available and used in the schools. Besides, children may be exposed to measuring tapes of different lengths used by carpenters and masons, iron rods used in measuring clothes in shops and such other scales.

At the early stage of measuring length with standard units in the classroom, scales or rulers of 30 cm length would be handy for use by children as this scale would be appropriate for measuring most of the familiar objects around them. Scales or tapes of larger lengths may be required, depending on the length of the objects to be measured. If the length of the classroom or verandah of the school is going to be measured meter scales of larger length need to be selected for use and when small lines drawn on the note books or small objects are required to be measured, then scale of 15cm length would be more appropriate.

Measuring lengths of objects is comparatively easier for children to understand and to perform. You can encourage children in this through different activities in and out of classrooms. Some such activities are as follows:

- Preparing non-standard scales using coloured sticks with equal parts marked with contrast colours.
- Preparing centimeter scales of different measures like 10cm, 15 cm, 20 cm etc. with each centimeter portion of the strip coloured differently from the other.
- Demarcating play field for playing games like Kabdadi, Kho Kho, Badminton etc.
- Participating in long jump, high jump, and measure distances or heights jumped for each jump.
- Drawing designs using straight lines of different lengths.

7.3.2 Measuring Area

Before going to measure the area of an object, let us see what is exactly meant by area. We have seen that the measurement is done on a straight line segment. The line segment which is one dimensional (1-D) could be represented by the edge of a table, a straight wire, lines in a map, a pencil etc. Now, let us consider the figures given below:

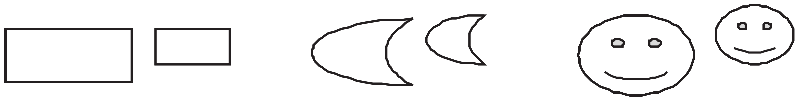
(a) (b) (c) (d)

Fig. 7.8

With our knowledge of measuring length, which aspects/parts of the figures shown above can we measure?

- "The lengths of the lines constituting the boundaries of the figures, of course."
- "But do the lengths of the boundaries describe the shapes and sizes of the figures?"

The boundary of a figure tells about the shape of the figure, but is not sufficient to give any idea of the size of the figure. As for example, compare the pairs of figures given below:



(a) (b) (c)

Fig. 7.9

In each pair of figures, you can find striking similarity in their shapes.

"But what is the difference between the two figures in each pair?"

The two figures in each pair are similar in shapes but are not of same size. If you ask children to identify the bigger of the two in each pair, nearly all of them can point to the bigger one correctly.

When a group of children were asked as to why they chose one to be bigger or larger than the other, some of the responses were:

"One looks bigger than the other."

"When I place one figure on the other, I can distinguish the larger one."

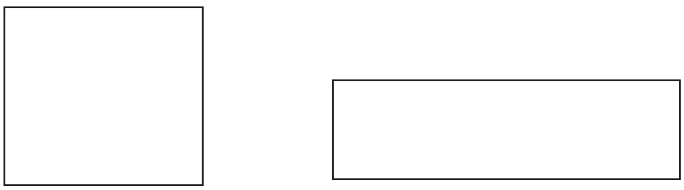
"The one that covers more portion of the paper is the larger one."

"One figure that spreads more on the table than the other is the bigger one."

From the responses, it seems that the children could perceive correctly about the largeness and smallness of figures and are very near to the idea of area.

Area is a characteristic of a plane figure. It is the *spread of the figure* on a plane or the portion of the plane covered by the figure ; the plane may be a paper, the surface of a table or a glass plate or of some such objects.

But, difficulty arises when the children are asked to compare two different figures which are not similar like the two figures given below:



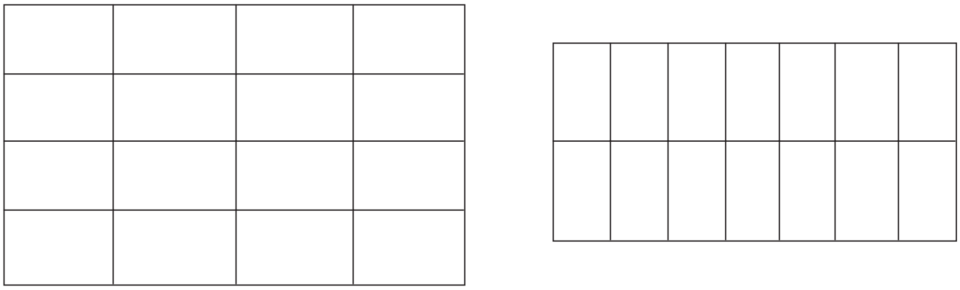
(a) (b)

Fig. 7.10

Notes

How to know which one of the above two figures has more space coverage or, to be specific, has more area?

By superimposing one on the other, we cannot compare their areas as we did in cases of similar figures in the previous example. In such cases, it becomes necessary to estimate the area of each figure by using an object which can be considered to be the unit for measuring area. Let us take match boxes of a particular size and of a definite brand (such a collection would have match boxes of nearly equal surface area). Now place the match boxes close to each other on the two figures such that there would be no gap or no overlapping in between the match boxes as shown below:

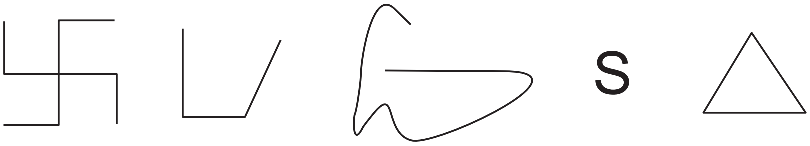


(a) (b)

Fig.7.11

You can see that the figure 7.11(a) is covered by 16 match boxes whereas the other figure is totally covered by 14 match boxes of same size. Therefore, you can now say with reason that the figure 7.11(a) has more area than the figure 7.11(b). Besides comparing the areas of the two figures, we could estimate the area of each figure with the help of a smaller unit like the surface area of a match box.

Can you estimate the area of the following figures?



(a) (b) (c) (d) (e)

Fig.7.12

We cannot calculate the area of these figures as there is no region in the plane bounded by any of these figures. Unlike our concepts of closed figures like triangles, quadrilaterals or circles, these are *open figures* and do not enclose any region.

At the initial stage of learning area of a plane figures, you should use familiar materials like match boxes of any particular brand, plastic squares of equal size, squares of equal size cut out of coloured papers, the books or note books of same size. Some activities of measuring with such objects that can be practiced in the classroom are given here:

- Covering the teacher's table with note books of equal size and calculate how many such note books are required for the purpose.
- Covering the top surface of the students' desks by books of equal size.
- Covering the surface of a book or a note book with the match boxes (you can change the style of placing the match box by changing the faces of the match box and ask the students to calculate the number of match boxes required in each case).
- Decorating a portion of the classroom floor or wall with coloured papers of equal size.

In estimating or comparing areas of plane figures we use smaller units like match boxes of same size such that the figures are entirely covered by the chosen unit. But what happens when some portion of the figure remains uncovered by the unit of measurement i.e. the match box in our example. For this purpose, we follow a common practice i.e. if the portion of the figure is less than the half of one unit we do not take that portion into calculation of area. And when the portion is equal to or more than half of the unit, then it is taken as a full unit for calculation of the area of the figure.

From the above discussion we can conclude that area relates to 2-D objects and to compare or estimate the area of the closed figures, we fill up the figures with smaller units in such a way that no unit should overlap another and no gap should be left in between. Several small plane objects of equal sizes can be used as estimating area measure. In calculation of the area of a figure, the portions of the figure left out after being covered by the full units of measurement are calculated as one unit if the portion is equal to half or more than half of a unit.

Standard Units of Area Measure: The area of a square is considered suitable for any standard unit for measuring area. As for example, a square, each side of which is of length 1metre, is shown in Fig. 7.13.

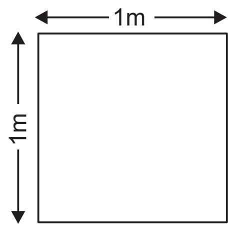


Fig. 7.13 A meter square

Area of such a square is taken as 1 square meter and denoted in symbol as ***1 sq m***, or $\mathbf{1m^2}$.

Note: The area of 1metre-square is *defined as* $\mathbf{1m^2}$.

If you are measuring area of your classroom, the surface area of the wall of your school, the area of a plot in the garden and such other large closed spaces, the unit of 1sq m is the appropriate unit for measuring the areas.

But, for measuring small areas like the figures drawn on your note books, size of the paper required for covering a book, or the size of your handkerchief, you need a

Notes

smaller unit of area as the unit of 1sq m would be too large to measure such small areas. For those purposes **1sq cm** or **1cm^2** would be a befitting unit.

How to determine the areas of geometrical figures using standard units?

Observe the rectangular figure *ABCD* of length measuring 5cm and breadth 3 cm. Since the lengths of sides are measured in cm–scale, the appropriate unit of measuring area in this case would be 1sq cm or 1cm^2.

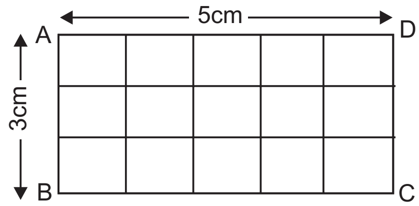


Fig. 7.14

Let us arrange the pieces of square-size papers of area 1sq cm each without overlapping each other nor would having any gap between them, then the picture wil be similar to the Figure 7.14. It requires three rows with 5 small squares (1cm^2 each) in each row. That means we require 15 small squares (5 small squares in each row × 3 rows) to completely cover the rectangle ABCD. Therefore, the area of the rectangle ABCD is 15 time of the unit of 1sq cm or simply 15sq cm or 15 cm^2.

The method of determining the area of a rectangular figure using 1cm^2 as the unit can be as follows:

In order to calculate the area of the rectangle ABCD (Fig.7.14), line- segments are drawn parallel to $\overline{AB}$ and parallel to $\overline{AD}$, such that the rectangle is divided into centimeter-squares.

No. of squares in 1 row = 5 and no. of squares in 3 rows = 5 × 3 = 15

Thus we see that the area of a rectangle = no of units along its length × no. of units along its breadth.

In short, we write:

$$\boxed{\text{Area of a rectangle} = (l \times b)\ \text{cm}^2}$$

Where '*l*' represents the no. of units along the length and '*b*' represents the no. of units along the breadth . In the above figure (Fig. 6.14), $l = 5$ (not 5cm) and $b= 3$ (not 3 cm)

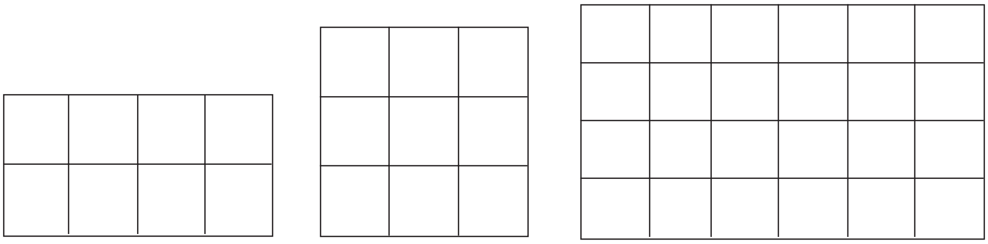
Hence, area of the rectangle ABCD = $(l \times b)$ cm^2 = (5×3) cm^2 = 15 cm^2

Similarly, area of a square = $(l \times l)$ cm^2

Notes

Check Your Progress:

E5. What is the area of each of the following figures where each small square represents the area of $1cm^2$?



(a) (b) (c)

Areas of other geometrical shapes like triangle, quadrilaterals, and polygons are calculated making use of the area-rule for a rectangle.

Area of Irregular Figures: For measuring area of irregular figures, you can use centimeter graphs or centimeter thread graphs. Both of these work on same principle. Place the irregular figure, say a leaf, on the graph and draw the outline of the leaf on the graph (or place the figure under the thread graph (Fig.7.15). To calculate the area of the leaf, count the squares covered in the outline of the leaf. If the outline of the leaf covers half the cell or more than the half, it should be taken as a full square. If it covers less than half of a square, then that square is not taken into account. The total number of squares counted gives an approximate area of the figure in centimeter squares. This is a crude approximation. One of the best ways of finding the area of a irregular closed region is to take the sum of the areas of very large number of vertical or horizontal rectangular strips partitioning the region.

(Please draw the picture as suggested).

Higher and lower area-units: ABCD is a metre-square (Fig. 7.16). It is divided into centi-metre-squares by drawing line segments parallel to $\overline{AB}$ and parallel to $\overline{AD}$ at intervals of 1cm.Thus there are 100 small squares along each of the sides $\overline{AB}$ and $\overline{AD}$ of ABCD.

Notes

$\therefore$ The total no. of small squares in ABCD = $100 \times 100 = 10{,}000$

Each small square is of area $1cm^2$ [as it is a centimeter-square]

Thus we see that $1m^2 = 10{,}000cm^2$

Similarly, it can be proved that $1cm^2 = (10 \times 10)\ mm^2 = 100\ mm^2$

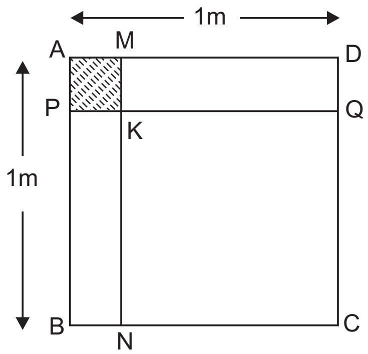


Fig. 7.16

Land measure*:* Usually the standard unit of measuring area of land in the traditional system of measurement is 'Acre'.

In metric system, 'Hectare' is the unit of measuring area of land along with 'Are'. hectare is equal 1hectare = $10{,}000m^2$ and 1 are = $100m^2$. From these you can calculate to find that 1 hectare = 100 are and 100 hectares = 1 km^2. Incidentally**,** 1 hectare = 2.471 acres

7.3.3 Measuring Volume

We have so far discussed about measurement of 1-D (measurement of length), and 2-D (measurement of length as well as area) objects. Let us discuss the measurement of different aspects of a 3-D object. Most of the objects around us have three dimensions – length, breadth and height or thickness. Name any object around you - be it a table, chair, desk, book, ball, bat, pencil, chalk- it must be a 3-D object and each occupies some definite portion of the space.

The amount of space occupied by an object is the volume of that object.

How to determine the quantity of space occupied by an object? Before trying to answer this question, first we should ensure whether children have grasped this sense of volume of an object by simple guessing through visual comparison of objects.

"Does a pencil occupy more space than a ruler?"

"Does a cricket ball occupy more space than a foot ball?"

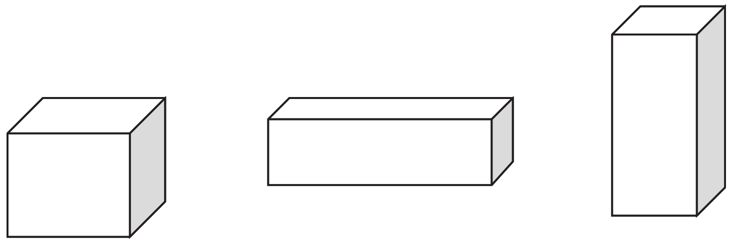
"Does a piece of chalk occupy more space than a duster?"

Notes

"Does a mango occupy more space than a lemon?"

"Does a cow occupy more space than a buffalo?"

After you become sure that children can compare the objects, you can show them some wooden blocks of cubes and cuboids as shown in the Fig 7.17 and ask them to compare the volumes of these blocks.

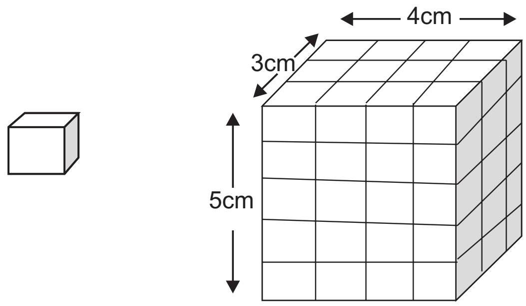


(a) (b) (c)

Fig. 7.17

Here, the children face the difficulty to estimate the volumes of the blocks. For this they need to know the ways of determining the volume of an object.

As in the cases of length and area, the standard unit for measuring volume needs to be introduced. Small unit of cube of 1cm× 1cm ×1cm is usually used as a standard unit for volume measurement.



(a) (b)

Fig. 7.18

This unit cube with each edge measuring 1cm is called a centimeter cube (Fig.6.18a). Its volume is taken as **one cubic centimeter** and is denoted by **1cm^3**.

For measuring large objects, larger unit of volume like **one cubic meter** or **1m^3** may also be used.

Let us see how we can measure the volume of a regular solid like a cuboid as shown in Fig. 6.18(b). As shown in the figure the cuboid is of 3cm× 4cm ×5 cm size. You can observe that there are the cuboid consists of 5 slabs of 1cm thickness and in each slab there are 3 rows with each row having 4 centimeter cubes. That means each slab has ($4 \times 3=$) 12 cm cubes and in 5 slabs there are altogether 60 cm cubes. Hence the volume of a (3cm $\times$ 4cm $\times$ 5cm) cuboid is 60 cubic cm or 60cm^3.

From this example we may deduce that

The volume of a cuboid = ($l \times b \times h$) cubic units, where l= the length, b= the breadth and h = the height of the cuboid.

In a cube, we know that $l = b = h$, hence the ***volume of a cube = l^3 cubic units***.

If we have solid objects in the form of cuboids or are regular 3-D objects, we can calculate their volumes using or appropriately modifying the formula for determining the volume of a cuboid.

There is a common method of measuring volume of any solid object. This is derived from the meaning of volume as the space occupied by the object. If you totally submerge the object in a liquid, it will displace equal volume of the liquid.

ACTIVITY - 5

Take a glass trough or a transparent plastic bucket and fill it with water. Mark the level of water on it. Then take piece of stone or pebble tied with a thread and lower it inside water in the pot. Mark the level of water when the object remains inside water (name it as w_1). Take another bigger piece and submerge it in water inside the pot and mark the level of water in the pot (w_2). Can you find the difference between levels of water (between w_1 and w_2)?

....

....

....

(Please give two diagrams as per the description given in this activity)

Using this property, the volumes of solid objects are measured using cylindrical glass jars with calibration of volume expressed in cubic centimeter (cc). In this method, the calibrated glass jar is filled with water or some other liquid to some extent and the initial level of the liquid is noted. Then the object which volume is to be determined is totally submerged in the liquid and the level of the liquid in the jar is noted. The difference between the two levels gives the volume of the object.

The other way is to fill any vessel completely with water or any liquid in such a way that any additional drop of the liquid will overflow out of the vessel. Then submerge the object into the liquid in the vessel and take care to collect every drop of the liquid displaced out of the vessel due to the submergence of the object. The volume of the displaced liquid is equal to the volume of the object.

Capacity and Volume: We discussed the ways to measure volume of the solid objects by the method of submergence in liquid. But, this method cannot be used for measuring the volume of liquids. Moreover, liquid substances have no definite shape. They take the shape of the vessel in which they are kept. The capacity of a vessel or a container refers to the volume of liquid, or sand, or salt or some such substance that it can hold. If a bucket can be filled completely with 20 bottles (of equal size) of water, then the capacity of the bucket is 20 such bottles of water. And if the capacity of each such bottle is 1 liter, then the capacity of the bucket is 20 liters.

At the initial stage of learning, the children should be given a lot of opportunities to use non-standard units of measuring capacities of different vessels. Some such activities that can be conducted by children in and out of school are:

- Using cups to fill the tea pots /kettles, or any other pots used as utensils with water.
- Filling pots with water using a fixed bottle.
- Filling drinking water storage in the school with water using a bucket.
- Filling sugar in the can using a small cup.
- Measuring sand with a tin can.
- Measuring volume of rice/paddy/wheat/any seeds with a small tin/plastic can.

While practicing with the non-standard units like spoon, cup, jug, tin can, plastic mug etc, always insist on the use of one measure in measuring a particular item. You can try different cups to measure an amount of rice, and also measure the same amount of rice with only one cup. You can find the difference in the two measures.

It is only when the children become quite adept in using non-standard units of measuring volumes, then the standard units of cubic centimeter (cc) and liter can be meaningfully introduced. They may be familiarized with the devices used to measure oil in the retail grocery shops.

The standard unit of measuring the liquid is a **litre** or **liter**. One litre is equal to 1000 cubic centimeters or 1000 cm^3.

E6. What is the standard unit that you will use while measuring the volume of a solid by measuring that of the displaced water by the solid?

E7. A water reservoir is 3 m long, 2 m broad and 1m high. How many litres of water does it hold?

7.3.4 Measuring Weight

The children, from a very young age, are familiar with the process of weighing the objects by experiencing heaviness of an object the activities in the market. The child might have experienced the weighing of rice, vegetables, groceries and other food items using standard weights and common balance. But, at the beginning stage, children should be familiarized with the process of weighing with a balance using non-standard weights like small stones, pieces of brick, wood, iron or any metal. At this stage children may be encouraged to perform the following activities:

- *Preparing a model of common balance with a beam and two pans hanging from the two extremes of the beam*: A thread is attached at the exact middle point of the beam. When equal weights are placed on the two pans the beam remains horizontal to the ground when it is raised with the thread at the middle of the beam.

- *Weighing with non-standard units:* Children should be encouraged to use the improvised balance they have made in weighing different materials like sand, leaves, seeds, etc. with non-standard units. Through such activities, they would develop the skills of using the balance properly in two main ways. First, they would master the mechanics of the balance like properly lifting the balance, putting the weight and the things to be weighed in proper pans, and keeping the beam horizontal by adjusting the materials being weighed. Second, using balance they can divide and subdivide the materials into two, four or eight equal parts.

When they become well versed with using the improvised scales and non-standard weights, they will feel the need of a proper balance and the standard weights. As they grow up, they should be exposed to different types of weighing devices like spring balance, electronic weighing machines, machines to weigh very small and very large weights.

Gram and kilogram are commonly used units to measure weights of familiar objects. Children are more exposed to weighing vegetables and groceries in kilograms, half-kilogram (500grams) and quarter kilograms (250 grams).

7.4 METRIC SYSTEM OF MEASUREMENT

There are two major systems of standard measuring units used in different countries. These are the Metric System and the British or Imperial System. In every system there are two types of measuring units – *the base units* and *the derived units.* The units for length, mass, and time, (along with those for temperature, electric current, luminous intensity and amount of substance) are called the base units because the units for other measures like area, volume, capacity, and velocity etc. can be expressed in terms of these base units. For example, the unit of area can be cm^2, that of volume can be cm^3 or that for velocity can be kilometer per hour (expressed shortly as km/hour). Using these base units of measurement, the Metric system is also called **c-g-s** (centimeter-gram-second) **system** or sometimes as **m-k-s** (meter-kilogram-second) **system.** Similarly, the British system is known as **f-p-s** (foot-pound-second) **system**.

Metric system is internationally accepted system and is known as ***'International System of Units*** or ***SI Units'*** (***SI*** stands for the French version of the International System of Units i.e. ***Système international d'unités).*** The SI includes two classes of units which are defined and agreed internationally. The first of these classes are the seven SI *base units* for length, mass, time, temperature, electric current, luminous intensity and amount of substance. The second of these are the SI *derived units*. These derived units are defined in terms of the seven base units. All other quantities (e.g. work, force, power) are expressed in terms of SI derived units.

In this section we shall be confining our discussion on the SI units of length, capacity and mass (commonly used as weight). The units of area and volume are closely associated with and are reported using the unit of length as indicated earlier in this unit. At times, we have tried to compare these measures with the corresponding units of traditional and the British system as in some areas of measurement in our country those systems are also used and makes sense to common people.

The History of Metric System

The Metric System was developed in France during the sixteenth and seventeenth century and Gabriel Mouton, the vicar of St. Paul's Church in Lyons, France, is the "founding father" of the metric system. He proposed a decimal system of measurement in 1670.

In 1790, during the hectic period of the French Revolution, the National Assembly of France requested the French Academy of Sciences to "deduce an invariable standard for all the measures and all the weights." The Commission appointed by the Academy created a system that was, at once, simple and scientific.

- The unit of length was to be a portion of the Earth's circumference.
- Measures for capacity (volume) and mass were to be derived from the unit of length, thus relating the basic units of the system to each other and to nature.

- The larger and smaller multiples of each unit were to be created by multiplying or dividing the basic units by 10 and its powers.

The Commission assigned the name "metre" (in the U.S. spelled "meter") to the unit of length. This name was derived from the Greek word, metron, meaning "a measure." The physical standard representing the meter was to be constructed so that it would equal *one ten-millionth of the distance from the North Pole to the equator along the meridian running near Dunkirk on English Channel in France and Barcelona in Spain.*

A surveying team under the direction of two men, Pierre-Francois-Andre Mechain and Jean- Baptiste-Joseph Delambre, spent 6 years in measuring the "arc" that the earth made in a line between Dunkirk in France on the English Channel and Barcelona in Spain. The surveyors underwent much harassment and even were jailed, at times, while making their measurements, because some of the citizens and area officials resented their presence and felt they were up to no good. It was later found that Delambre and Mechain had not properly accounted for the earth's flattening in correcting for oblateness. However, the meter remains the invariable standard for the metric system, and its length has not changed even though the official expression of the definition the meter has changed several times to improve the accuracy of its measurement.

Meanwhile, scientists were given the task of determining the other units, all of which had to be based upon the meter.

The initial metric unit of mass, the "gram," was defined as the mass of one cubic centimeter (a cube that is 0.01 meter on each side) of water at its temperature of maximum density (about 4^0C). For capacity, the "litre" (spelled "liter" in the U.S.) was defined as the volume of a cubic decimeter — a cube 0.1 meter on each side.

After the units were determined, the metric system underwent many periods of favor and disfavor in France. Napoleon once banned its use. However, the metric system was officially adopted by the French government on 7 April 1795. A scientific conference was held from 1798 to 1799 (with representatives from the Netherlands, Switzerland, Denmark, Spain, and Italy) to validate the metric system's foundation and to design prototype standards. Permanent standards for the meter and the kilogram were made from platinum. These standards became official in France by an act of 10 December 1799.

Although the metric system was not accepted with enthusiasm at first, adoption by other nations occurred steadily after France made its use compulsory in 1840. Most of the countries around the world adopted this system around 1950 and 1960's. There are a few countries including U.S.A. which have not adopted the metric system as yet.

The metric system in weights and measures was adopted by the Indian Parliament in December 1956 with the *Standards of Weights and Measures Act*, which took effect from October 1, 1958.

The Units in the Metric System: In metric system, the sub-units (smaller units) are taken as $\frac{1}{10}, \frac{1}{100}$ etc. of the unit and the super-units (higher units) are taken as 10 times, 100 times etc. of the basic unit. This makes the calculations convenient.

The system is as follows:

Higher Units

Kilo unit = 1000 units

Hecto unit = 100 units

Deca unit = 10 units

Unit

Sub-Units

Deci unit = $\frac{1}{10}$ units

Centi unit = $\frac{1}{100}$ units

Milli unit = $\frac{1}{1000}$ units

Let us see how this structure of the metric units are uniformly used in defining the units of measurement in length, capacity and mass (or weight)

***Unit for Length Measure*:** Unit of length measure the metric system is **metre or meter.**

Higher Units

Kilo metre (km) = 1000 m

Hecto metre (hm) = 100 m

Deca metre (dam) = 10 m

Unit : Metre (m)

Sub-Units

Deci metre (dm) = $\frac{1}{10}$ m

Centi metre = $\frac{1}{100}$ m

Milli metre = $\frac{1}{1000}$ m

From among these, kilometer, metre, and centimetre are frequently used in measurement of lengths of varying distances and commonly understood.

Notes

Although metric system has been adopted in our country since 1958, yet the unit of length like inch (=2.54 cm), foot (=12 inch) and yard (=3 feet) are still used in several events of measurement like in land measurement and also by tailors for measuring cloth for making dress.

***Units for capacity measure*:** It is a common experience that liquid has no shape. It takes the shape of the container in which it is kept. Thus for deciding the unit for measuring the quantity of liquid we either use weight measure unit or we use volume measure unit.

Litre is the unit of capacity measure (i.e., volume-measure).

The volume-unit is known as 1 litre (equal to 1000 cm^3). Different containers are made of different capacity measures.

Higher Units ↑

Kilo litre (*kl*) = 1000 *l*

Hecto litre (*hl*) = 100 *l*

Deca litre (*dal*) = 10 *l*

Unit: Litre (*l*)

Sub-Units ↓

Deci litre (*dl*) = $\frac{1}{10}$ *l*

Centi litre (*cl*) = $\frac{1}{100}$ *l*

Milli litre (*ml*) = $\frac{1}{1000}$ *l*

Litre is the most frequently used unit to measure the liquid substances like milk, water and oils

Units for measure of Mass (Weight): The basic unit is **gram**. Other higher and sub-units are given below.

Higher Units ↑

Kilogram (kg) = 1000gm

Hectogram (hm) = 100g

Decagram (dag) = 10g

Unit: Gram (g)

Lower Units ↓

Decigram (dg) = $\frac{1}{10}$ g

Centigram (cg) = $\frac{1}{100}$ g

Milligram (mg) = $\frac{1}{1000}$ g

From among these, kilogram, gram and milligram are commonly used.

Besides these units of mass, 'quintal' and 'metric ton' are used to weigh very heavy materials. One quintal is equal to 100 kilograms and 1 Ton = 1000 kg or 10 quintals.

Check Your Progress:

E8. State the major advantage of the Metric system over the British system of measurement.

E9. If 1 kg of rice costs Rs.25, what is price of 5 quintals? If this quantity of rice is packed in smaller packets each containing 20kg, how many packets can be made?

7.5 MEASUREMENT OF TIME

Time-measure is related to the revolution of the earth about its axes and rotation of the earth around the sun.

The duration between two consecutive sunrises is commonly known as **a day**. But the scientists count a day as the duration between two **consecutive mid-nights**. Thus, a day starts at a mid-night and ends at the next midnight. This duration is known as a **solar-day**.

A solar day is the time which the earth takes to rotate once about its axis.

A complete rotation in degree- measure system, has a measure of 360 degrees.

To bring in coherence between the time-measure and degree measure (as these are both related to rotation), unit divisions are made on the basis of sub-multiples of 360.

Thus, the duration of a solar day is divided into 24 equal divisions and each division is known as an hour. 1 hour = 60 minutes and 1 minute has been taken to have 60 seconds.

Thus :

1 solar day = 24 hours

1 hr = 60 minutes

1 minute = 60 seconds.

Solar year : *The duration in which the earth completes one rotation about the sun is known as a*

Relation between a solar year and a solar-day.

1 solar year = 365 days 5 hours 48 minutes 47 seconds
Roughly, 1 solar year = 365¼ days

A calendar year is taken to have 365 days

Thus every year we lose about 6 hrs i.e., ¼ of a day.

Hence in 4 years we lose 1 day. To make it up, in every 4 years, one year is taken to have 366 days and this year is known as a leap year.

The year number which is divisible by 4 is a leap year and the month of February is taken to have 29 days instead of 28 as in case of other years. So in a calendar, we have 31 days in each of the moths of January, March, May, July, August, October and December; 30 days in each of the moths of April, June, September and November, 28 days in February with one extra day (i.e. 29 days) in every leap year. The children can easily remember the days of the calendar months in different ways. Singing a song on the months may be one interesting way.

A Song of the Months

Thirty days hath September

April, June and November

All the rest have thirty one

Except for February alone

And that has twenty-eight days clear

And twenty-nine in each leap year

Exceptions :

The year numbers that have zeroes at the ten's place and unit's placeand are merely divisible by 4 only are not leap years. But from among them which are multiples of 400 are leap years.

Thus, 2000 was a leap year, whereas 1900, 1800, 2100, 2200, 2300 etc. are not leap years.

Clock time : There are two types of clock: 12 hour-clock and 24 hour-clock.

- Ordinarily we use 12 hour- clocks. In such a clock, the numbering of hours on the dial of the clock are limited within 1 to 24. The hour hand rotates once over the clock-face in 12 hours and the minute hand rotates once in 1 hour.
- Mid-night and noon are indicated by 12. We say: 12 Mid-night, and 12 Noon.

  The time strictly between 12-midnight and 12-noon is indicated as **am,** such as in the morning we say 5 am or 6.30am or 8 am and the time strictly between 12-

noon and 12-midnight is indicated by **p.m.** like 4 pm in the afternoon**,** 7pm in the evening.

- 24 hour clocks are used in railways and airways.

  Midnight is indicated as 24hr and the subsequent hours are counted as 1 hr., 2 hr., 3hr., and so on till the following midnight. There is no use of *am* and *pm*. In this system, the clock-face shows the numbers from 1 to 24 and the hour hand rotates once over the clock face in 24 hours.

Time Sense: Before the child could read the clock and calendar, he/she should have developed a sense of time i.e. the sense of past, present and future. When the children come to the school for the first time at about 6 years of age, they have developed such sense to a considerable extent. They can talk in terms of what happened yesterday or last week, what they are doing now and what they are going to do tomorrow. Beginning with the idea of yesterday, today and tomorrow, you can lead them to express in terms of last month or last year or some years back arrange the events in terms of the date of their occurrence. Similarly, you can discuss with them regarding the events that are going to happen tomorrow, next week, next months and in years to come. For all these purposes, you can in different times involve them in formal or informal discussions on the questions related to events occurring in past, present and future. Some such questions are:

- What is the day to-day?
- What was it yesterday or what it will be tomorrow?
- Who are absent in your class to-day?
- Who were absent yesterday?
- In which period tomorrow mathematics is going to be taught in your class?
- What day is to-day?
- What day will be tomorrow?
- Who is older, you or your friend?

The more you discuss with children along these lines, you will be able to sharpen their sense of time. In addition to such discussions, provide situations when they can be able to arrange three or more events in serial order of time, beginning from the distance past to immediate past and proceed to immediate and distant future. In other words, they try to arrange events on a time line. Once they understand and successfully arrange the events chronologically, you can then proceed to acquaint them with clock and calculation of time.

Instant and Duration of Time

The recording of time depends on your requirement of the type of period of events you want to record. For example study the three situations:

1. How many years have passed since India got independence?
2. What is the difference in the ages of two of your friends Seema and Sehna?
3. How much time did Rohit take in answering the classroom test completely in Mathematics?

The first question requires knowing the year in which India got its independence (1947A.D.) and the present year from which the difference is to be calculated (say 2011A.D.). Simply deducting 1947 from 2011 you get the answer.

The second situation requires the exact dates of birth in terms of the year, month, and date of birth of each of the two friends. It is slightly more complex than the first one.

Suppose the dates of birth of the two friends are as follows:

Seema was born on 18th Ocober 1999 and Sehna was born on 12th September 2000.

You can calculate the difference in their ages in two different manners: (i)Calculate the present age as on a definite date (say, 1st. April, 2012) and find the difference between their present age, (ii) Calculate the differences between their dates of birth which would be equal to their age at any instant of time. We have taken the second method to calculate the difference in their ages.

To find out the difference between their dates of birth you arrange their dates of birth as shown below and calculate the difference in their

	Year	M	Day
Sehna:	2000	09	12
Seema:	1999	10	18
	00	10	24

You can explain how to calculate the difference when months and days are involved. Since in this case 12 days are less than 18 days, therefore, we have to borrow 1 month (=30days) from the 9 months and calculate the difference between(30+12 =) 42 days and 18 days to be 24 days, Again, we are left with 8 months from which 10 months cannot be deducted and hence we borrow 1 year(=12 months) and subtracting 10 months from (12 + 8 = 20) months we get 10 months. Thus we get the difference between the dates of birth of two friends and on that basis we can say that Seema is 10 months 24 days older than Sehna.

In brief, in order to help your students to develop skill in measuring time, you need to plan several types of activities that include reading the clock accurately, stating the time in am and pm, calculating difference of time between two events.

Check your progress

E10. Rama left for her hostel in her college on the morning of 11.11.1911 and is likely to come back at the night of 12.12.2012. How many days would she be absent from her home?

E11. From among the following years, identify the leap years.

1536, 1600, 1682, 1700, 1820, 1980, 2000, 2006, 2012.

E12. On the 10^{th} January 2008, the repair work of a school building started and continued for a period of 65 days. On which day was the work completed?

7.6 LET US SUM UP

- Measurement is quantification of some particular aspects of objects arrived at through comparison of the particular aspect with another similar object of fixed dimension(a unit).
- Length is one dimensional, while area and volume are associated with 2-D and 3-D objects respectively.
- While observation, superimposition and indirect methods are used for measuring objects, non-standard and standard units are predominantly employed for the purpose.
- Non-standard units using body parts or locally available materials may not be as accurate as the standard units, but are more meaningful and beneficial for the beginners to anchor the concepts of units and processes of measurement.
- Before acquainting with the standard units of measuring length, area, volume, capacity and weight, children should be made familiar with the processes of measuring the attributes. When standard units are introduced, children should be encouraged to employ their experiences gained by using non-standard units.
- Every child in elementary schools should be facilitated to acquire skills in using the standard metric units of length, area, volume, capacity and weight after gaining knowledge of the units, sub-units, and higher units in each of the aspects.
- Metric system has been accepted as the international standards (SI) of measurement with seven *base units* for length, mass, time, temperature, electric current, luminous intensity and amount of substance.
- Time is measured in terms of solar year, solar months, days, clock hours, minutes and seconds.
- Children should be provided scope for developing skills in recording time of events and calculating duration of events.

7.7 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. State any three differences

E2. They are accurate, uniform across the world, and are used in scientific measurements.

E3. State the examples

E4. Non-standard units are familiar to children, hence they are meaningful and can be easily handled by children before they could try standard scales.

E5. (A) 8cm^2 (B) 9cm^2 (C) 24cm^2

E6. Cubic centimeter (not in litre as it is the unit of capacity measurement or of measuring liquids)

E7. 6000 litres

E8. In Metric system the higher units and sub-units are stated in multiples or sub-divisions of 10 of the main unit of measurement.

E9. Rs. 12500.00; 25 packets

E10. 398 days

E11. 1536, 1600, 1820, 2000, and 2012.

E12. On 14th March, 2008

7.8 SUGGESTED READINGS AND REFERENCES

IGNOU (2008). *Teaching of primary school mathematics(vol.5): Measurement*. New Delhi: IGNOU.

7.9 UNIT-END EXERCISE

1. Describe the methods of comparison as a means to measuring different objects.
2. Compare nature and utility of the non-standard and standard units of measurement with suitable examples.
3. Why the metric units of length, mass and time are considered as the SI base units and other units like those of area, volume, capacity and mass as derived units?

UNIT 8 DATA HANDLING

Structure

8.0 INTRODUCTION

You know that for planning developmental activities, information or data on several aspects like land, revenue, agricultural products, manpower etc. are collected in regular intervals and records in respect of those information are maintained at different levels. In your daily life, you must have seen several kinds of data consisting of numbers, figures, names etc are also collected and used. Collecting data, arranging or processing them and drawing inferences from these data and using those to solve different problems have been regular features in the management of all developmental activities including school education. Children too are often engaged in activities that require basic knowledge of handling data. They also often generate data through their own games and activities. Thus, we need to help children acquire some techniques which will be useful in handling data.

In this unit, we will discuss about the basic ideas involved in introducing the handling of data required for primary classes.

For completing this unit, you shall need about 07 *(seven) study hours*.

8.1 LEARNING OBJECTIVES

After studying this unit, you will be able to

- describe how to collect data and classify these data;
- represent data in tables and graphs;
- prepare different types of graphs of data such as Pictograph, Bar graphs, Histogram and Pie chart; and
- draw inferences from representation of data.

8.2 COLLECTION AND REPRESENTATION OF DATA

In our daily life, there is a need to collect information to fulfill our various requirements. You need to plan for your monthly expenditure when you get your monthly salary. Usually you list the different items of expenditure like purchasing daily commodities, paying electricity bills, purchasing dresses, tuition fees of children, medical expenses, and travel expenditures. For preparing the estimates on these items you usually go through the expenditures on these aspects during last few months. In short you are collecting information on the expenditure on different heads for the last few months to plan for the recent month.

ACTIVITY -1

Prepare a list of information that you need to collect in your school daily and annually.

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As a teacher you are collecting some information several data on daily, weekly, monthly, quarterly and annual basis like number of students enrolled in each class during an

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academic year, number of boys, girls in each class, number of SC and ST children in each class, number of students present on each day, number of working days in a month, monthly expenditure on salaries etc. All these information are expressed in numbers. Such numerical description of the required information is called data. Thus a data is a collection of numbers gathered to give some useful information.

The data may be collected from different sources directly and indirectly. The collected data are to be recorded in a systematic manner which would be convenient for further use. We have to learn how to collect data, tabulate and put them in pictorial form. The collection, recording and presentation of data help us in organizing our experiences and draw inferences from them.

8.2.1 Collection of Data

Data are information collected in a systematic manner with the aim of deriving certain related conclusion. Let us discuss the sources for collection of data. Consider the following examples:-

Ex.1: The students of classVII are preparing for a picnic. The class teacher asked them to give their choice of fruits out of apple, orange, banana or guava. The teacher prepared a list by asking individually the choices which will help the teacher to distribute fruits according to the choice.

Here the data are collected directly from the source i.e., from the students. This is an instance of the data being collected from the *primary source*.

Ex.2: Suppose we want to know the number of persons in various income groups in a town/village.

Here the sources of information about the income groups are the records (census report) available in the Municipality/ Panchayat office. This is not a direct source of information. Thus the data which are collected indirectly i.e., from the documents containing the information collected for some other purpose. Such indirect sources are called *secondary source.*

The collected data from different sources which are not arranged or organized in any manner are called *raw data*.

Ex .3: Runs scored by Tendulkar in different innings of a Test series are 16, 56, 25, 8, 3, 33, 23, 107.

Here the runs are not organized in any manner and hence are examples of raw data.

The data are arranged in groups on the basis of certain aspect is called distribution. The number indicating different aspects of information such as marks, age, height, income etc. are called the scores of the distribution. The raw data is to be organized after it is collected.

8.2.2 Tabular Representation of Data

If the raw data consists of few scores, then we arrange them in ascending (increasing) or descending (decreasing) order. Then it is called *arrayed data*.

Ex.4: Raw data: Marks secured by 12 students in a Unit Test (Full mark- 25)

16, 7, 23, 10, 18, 9, 21, 20, 12, 17, 16, 21

Arrayed data: 7, 9, 10, 12, 16, 16, 17, 18, 20, 21, 21, 23 (arranged in an increasing order).

ACTIVITY - 2

(1) Arrange the above data in decreasing order.

(2) Arrange 88, 25, 16, 43, 7, 70, 16, 34, 61, 52, 97 in ascending and descending order.

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N.B. Equal marks are kept successively.

It is difficult to draw inference when a large no. of data are arranged in an arrayed form. So, we have to arrange the scores in different ways where a precise picture of the distribution becomes clear and drawing inference becomes easier. Let us now think of the following example.

Ex. 5: Esma collected the data for size of shoes of students of her class. The different sizes of shoes are as below:

5	4	7	5	6	7	6	5	6	6
4	5	6	8	7	4	6	5	6	4
5	7	6	7	5	6	6	4	8	7

Esma prepared the following table using tally marks

Table 8.1 Size of Shoes

Size of Shoes	Tally mark	No. of students (f)
4	卌	5
5	卌 ||	7
6	卌 卌	10
7	卌 |	6
8	||	2
	Total	**30**

Now drawing inference from the table will be easier.

Do you know?

The mark (bar) used to represent the no. of occurrence of the score is called tally mark.

The no. of occurrence of a particular score or a group of scores is known as ***frequency (f)*** of the score or group of scores.

The above is an example of an ***ungrouped frequency distribution***.

ACTIVITY - 2

Collect the following data and prepare frequency distribution table:

(a) Age of the students in a class.

(b) Height of the students of a class

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Let us observe another type of arranging the raw scores.

Ex. 6: The marks secured by 40 students of Class VI in a test of Mathematics are:

8, 48, 55, 52, 78, 42, 93, 85, 7, 37, 94, 66, 72, 73, 66, 91, 52, 78, 85, 9, 68, 81, 64, 60, 75, 84, 78, 10, 63, 21, 14, 30, 19, 25, 93, 33, 15, 29, 25, 13

Arranged in an ascending order, the distribution of scores become (in Arrayed form):

7, 8, 9, 10, 13, 14, 15, 19, 21, 25, 25, 29, 30, 33, 37, 42, 48, 52, 52, 55, 60, 63, 64, 66, 66, 68, 72, 73, 75, 78, 78, 78, 81, 84, 85, 85, 91, 93, 93, 94.

Arranging the large number of scores in such a fashion does not help us very much for observing any trend. Similarly, arranging these scores in an ungrouped frequency distribution would be of very limited utility for us.

Let us try another mode of arrangement of these scores:

In this arrangement, instead of arranging the scores individually with their respective frequencies, we make groups (or pockets) of scores and place the scores of this distribution in those groups. For example, let there be a group of 10 scores as 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19 or shortly we can represent the group as 10-19 otherwise called a ***class interval (C.I.).*** The *size* or the *length* of this class interval is 10 i.e. the number of consecutive scores constituting the C.I.

How many scores of our distribution are there within this class interval? There are only 5 scores of the distribution i.e. 10, 13, 14, 15, and 19 within the class interval of 10-19.

The process of arranging the scores of the distribution is as follows:

In this distribution, the scores range from minimum 7 to maximum of 94.

Range of scores in the distribution or simply Range = Highest score – Lowest score = 94-7=87

The range helps us to decide the no. and length of the class intervals (C.I).

If we take the length of the C.I. to be 10, then the no. of C.Is. would then be 10.

Table 8.2 Grouped Frequency Distribution

<table>
<tr><th>Class Interval (C.I.)</th><th>Tally Marks</th><th>Frequency (f)</th></tr>
<tr><td>0-9</td><td>|||</td><td>03</td></tr>
<tr><td>10-19</td><td>~~||||~~</td><td>05</td></tr>
<tr><td>20-29</td><td>||||</td><td>04</td></tr>
<tr><td>30-39</td><td>|||</td><td>03</td></tr>
<tr><td>40-49</td><td>||</td><td>02</td></tr>
<tr><td>50-59</td><td>|||</td><td>03</td></tr>
<tr><td>60-69</td><td>~~||||~~ |</td><td>06</td></tr>
<tr><td>70-79</td><td>~~||||~~ |</td><td>06</td></tr>
<tr><td>80-89</td><td>||||</td><td>04</td></tr>
<tr><td>90-99</td><td>||||</td><td>04</td></tr>
<tr><td></td><td>Total (N)</td><td>40</td></tr>
</table>

This is an example of a ***grouped frequency distribution***. Here the length of the class interval, as explained above, is 10. This can be determined in another way. For example, let us take any class interval say, 60-69.

The upper and lower limits of the C.I. are respectively 69 and 60.

$\therefore$ The length of the C.I. = 69-60+1= 10.

You can observe that the lengths of all class intervals in one grouped frequency distribution are equal.

Sometimes, the class intervals are represented in the following manner:

0-10 in place of 0-9

10-20 in place of 10-19

20-30 in place of 20-29

30-40 in place of 30-39 and the like.

In such type of arrangement, the score indicating the upper limit of each C.I. is not included in that C.I.

Further, the length of the C.I. in this case is equal to the difference of the upper limit and the lower limit of the C.I. For example, the length of the C.I. 30-40= 40-30=10.

ACTIVITY -3

Measure the weights of the students of Class VII of your school. Prepare a suitable of grouped frequency distribution table of their weights.

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8.3 PICTORIAL DEPICTION OF DATA

You know pictures are generally eye-catching, easier to understand and leave a more lasting impression on the mind of the observer. Hence a variety of methods have been developed to present numerical data pictorially/graphically. In this section we discuss four such representations i.e. the pictographs, bar graphs, histogram, and pie charts.

8.3.1 Pictograph

We can use picture symbols to denote numerical data. These pictographs have an immediate visual impact. Let us present the following numerical data in a pictograph.

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Ex.7 The number of girls studying in different classes of a school are given below.

Table 8.3 No. of Girls in the School

Class	I	II	III	IV	V
No. of girls	25	20	30	10	15

The above data is presented by pictograph as follows:

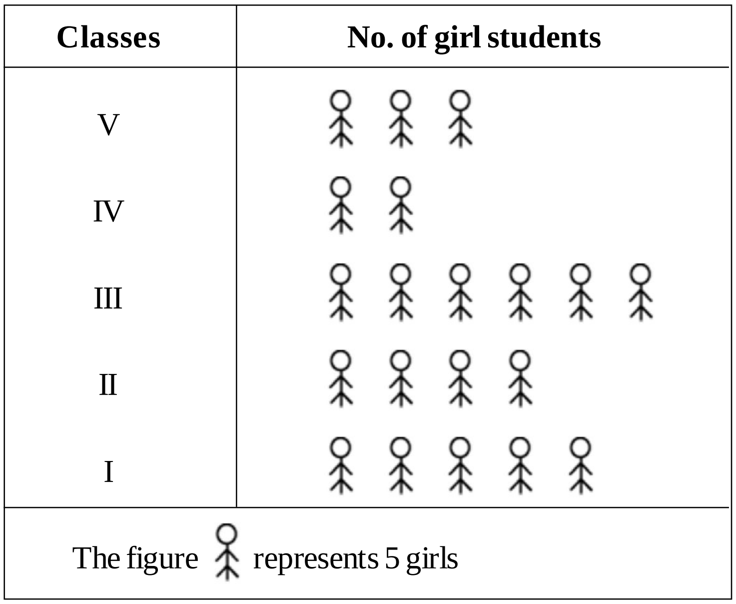

Fig. 8.5 Pictograph of No. of Girl Students in the School.

Pictographs give a visual impression about the distribution.

N.B : (i) Pictographs are often used in magazine, newspaper etc.

(ii) Drawing pictograph is time consuming.

Drawing a Pictograph: Sometimes in a pictograph, a symbol is used to represent one or more objects and may be difficult to draw. In such cases simple symbols may be used. For example, if represents 5 students then will represent 4 students, represents 4 students, represent 2 students and represents 1 student.

Thus, the task of representation will be easier.

ACTIVITY - 4

(*a*) *Collect 2 to 3 pictographs from the newspapers or magazines and display those in your class.*

(b) *Draw a picture of no. of absentees in different classes of your school on any day.*

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8.3.2 Bar Graph

Drawing pictograph is sometimes difficult and time consuming. Let us look for some other way of representing numerical data visually. The bar graph on the data used in Ex-7 can be shown as :

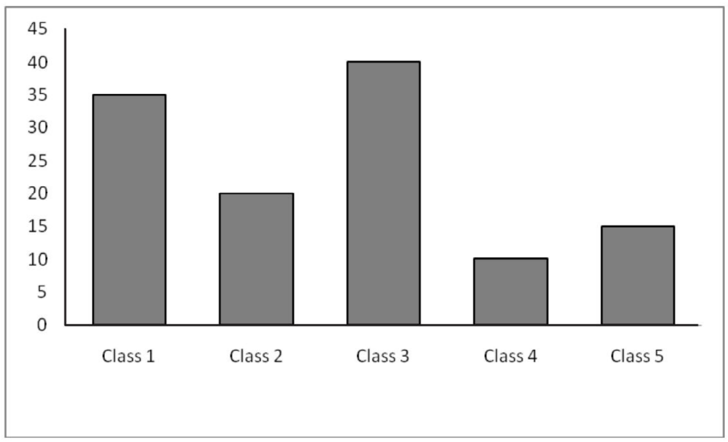

Fig. 8.6 Class-wise No. of girls in the school

Let us observe the above graph keenly. You may have the following observation:

Bars of uniform width are drawn vertically.

Gaps left between the pairs of consecutive bars are of equal width.

The length of each bar represents the given nos.

Such a figure of representing data is called bar graph or bar diagram.

Do you know?

Bars can be drawn horizontally also by changing the axes.

All bars shall have same colour or shaded in the same way.

Steps to draw a bar graph :

Step-I : Draw a horizontal line and a vertical line.

Step-II : On the horizontal line put marks (scores) to draw bars representing each item.

Step-III: On the vertical line write numerals representing no. of items. (after selection of suitable scale)

Step-IV: Use columns of equal width to represent the item.

Step-V: The column representing the same item should be shaded in the same way.

ACTIVITY - 5

The number of students who remained absent in your school during a week is given below. Prepare a bar graph on this data.

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of Absentees	*25*	*7*	*16*	*11*	*9*	*18*	*20*

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Multiple bar diagram:

Sometimes two or more sets of data are given for visualizing a comparison among those three. Let us discuss the following example where the performance of a student on different subjects taught in Class VII in Half-yearly and Annual Examinations have been given. The graph would demonstrate the change in performance on the subjects during two examinations.

Ex.8

Table 8.4 Marks in Different Subjects

Subjects / Exam.	**M.I.L.**	**English**	**Maths.**	**Science**	**Social Study**
Half Yearly	61	65	90	70	58
Annual	68	72	80	57	76

The bar graph of the data consists of two bars for each subject. To differentiate the two exams, we use different colours or different shades for two bars under each subject.

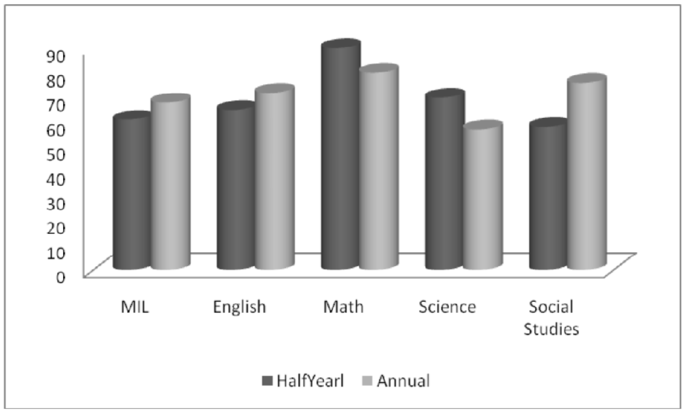

Fig. 8.7 Marks in Half- yearly and Annual Examinations

N.B. You can also use graph paper to draw this graph.

8.3.3 Histogram

If the data is a grouped frequency distribution where the frequencies of the groups (classes) with continuity are given, then there is no justification of having a gap between two successive bars. So the bars are drawn consecutively. Such type of graph is known as Histogram. Let us discuss the following example.

Ex. 9. A histogram has been drawn using the frequency distribution of marks of a Unit Test in Maths of ClassV students.

Table 8.5 Distribution Marks on Class V Mathematics Test

C.I. of Marks	0-5	5-10	10-15	15-20	20-25
No. of students	5	7	10	12	6

The Histogram for the given data is shown below

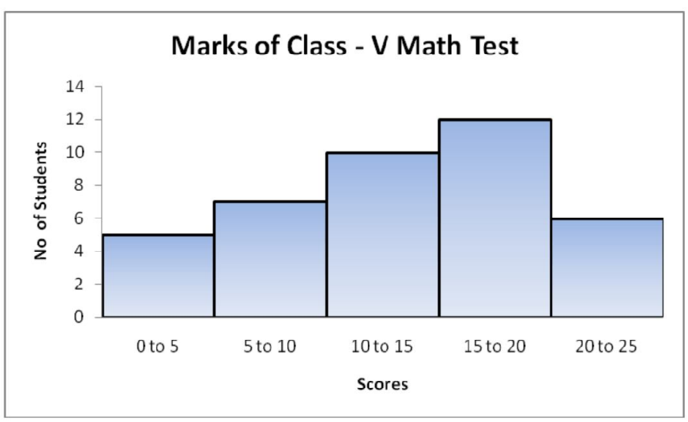

Fig. 8.8 Histogram of Scores on Maths. Test

N.B. : Histogram can also be prepared on a graph paper.

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8.3.4 Pie-chart

We can also draw a pie chart to represent an ungrouped data. In a pie-chart, a circle represents the whole data. Each division of the data is shown by a sector. We have to find the central angle for each sector which is given by

$$\theta = \frac{f}{N} \times 360°$$

where f = frequency of the division, and N = Sum of the frequencies (total).

Ex.10. Let us draw a pie-chart for the following example.

Table 8.6 shows the monthly expenditure of Mitali's family on various items.

Table 8.6 (Monthly Expenditure)

Items of Expenditure	Expenditure (*in hundreds of Rupees*)
House rent	21
Electricity	03
Education	36
Transport	06
Food	42
Miscellaneous	12
Total	**120**

We have to prepare a pie chart to represent the given data. So we have to calculate the sector angle of each item. Let us construct the following table showing this relationship.

Table 8.7 Monthly Expenditure

Items of Expenditure	Expenditure (in hundreds of Rs.)	Central Angle θ
House Rent	21	$\frac{360}{120} \times 21 = 63°$
Electricity	03	$\frac{360}{120} \times 03 = 9°$
Education	36	$\frac{360}{120} \times 36 = 108°$

Transport	06	$\frac{360}{120} \times 06 = 18°$
Food	42	$\frac{360}{120} \times 42 = 126°$
Miscellaneous	12	$\frac{360}{120} \times 12 = 36°$

The pie-chart of the above data is shown below.

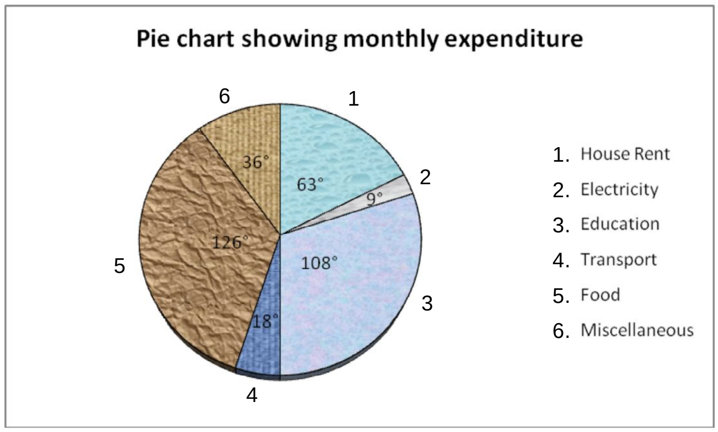

Fig. 8.9 Pie chart showing monthly expenditure

Steps to draw a pie-chart :

Step-I: Calculate the central angle θ of each sector.

Step-II: Draw a circle with suitable radius.

Step-III: Draw radii showing the central angles either in clockwise or in anti clockwise manner

Step-IV: Shade (or colour) the sectors differently.

Before proceeding further, try to respond to the following:

E1. Total number of animals in five villages are as follows :

Naraharipur	Purunakote	Patla	Sanamunda	Kantimili
80	100	60	120	50

Prepare a pictograph of these animals using symbol $\otimes$ to represent 10 animals

Notes

E2. The no. of books sold by a shopkeeper in a week is shown below :

Days	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
No. of books	60	40	35	50	25	70	30

Draw a bar graph choosing a proper scale.

E3. Number of persons in different age groups in state like Odisha e.g. Dhenkanal town is given below:

Age group	1-15	15-30	30-45	45-60	60-75	75 and above
No. of persons	24	30	42	36	18	12

Draw a histogram to represent the above information.

E4. The following table shows the no. of students of a school who came to the school on a certain day by using different kinds of conveyances. Draw a pie chart for this.

Kinds of conveyance	School Bus	Cycle	Motor cycle/Scooter	Car	Others
No. of students	160	80	60	20	40

8.4 DATA ANALYSIS

We have learnt the ways one can record the collected data and display those data using pictorial forms and graphs. Apart from visual representation of data, can we make any other analysis of data in order to comprehend the trend of the distribution of scores?

If you study closely any frequency distribution discussed so far (either ungrouped or grouped), you would observe two features of the distribution:

The scores or observations tend to cluster around a specific value which is nearer to the middle of the distribution. This characteristic of the distribution of data is called its **central tendency**. A measure of central tendency refers to a central value which is representative of the entire set of data to which it belongs.

Only knowing the measure of central tendency is not sufficient to study the nature of the distribution of data. Besides the central value, we have to know the nature of spread of individual observations around the central value. Look at the two distribution shown below:

(a) 10, 11, 12, 13, 14, 15, 16

(b) 5, 9, 10, 13, 19, 17, 18

In both the above distributions, the central value is 13. But , in the distribution (a), the scores are clustered nearer to the central value 13 than the score in the distribution (b). This is a very simple example to show that we need to know the spread of scores or variation of scores around the central value for a complete analysis of the distribution. The spread or variability nature of data is otherwise called *variation* or *dispersion*. A measure of ***variability*** or ***dispersion*** of data relates to quantifying how closely individual observations are scattered around the central value.

In this section we shall discuss about some measures of central tendency and dispersion.

8.4.1 Measures of Central Tendency

Central tendency is a statistical measure which identifies a representative value (score) of an entire distribution. The literal meaning of the term central tendency is the score where all the scores tend to centralize. The goal of central tendency is to find the single score that is most typical or most representative of the entire group. In this section we shall discuss about three principal measures of central tendency. These are (i) **Arithmetic Mean**, (ii) **Median**, and (iii) **Mode**

Let us now discuss the methods of determining these three measures.

8.4.1.1 ARITHMETIC MEAN

The most common measure of central tendency is the *arithmetic mean* or simply *mean*. It is also commonly known as *average*.

(a) **Mean of Ungrouped Data** : The mean of some given observations ($x_1, x_2, x_3, \ldots, x_n$) is defined as

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum x}{n} \quad (\sum \text{ is symbol used for summation})$$

$$\text{Mean} = \frac{\text{sum of the observations (scores)}}{\text{no. of observations (scores)}}$$

Ex.11: Dhoni scored 33, 17, 60, 25 and 45 runs in a one-day series. Calculate the mean score.

Total runs = 33 + 17 + 60 + 25 + 45 = 180

No. of matches = 5

$$\text{So, mean score} = \frac{\text{Total runs}}{\text{no. of matches}} = \frac{180}{5} = 36$$

Thus the mean runs per match is 36.

ACTIVITY -6

Calculate the mean of your study hours in a particular week.

Find the mean of your sleeping hours during one week

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(b) **Mean of an Ungrouped Frequency Distribution**:

Let us discuss the following example.

Ex.12. The daily wages of Ranjit in a month are shown in the following table. Calculate the mean of his daily wages.

Daily wages (in `)	**120**	**130**	**140**	**145**	**150**
No. of days	5	4	7	6	8

For calculating the mean daily wage, we have to prepare the following table:

Table 8.8 Daily wages of a month

Daily wages (in `) (x)	**No. of days (frequency = f)**	$f \times x$
120	5	600
130	4	520
140	7	980
145	6	870
150	8	1200
	$\sum f = 30$	$\sum fx = 4170$

$$\text{So, mean} = \frac{\text{sum of the wages}}{\text{No. of days}} = \frac{\sum fx}{\sum f} = \frac{4170}{30} = 139$$

$\therefore$ The mean daily wages of Ranjit is ` 139.00.

ACTIVITY - 7

Calculate the mean of the following distribution of marks:

Marks obtained	*15*	*17*	*20*	*22*	*24*	*25*
No. of students	*3*	*5*	*9*	*4*	*7*	*2*

(c) **Mean of Grouped Frequency Distribution**: In case of a grouped frequency distribution, each group (class interval) is replaced by a single number which is the mid point of the group.

So, midpoint of a class interval = $\frac{l_1 + l_2}{2}$, where l_1 and l_2 are the lower and upper limit respectively of the class interval.

Ex.13. Let us now calculate the mean of the following distribution:

Table 8.9 Grouped Frequency Distribution

Scores (C.I.)	**Frequency (f)**	**Midpoint (x)**	$f \times x$
0-10	5	5	25
10-20	6	15	90
20-30	12	25	300
30-40	8	35	280
40-50	10	45	450
50-60	6	55	330
60-70	6	65	390
	$\Sigma f = 50$		$\Sigma fx = 1865$

So, mean. $= \frac{\Sigma fx}{\Sigma f} = \frac{1865}{50} = 37.3$

Note: Replacing the classes by the midpoints, the grouped distribution changes to ungrouped distribution, and the midpoints of the classes take the place of scores.

ACTIVITY - 8

Calculate the mean of the following distribution:

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	12	15	9	11	5

E5. What is the mean of the first ten natural numbers?

Do you know?

In a normal distribution without having any unusual extreme scores, mean is the most reliable measure of central tendency.

Mean is influenced by the size of every score in the group. If any one score is increased (or decreased) by *c* units, the mean is increased (or decreased) by *c/n* units.

If a constant, say *c* is added to each score in a group which means $\overline{X}$, the resulting scores will have a mean equal to $\overline{X} + c$. (You can verify this property of mean taking small distribution). Verify what will happen to mean when each score is multiplied by a constant *c*.

8.4.1.2 Median

The middle value of the data when arranged in ascending (increasing) or descending(decreasing) order is called as Median. Thus Median refers to the value which lies in the middle of the data with half of the observations above it and the other half below it. In other words, median divides the distribution in two equal halves.

(a) Median from ungrouped data:

After arranging the given data either in ascending or descending order of magnitude, the value of the middle observation is called the median of the data.

Method for finding the median of an ungrouped data: Arrange the scores in ascending or descending order. Suppose the total no of observation is 'n'.

Case – (i) when n is odd

$$\text{Median} = \text{Value of} \frac{1}{2}(n+1)^{th} \text{ observation.}$$

Case – (ii) when n is even, there occur two midpoints

$$\text{Median} = \frac{1}{2}\left[\frac{n}{2}^{th} \text{ Observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}\right]$$

Let us study the following examples:

Ex.14. The ages (in years) of 10 teachers in a school are 37, 34, 52, 45, 50, 41, 31, 40, 36, and 55. Find their median age.

Arranging the ages in ascending order, we have 31, 34, 36, 37, 40, 41, 45, 50, 52, and 55

Here n = 10 which is even.

So the scores in the middle of the distribution are 40 and 41.

$\therefore$ Median = mean of 40 and 41

$$= \frac{1}{2}(40 + 41) = \frac{1}{2} \times 81 = 40.5$$

Hence the median age of the teachers is 40.5 years.

Ex 15. The runs scored by the Indian cricket team in a one-day match are 95, 40, 2, 55, 10, 38, 33, 22, 0, 18, 8 find the median score.

Arranging the runs in descending order we have 95, 55, 40, 38, 33, 22, 18, 10, 8, 2, 0

Here n = 11 which is old.

So, the median score $= \text{value of } \frac{1}{2}(11+1)^{\text{th}}$ observation

= value of the 6^{th} observation = 22

Hence the median score is 22.

(b) Median from the ungrouped frequency distribution: The steps of finding the median of an ungrouped frequency distribution are:

Step I: Arrange the scores in ascending or descending order.

Step II: Determine the cumulative frequency which is the added up frequencies from the beginning upto the score concerned.

Step III: Find the sum of the frequencies N.

If N is odd, then median = size of the $\left(\frac{N}{2}\right)^{\text{th}}$ score

Let us observe the following example

Ex. 16. Find the median of the following distribution.

Marks obtained	**17**	**20**	**15**	**22**	**18**	**25**
No. of students (f)	5	8	6	7	10	3

Now, preparing the cumulative frequency table, we have

Table 8.10 Cumulative Frequency Distribution

Marks obtained (x)	**No. of students (f)**	**Cumulative frequency (cf)**
15	6	6
17	5	6 + 5 = 11
18	10	11 + 10 = 21*
20	8	21 + 8 = 29
22	7	29 + 7 = 36
25	3	36 + 3 = 39
	$N = \sum f = 39$	

*The 20^{th} position

So, Median position is the $\frac{N+1}{2}$ th position i.e. the $\frac{39+1}{2}$ th or 20th position. This means the mid-point falls in the 20^{th} position of the distribution. In this example the score 18 occupies all the positions from 12^{th} to 21^{st},i.e. the mark which each one in the 12^{th}, 13^{th}, 14^{th}……21^{st} rank gets is 18.Hence the median of marks = score of the median position = 18

(c) Median from Grouped Frequency Distribution:

Consider the following grouped distribution of marks obtained in Mathematics of class VII students:

Table 8.11 Median for Grouped Data

C.I. of Marks Obtained	**Number of Students(f)**	**Cum. f**
30-39	1	1
40-49	3	4
50-59	8	12
60-69	15	27
70-79	10	37
80-89	9	46
90-99	4	50
N = 50		

For a grouped frequency distribution, the median is calculated using the following formula:

$$\text{Median} = L_m + \frac{\left(\frac{N}{2}\right) - F}{f_m} \times i$$

Where, L_m = The exact lower limit of the class interval in which the median lies.

N = Total number of scores,

F = Total frequency below L_m,

f_m = frequency within the class interval in which the median lies, and

i = length of the class interval.

In the above example, N= the total number of scores = 50.

Hence, the median lies in between 25th and 26th positions i.e. within the class interval 60-69.

Here, L_m = exact lower limit of 60-69= 59.5,

$F = 12$, $f_m = 15$, and $i = 10$.

$$\therefore \quad \text{Median} = 59.5 + \frac{\left(\frac{50}{2}\right) - 12}{15} \times 10 = 59.5 + 8.67 = 68.17$$

Steps of calculating median in grouped distribution:

1. Calculate the Cum. f and locate the C.I. in which the median lies.
2. Determine the exact lowest limit of the C.I.(L_m) in which the median lies.
3. From the calculations of the Cum. f , determine the total frequency below the exact lower limit of the C.I.(F) in which the median lies.
4. Find the difference N/2 - F , divide the difference with the frequency of the scores within the C.I. in which the median lies and multiply it with the length of the C.I.
5. Add the result in 4 with the exact lower limit Lm to get the median.

Before proceeding further answer find the median in the following two cases:

E6. Find the median of the first 8 odd natural numbers.

E7. Find the median weight for the following data.

Weight (in kg)	40	42	45	46	48	50	52
No. of students	8	5	6	9	7	4	2

Notes

Do you know?

The median is the score that occupies the exact middle position of the distribution when the observations are arranged in ascending or descending order.

Median is a locational or positional observation and hence not affected by any extreme score. If you increase or decrease the scores that lie at the extremes, the median does not change.

Median is preferred when the distribution has number of extremely small and/or large scores which would affect mean.

Median is used when a quick estimate of central tendency is required.

8.4.1.3 Mode

Other than Mean and Median there is also another measure of central tendency which is called '*Mode*'. For different requirements relating to a data, this measures of central tendency is used. Now look at the following example.

Ex.17. To find out the monthly demand for different sizes of shoes a shopkeeper kept records of sales. Following is the record for a month:

Size	5	6	7	8	9	10	Total
Pair of shoes sold	20	51	70	35	10	6	192

$$\text{Mean of the shoes sold} = \frac{\text{Total shoes sold}}{\text{No. of different size}} = \frac{192}{6} = 32$$

Should the shopkeeper obtain 32 pairs of shoes of each size? If he does so, will he be able to meet the needs of largest group of the customers?

On looking at the record, the shopkeeper decides to procure shoes of sizes 6 and 7 and 8 more.

The size of shoes that has maximum number of customers is 7.

Observe that the shopkeeper is looking at the shoe size that is sold most.

This is another representative value for the data.

This representative value is called the Mode of the data. Thus the mode of a set of scores is that which occurs most often.

Mode : Mode is the score which occurs most frequently in the distribution.

Let us discuss some more examples.

Ex.18. The ages of cricket players of India are given below. Find their modal age.

29, 38, 19, 24, 34, 29, 24, 38, 23, 28, 24, 25, 21, 26, 24

Arranging the ages with same age together we get 19, 21, 23, 24, 24, 24, 24, 25, 26, 28, 29, 29, 34, 38, 38

Mode of this data is 24, because it occurs more frequently than other observations.

Ex.19. Determine the mode of the following data.

Marks secured	**10**	**12**	**13**	**15**	**18**	**19**	**20**
No. of students	3	4	5	2	5	2	1

Mode of marks i.e. mark(s) of highest frequency are 13 and 18

Note : A distribution can have multiple modes i.e. more than one mode.

If the no. of observations is large, we have to prepare an ungrouped frequency distribution table by putting tally marks. Thereafter, we can determine the mean, median and mode of the distribution.

Do you know?

Formula for calculating an approximate value of mode in a perfectly symmetrical distribution is : **Mode = 3 × median – 2 × mean**

Mode is used when we wish to know what the most typical case is.

Mode is most unstable measure of central tendency.

Mode is a score of the distribution and not a frequency.

8.4.2 Measures of Variability

There are two types of measures of variability or dispersion – ***distance measures*** and ***measures of average deviation.***

Distance Measures: Distance measures describe the variation in data in terms of the distance between selected measurements. Here, we discuss two such measures-Range and Inter-Quartile Range.

Range: Range is the simplest measure of variability both from the point of view of calculation and understanding. It is the difference between the largest and the smallest observations in the data.

For example, the range of scores 2, 5, 6, 4, 12, 10, 9 and 8 is 12-2 = 10.

Similarly, -2, 0, 3, 7 and 9 has the range = 9-(-2) =11.

This type of range is called '***Exclusive Range'***. But when we determine the range through the difference between the *upper real limit* (URL) of the highest score and the *lower real limit* (LRL) of the lowest score, we call it '***Inclusive Range'***.

Notes

In the above example, the URL of the highest score 12 is 12.5 and the LRL of the lowest score 2 is 1.5 (Every observation in social science is not absolute and fixed. Rather it is assumed to be stretched over an interval of 0.5 before and after the score. Therefore the score 12 is any value in the interval of 11.5 - 12.5, the extreme points of the interval determines the upper and lower real values of 12). Hence, the inclusive range of the distribution is 12.5 – 1.5=11 while its exclusive range is 10.

E8 . Calculate the inclusive range of - 1, 2, 5, -5, 4, 6, and 7.

E9 . What is the difference between the inclusive and exclusive ranges of a distribution?

Although range, either exclusive or inclusive, is easier to calculate and the simplest measure of variability, it does not reflect the spread or scatter of scores. For example, if 100 scores are spread evenly between 1 to 10, the inclusive range is 10. But if one score is 1 and another is 10 and in between these two scores the score 5 is repeated 98 times, then the total number of scores remaining 100 the inclusive range would still be 10. This goes to show that range may be the quickest estimate of variability but not an accurate measure of it.

Interquartile Range: Interquartile range or specifically semi-interquartile range is another distance measure of the variability of distribution. Earlier, we had defined median as the score that divides a distribution exactly in half. In a similar way, a distribution can be divided into four equal parts using quartiles. By definition, the first quartile (Q1) is the score that separates the lower 25% of the distribution from the rest. The second quartile (Q2) is the score that has exactly two quarters, or 50%, of the distribution below it. Notice that the second quartile and the median are the same. Finally, the third quartile (Q3) is the score that divides the bottom three-fourths of the distribution from the top quarter.

The interquartile range is defined as the distance between the first and the third quartile.

$$\text{Interquartile Range} = Q3 - Q1$$

When the interquartile range is used to describe variability, it commonly is transformed into the *semi- interquartile range*. It is simply one-half of the interquartile range:

$$\text{Semi-interquartile Range} = \frac{(Q3-Q1)}{2}$$

Ex. 20 Find the semi interquartile range for the following data:

3, 4, 6, 9, 11, 12, 14, 15.

The total number of scores in the distribution is 8. Its 25% is 2 and 75% is 6. That means Q1 is a point under which the first two scores i.e.3 and 4 will lie. Such a point will be the mid point of the distance between 4 and 6, i.e. 5. Hence Q1=5, Similarly Q3, below which first 6 scores will lie, is equal to 13 the mid point between 12 and 14.

Hence, semi-interquartile range = $\frac{(Q3-Q1)}{2} = \frac{(13-5)}{2} = 4$

Since, semi-interquartile range focuses on the middle 50% of a distribution; it is less likely to be influenced by extreme scores. But, since it does not take into account the actual distance between individual scores, it does not give a complete picture of how scattered or clustered the scores are. Therefore, like the range, the semi-interquartile range is considered to be a somewhat a crude measure of variability.

Measures of Average Deviation: Deviation of a score is its distance from the mean. When the average of such deviation scores are taken into consideration, the measure of variability becomes more accurate and reliable. We discuss here two such measures based on the average deviation scores – Average Deviation and Standard Deviation.

When the scores are nearer to the mean i.e. when the distribution is compact, the measure of average deviations would be small and vice versa. This helps us to understand the nature of distribution. For example, in a class examination, the mean of scores in Mathematics test was 65 and the standard deviation was 10, while the mean of Language test scores was 60 and the standard deviation was 5. We find majority of students scored nearer to 60 in Language. But in Mathematics the scores are spread much wider than that of the scores on Language. There would be quite a number of students securing lower marks in Mathematics ; below the minimum score in Language. Without the measures of average deviations, we cannot draw any valid conclusions from the measures of central tendency alone.

Average Deviation: Deviation score (x) = Score – Mean = $X - \overline{X}$

If in a distribution the mean is 50, the deviation of the score 55 is given by

$$X - \overline{X} = 55 - 50 = 5,$$

And the deviation of the score 45 = 45-50= - 5.

Notice that there are two parts to a deviation score: the sign (+ or -) and the number. The sign tells whether the score is above the mean (+) or below it (-). The number gives the actual distance of the score from the mean.

Average or mean deviation is the mean of deviations of all scores from the mean of the scores in the distribution. In calculating the mean deviation, no account is taken of signs, and all deviations +ve or –ve are treated as positive.

Let us calculate the average deviation (AD) of five scores 5, 8, 11,12, 14 which means 10.

Notes

Table 8.12 Average Deviation of Ungrouped Data

Score (X)	Deviation $(X - \bar{X})$	$\lvert X - \bar{X} \rvert$
5	5-10= -5	5
8	8-10= -2	2
11	11-10=1	1
12	12-10=2	2
14	14-10=4	4
$\sum \lvert X - \bar{X} \rvert = 14$		

Average Deviation (AD) = $\dfrac{\sum \lvert X - \bar{X} \rvert}{N} = \dfrac{14}{5} = 2.8.$

Notice the sum of deviation scores is 0. Why?

Average Deviation for Grouped Data: Observe the process of calculation of AD for the grouped frequency distribution shown below:

Table 8.13 Average Deviation of Grouped Data

C.I.	f	Mid-point of C.I.(X)	$\lvert X - \bar{X} \rvert$	$f \lvert X - \bar{X} \rvert$
30-34	3	32	10.38	31.14
35-39	9	37	5.38	48.42
40-44	15	42	0.38	5.70
45-49	8	47	4.62	36.96
50-54	5	52	9.62	48.10
	N = 40			170.32

Here N = 40, and Mean (X) = 42.38 and $\sum f \lvert X - \bar{X} \rvert = 170.32$.

The average deviation (AD) of the grouped frequency distribution is calculated by using the formula:

$$\text{AD} = \frac{\sum f \lvert X - \bar{X} \rvert}{N}$$

In the above example, AD = $\frac{\sum f\left|X-\bar{X}\right|}{N} = \frac{170.32}{40} = 4.26.$

Standard Deviation: The widely used measure of variability is the standard deviation.

From the calculation point of view the standard deviation is called ***'Root-mean-squared-deviation'***. From this we can state the steps of computation of standard deviation (SD):

1. Calculate the mean of the distribution for which you have to find the SD.
2. Determine the deviations of the scores from the mean.
3. Square each deviation. The deviation may be positive of negative, but the square of the deviation is always positive.
4. Find out the mean of the squared deviations. This mean of the squared deviation is called 'variance' which has wider use in higher statistics.
5. Determine the positive square root of the variance and the result is the standard deviation.

Let us calculate the SD, usually signified by the Greek letter '?'(sigma) or s, of the following set of scores: 5. 6, 8, 10, 11,14 with mean of 9.

Table 8.14 Standard Deviation of Ungrouped Data

Score	Deviation ($x = X - \text{X}$)	Squared Deviation (x^2)
5	-	16
6	-	9
8	-	1
10	1	1
11	2	4
14	5	25

Sum of the squared deviations = $\sum x^2$ = 16 + 9 + 1 + 1 + 4 + 25 = 56

Variance = Mean of squared deviations = $\frac{\sum x^2}{N} = \frac{56}{6}$ =9.33

Standard Deviation (SD)= $\sqrt{\frac{\sum x^2}{N}} = \sqrt{9.33} = 3.05$

Notes

Standard Deviation for Grouped Data: Let us take the grouped data of Table 8.13 and calculate the standard deviation.

Table 8.15 Standard Deviation of Grouped Data

C.I.	f	Mid-point of C.I. (X)	Deviation $x = X - \bar{X}$	fx	fx^2
30-34	3	32	-10.38	-31.14	323.2332
35-39	9	37	-5.38	-48.42	260.4996
40-44	15	42	-0.38	-5.70	2.166
45-49	8	47	4.62	36.96	170.7552
50-54	5	52	9.62	48.10	462.722
$\sum f$ =N=40			$\sum fx^2$ =1219.3760		

Standard deviation of the grouped frequency distribution is calculated using the formula:

$$\text{SD} = \sqrt{\frac{\sum fx^2}{N}}$$

Where x is the deviation of the midpoint of C.I. from the mean,

f *is* the frequency of scores in the respective C.I., and

N is the total number of scores.

From the above table, we get

$$\text{SD} = \sqrt{\frac{\sum fx^2}{N}} = \sqrt{\frac{1219.3760}{40}} = \sqrt{30.4844} = 5.52$$

Use of Measures of Variability

Range is used when we want a quick estimate of the spread of scores which are very small in number and there are no extreme scores.

When there are some extreme scores in a distribution and median is the measure of central tendency, semi-interquartile range is preferred.

When the scores are too scattered which would influence SD unduly, average deviation is used to give us a reasonable estimate of the spread of scores.

Among all the measures of variability, SD is most stable and accurate.

Notes

Try this

E10. Calculate the SD of the following scores

11,13,15,17,19,21,23

8.5 LET US SUM UP

Some data can be collected directly from the primary sources while some are collected from secondary sources like records, journals, census etc.

Scores can be arranged in the form of ungrouped and grouped frequency distributions.

Data can be presented through pictures and graphs for easy understanding and analysis Pictographs, bar graphs, histogram and pie charts are simple graphical forms.

Arithmetic mean, median and mode are the three measures of central tendency used for basic statistical treatment of data.

As measures of variability, range and semi-interquartile range are measure of distances while mean deviation and standard deviation are measures on average deviations.

8.6 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1: Draw the pictograph taking name of the villages on the Y axis and no. of animals of the X axis.

E2: Draw a bar graph choosing the scale 1 c.m. $\equiv$ is 10 units. You can draw horizontal or vertical bars.

E3: Prepare histogram taking C.I. for age groups on X axis and no. of persons on Y axis.

E4: Prepare pie-chart by drawing a circle of radius 3 or 4 c.m. Here central angle

$$\theta = \frac{360^0}{360} \times (\text{no. of students})$$ for each conveyance mode.

E5: Mean = 5.5

E6: Median = 8

E7. Median weight = 45kg

E8. 13

E9. 1

E10. 4

8.8 SUGGESTED READINGS AND REFERENCES

NCERT Maths Text Book for Classes VI, VII, VIII

Teaching of Maths of Upper Primary Level Vol-I, An IGNOU Publication

8.7 UNIT-END EXERCISES

1. Prepare (i) Pictograph (ii) Histogram and of the data on the marks secured by Lily on the subjects shown below:

Subject	MIL	English	Math	Science	S.Study
Marks Secured	60	55	80	55	50

2. Determine the (i) Mean (ii) Median and (iii) Mode of the following data :

   16, 24, 14, 10, 20, 14, 15, 21, 15, 12, 13, 15, 16, 19, 17

3. Determine the mean, median and mode of the following distribution:

Scores	5	6	7	8	9	10	11	12
Frequency	8	10	15	20	16	12	9	10

4. Calculate the standard deviation of the following group data:

C.I.	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Scores	6	9	15	25	13	7	5

Notes

UNIT 9 ALGEBRA AS GENERALIZED ARITHMETIC

Structure

9.0 INTRODUCTION

You are quite familiar with Arithmetic, a branch of Mathematics. It deals with concrete numbers like 1, 2, 25, 37, 456,…. and various operations on the numbers like addition, subtraction, multiplication, division. But instead of numbers if we use symbols like letters for indicating quantity and perform different arithmetic operations with these letters as was done with the numbers, we are generalizing Arithmetic and name it as Algebra.

Thus, Algebra is a branch of Mathematics where the principles of Arithmetic are generalized by using letter symbols representing numbers. Algebra is very interesting and useful when we try to express a quantity without necessarily attaching any numerical value and solving problems with the operations as we used in Arithmetic.

The Origin of Algebra

The word "Algebra" is derived from an Arabic word 'al-jabar' means reunion, used in a mathematical treatise entitled "Al-Kitab al-mukhta ? ar fi hisab al-gabar wa'l-muqabala" (Arabic for "The compendious Book on calculation by completion and Balancing") written by the Persian Mathematical Muhammed ibn Musa al Khwarizmi of Baghdad in 820 A.D.

The famous Greek mathematician Diophantus living in Alexandria in 3rd century A.D. is regarded as the "Father of Algebra" for his seminal work entitled "Arithmetica".

For completing this unit you will need approximately 7 *(seven) study hours*.

9.1 LEARNING OBJECTIVES

After going through this unit, you will be able to :

- Explain algebraic terms, expressions and categorize algebraic expressions;
- Differentiate between variable and constant, like and unlike terms;
- Perform different operations on algebraic expressions;
- Solve linear algebraic equations in one variable;
- Apply algebraic methods in solving mathematical problems.

9.2 USING SYMBOLS FOR NUMBERS

The main feature of Algebra is to use symbols to represent numbers, quantities or mathematical relations in a general situation rather than only in a particular case as in Arithmetic. Use of letters will allow us to write rules and formulae in a general way. Consider the following examples:

Ex.1. Ayesha has 3 pens and her brother Arvin has 2 pens. So they both have $3 + 2 = 5$ pens.

In this example, numbers are involved in representing the quantities and in the calculation process. Let us suppose that Ayesha has x number of pens and Arvin has y no. of pens. Can we find the total no. of pens they have? We can say that they have $(x + y)$ no. of pens

Here x and y represent two definite numbers.

Ex. 2. (a) Fig 9.1 is a square whose length of each side is *2 cm*.

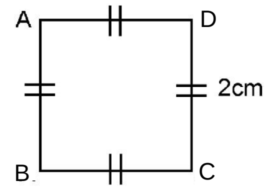


Fig. 9.1

Its perimeter $= AB + BC + CD + DA$

$= (2 + 2 + 2 + 2)\ cm$

$= 2 \times 4$ cm $= 4 \times 2$ cm $= 8$ cm

(b) Find out the perimeter of the square in fig 9.2

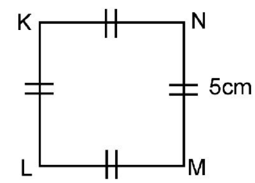


Fig. 9.2

Perimeter of $KLMN$ $= KL + LM + MN + KN$

$= (5 + 5 + 5 + 5)$ cm

$= 5 \times 4$ cm $= 4 \times 5$ cm $= 20$ cm

Thus we can find out the perimeter of any square of given length.

From the above example we can conclude that the perimeter of a square is 4 times the length of each side of it.

Perimeter of square = 4 × length of a side.

If the length of a sides of a square is 'a', then its perimeter 'P' can be expressed as

$$\boxed{P = 4 \times a}$$

Here 'a' represents the number that denotes the length of a side of a particular square and we can find out the perimeter of any square for different values of a.

Thus, with the help of symbols as representation for numbers, a number relation can be generalized. Letters a, b, c ….. x, y, z are used as symbols to represent numbers which are unknown quantities. As a result, various word problems can be expressed as symbolic statements. Since letters represent nos. they follow all the rules and properties of the four arithmetic operations.

9.3 ALGEBRAIC TERMS AND EXPRESSIONS

9.3.1 Algebraic Expressions

In arithmetic we have come across expressions like

$(3 \times 8) + 2$; $(10 \div 5) + (3 \times 20) - 7$ etc.

In these examples we can observe :

(i) Expressions are formed from nos.

(ii) All the *4* fundamental operations such as addition, subtraction, multiplication and division or some of them are used in an expression

We can form an expression using variables also.

Let us discuss the following examples :

Ex. 3: Babulu is in Class-VI. In his class there are '*m*' girl students. Number of boys is *7* less than the girls. Calculate the total no. of students in his class.

No. of girl students = m

No. of boy students = $m - 7$

Total no. of students = $m + (m - 7) = 2m - 7$

Here $2m - 7$ is an expression which is formed using variable m and constant *2* and *7*. Subtraction and multiplication operations are used here.

$2x + 3$ is an expression, variable x and constants *2* and *3* are used in forming the expression. Addition and multiplication operations are also used.

The expressions, as we got in both the examples above are called algebraic expressions since both variables and constants are used in their formation.

So, a combination of constants and variables connected by some or all of the four fundamental operations +, -, × and ÷ is called an **algebraic expression**.

For example : $7m$, $2py + 1$, $\frac{a}{2} - 5$, $m + n - 2$ are algebraic expressions.

We can write an expression when instruction about how to form it is given. Now observe the example and fill up the table.

Instruction	Expression
16 added to p	P + 16
25 subtracted from r	
P multiplied by (-6)	
X divided by 3	
'm' multiplied by 3 and 8 is added to the product	

Also, when an algebraic expression is given we can tell how it is formed. Now read the example and fill up the blank boxes

Expression	How it is formed
s – 1	1 subtracted from s
t + 25	
11a	
$\frac{2b}{5}$	
2n – 4	

We have seen that the expression have one or more than one term. An algebraic expression is classified into different categories depending upon the no. of terms contained in it.

Monomial : An expression which contains only one term is called a monomial. For example: $7xy, 2x, -4n, -8, 3a^2b$

Binomial : An expression which contains two unlike terms is called a binomial. For example: $x + y$, $2p – 3q$, $z + 1$, $3xy + 2x$

Trinomial : Expression having 3 unlike term is called a trinomial. For example: $2a – 5b + 3c$, $x + y – 3$, $pq + p – 2q$

Polynomial : An expression with one or more terms is known as a polynomial in general. For example: $5x$, $2a + 3b$, $m + n – 3$

Do you know ?

3xy is not a binomial. It is rather a monomial

$m + n – 3$ is not a binomial. It is a trinomial.

$2a + 3a$ is not a binomial. Here the terms are not unlike term.

Monomial, binomial, trinomial are all polynomial.

9.3.2 Variables and Constants

Let us examine mathematical statement $P = 4a$

Here, when $a = 1$, then $p = 4 \times 1 = 4$

when $a = 2$, then $p = 4 \times 2 = 8$

when $a = 3$, then $p = 4 \times 3 = 12$

Thus, we see that for different values taken for *'a'*, the value of *'p'* changes. I.e. *P* varies with the change in the value of a. We say that *'a'* and *'p'* both are changeable or variable. Hence we can say:

> *A symbol which does not have any fixed value for it, but may be assigned any numerical value according to the requirement is known as a **variable**.*

Well, can the number of sides of a triangle be anything other than 3 ? Definitely not. Therefore, the number of sides of a triangle is a fixed number and thus it is a constant. Thus we can say:

> *A symbol having a fixed numerical value is called a **constant**.*

In the statement $P = 4a$, '*a*' and '*p*' are called variables and '4' is a constant.

Do you know ?

A variable has no fixed value.

The letters *x, y, z, p, q, r* are usually taken to represent variables.

All real nos. are constant.

Algebraic expressions are formed from variables and constants.

Let us discuss the following examples :

Ex. – 5: Papulu purchased 2 similar pens costing 10 rupees each. How much he has to pay to the shopkeeper?

Clearly the cost of *2* pens = *Rs.* 10×2 = *Rs. 20.*, which Papulu has to pay

Here we have, Total cost = cost of each item × no. of items

If we take *c* for total cost and *n* for no. of items, the above statement can be written as $C = 10n$

Here n and *c* both are variables and *10* is a constant.

Ex. – 6: Esma and Reshma are sisters. Esma is older than Reshma by *4* years. Now read the table and fill the blank boxes.

Reshma's Age in Years	Esma's Age in Years
7	7 + 4 = 11
9	
x	

Your answer in the last box will be $(x + 4)$ means Esma's age will be $(x + 4)$ years when Reshma's age is '*x*' years. Here $(x + 4)$ is an algebraic expression where *x* is a variable and *4* is a constant.

Now write the variables and constant involved in the following expressions :

Expression	Variables	Constant
$y - 7$		
$\frac{s}{2} + 3$		
$2p + 3q$		

9.3.3 Terms of an Algebraic Expression

From earlier discussion we know that an algebraic expression consists of one or more terms. Let us consider the following examples :

Ex. 7 $2p + 3$ is an expression.

In forming the expression we first formed $2p$ separately as a product of 2 and p and then 3 is added to it.

Ex. 8 $xy + 3z - 5$ is an expression.

To form this expression we first formed xy separately as a product of x and y. Then we formed $3z$ separately as a product of 3 and z. We then added them *(xy and 3z)* and then added *(-5)* to it to get the expression.

You will find that the expressions have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are called as terms.

In Example 7, 2*p* and 3 are terms and in Example 8, *xy*, 3*z* and 5 are terms

The different parts of the algebraic expression separated from each other by the sign **'+'** *or* **'-'** *are called the* **terms** *of the expression.*

Do you know?

Terms are added to form expressions.

The sign before a term belongs to the term itself.

Can you find the terms and their number in the following expressions :

Expression	No. of term	Name of the term
2ab – 3	2	2ab, -3
$\frac{k}{3}+1$		
$\frac{-xyz}{2}$		
$16 - x + 3y^2$		

Try this

Write two expressions each having four terms.

9.3.4 Product, Factor and Coefficient

We know from the previous chapters that in the multiplication $2 \times 5 = 10$, 10 is the product and 2 and 5 are the factors of 10.

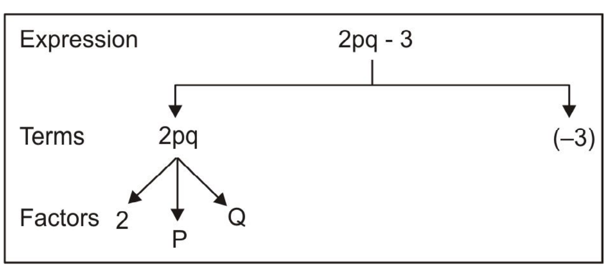
When two variables are multiplied, what is the product ?

We write the product of 3 and $z = 3 \times z = 3z$

And product of y and $z = y \times z = yz$

We saw above that an expression consist of one or more terms. For example expression $2ab – 3$ has 2 terms namely $2ab$ and -3. Here $2ab$ is the product of 2, a and b. We say that 2, a and b are the factors of the term $2ab$.

We can represent any algebraic expression into its constituent terms and terms into the factors by a tree diagram.



Try these : Draw the tree diagram for

(i) 3xy + 5y

(ii) 7ab – 5a + 2

Do you know ?

A constant factor is called a **numeric** factor.

A variable factor is called is a **literal (algebraic)** factor.

Ex.- 9: The expression 3xy-5y has two terms 3xy and -5y. The numerical factor of a term calls it numerical coefficient or simply coefficient. 3 is the coefficient of 3xy and -5 is the coefficient of y.

Try these : Identify the coefficient of the terms in the following expressions:

(i) -6ab (ii) $-\frac{pq}{3}$

9.3.5 Like and Unlike Terms

Let us examine the factors of the terms *2pq. –pq, 5pq,* $\left(\frac{1}{2}\right)pq$. These terms have same literal (algebraic) factors *pq* but different numerical factors. But the terms *2p, 3pq, -5q* have different literal factors. We say the terms having the same algebraic factors as **similar or like terms**. When the terms do not have the same algebraic factors are called **unlike terms**.

Ex - 1: In the expression $2a + 5ab - 3a - b$, the terms *2a, -3a* have same algebraic factor *a*. So they are like terms. But the terms 2a, 5ab have different algebraic factors, they are unlike terms. Similarly the terms *5ab* and *-4* are unlike terms.

Try these :

1. Group the like terms together from. : 7x, 7, -8x, 8y, x, -y, 15y
2. Fill up the table for each group.

Terms	Factors	Algebraic Factor Same or different	Like/Unlike Term
15x 12y	15, x 12, y	Different	unlike
9z –13z			
6xy 2x			
–2ab 5ba			

Check Your Progress :

Now try to assess your understanding of the concepts covered so far by answering the following questions:

E1. Write any 2 algebraic expressions using the variables p and q.

E2. Write down the expression in the following cases.

(i) 3 added to 5 times the product of x and y

(ii) Sum of a and b subtracted from their product.

E3. Identify the variables and constant in $2y - 3z + 5$

E4. Write down the coefficient of x in each term.

(i) $-x$

(ii) $2xy+y2+z2$

(iii) $\frac{2}{3}x^2y$

E5. Identity the like terms in the following:

$$2p, -3pq, \frac{1}{2}pqr, -5, \frac{p}{3}, 3pqr, 5pq$$

E6. Write the factors in each of the following monomials:

(i) $5xy$

(ii) $-3abc$

E7. Identity the terms and factors in each expression:

(i) $3xy - 5y$

(ii) $ab + 2a - 3y$

9.4 OPERATION ON ALGEBRAIC EXPRESSIONS

We are acquainted with the four fundamental operations – addition, subtraction, multiplication and division on numbers. Here we will move systematically from arithmetic to algebra and learn how these operations are worked on letters representing numbers. Since letters represent nos. they follow all the rules and properties of addition, subtraction, multiplication and division of number.

We will perform different operations in algebra in two phases :

(i) Operations on letters

(ii) Operations on expression

9.4.1 Addition

There are a number of real life problems in which we need to use algebraic expressions and apply arithmetic operations on them. Let us see how expressions are added.

(a) Addition of letters/monomials

We have learnt that $2 + 2 + 2 = 2 \times 3 = 3 \times 2$

similarly we can have $x + x + x = x \times 3 = 3 \times x = 3x$

and $x + x + x + x + x = 5 \times x = 5x$

Now sum of 3x and $5x = 3x + 5x$

$= (x + x + x) + (x + x + x + x + x)$

$= x + x + x + x + x + x + x + x = 8x$

Also $3x + 5x = (3 \times x) + (5 \times x)$

$= (3 + 5) \times x$ (distributive law)

$= 8 \times x = 8x$

Let us see the following example :

Ex-10: Find the sum of $5ab$, $7ab$ and ab

Solution : The sum $= 5ab + 7ab + ab$

$= (5 \times ab) + (7 \times ab) + (1 \times ab)$

$= (5 + 7 + 1) \times ab = 13 \times ab = 13ab$

So, $5ab + 7ab + ab = 13ab$

Try these :

Find the following sum.

(i) $3p$, p and $7p$

(ii) $6xyz$ and $12xyz$

Thus we know how two or more like terms can be added. Now think of the addition of unlike terms.

Let us find the sum of *5* mangoes and *3* oranges.

We can not say that the sum is *8* mangoes or *8* oranges

Similarly, the sum of $5x$ and $3y$ can not be a single term. We write the result as $5x + 3y$ where both the terms are retained.

(b) Addition of Algebraic Expressions : Let us discuss the following examples :

Ex-11: Find the sum of 5a + 7 and 2a – 5.

The sum $= 5a + 7 + 2a - 5$

$= (5a + 2a) + (7 - 5)$ (putting the like terms together)

$= 7a + 2$

Ex-12: Add $4x + 3y$, $8 + 2x$ and $2y - 5$

The sum $= 4x + 3y + 8 + 2x + 2y - 5$

$= (4x + 2x) + (2y + 3y) + (8 - 5)$ (rearranging terms)

$= 2x + 5y + 3$

Try these :

Add the expressions : (i) $mn + 5$ and $2nm - 7$ (ii) $2a + 3b - 1; 3a + 7$ and $5b - 3$

Note : Expressions obey closure, commutative, associative, properties in addition. It also has additive identify and additive inverse

9.4.2 Subtraction

We know how to subtract integers earlier. The same principle also works with algebraic expressions.

(a) Subtraction of Monomials : Let us subtract $2x$ from $5x$

$5x - 2x = (x + x + x + x + x) - (x + x)$

$= x + x + x + x + x - x - x$

$= x + (-x) + x + (-x) + x + x + x$ [As x and –x are additive

$= 0 + 0 + x + x + x$ inverse of each other]

$= 0 + 3x = 3x$

In brief, we can also do the work as follows:

$5x - 2x = 5 \times x - 2 \times x$

$= (5 - 2) \times x = 3 \times x = 3x$

Let us see another example:

Ex-13 Subtract $7mn$ from $16mn$.

$16mn - 7mn = 16 \times mn - 7 \times mn = (16 - 7)mn = 9 \times mn = 9mn$

But the difference of two unlike terms is not a monomial rather it will be a binomial. For example, difference of $5x$ and $3y = 5x - 3y$

Try these :

Subtract

(i) 5 m from 11 m, (ii) 6ab from 10ab (iii) 5xy from 3xy

(b) Subtraction of Algebraic Expressions: The process of subtraction is similar to that of addition. Let us observe the following examples.

Ex-14 Subtract $3a + 2b$ from $4a + 5b -2$

Solution: $(4a + 5b - 2) - (3a + 2b)$

$= 4a + 5b - 2 - 3a - 2b$

$= (4a-3a) + (5b-2b) - 2$ (Putting the like terms together)

$= a + 3b - 2$

Alternative method **:** We shell write the expressions one below the other with the like terms remaining in one column and perform subtraction on each of the terms separately as shown below:

$$\begin{array}{r} 4a + 5b - 2 \\ -\quad 3a + 2b \quad\;\; \\ \hline a + 3b - 2 \\ \hline \end{array}$$

Try these :

Subtract (i) $5x - 9$ from $10x + 7b - 3$

(ii) $4pq - 5p + 2$ from $6pq - 1 + 3q$

9.4.3 Multiplication

(a) Multiplication of monomials

$a \times a = a^2$ where 2 is the number that represents the number of a's.

The number 2 in a^2 is known as **index** or **exponent** or **power** of a and 'a' is the **base**.

Try to fill up the table

No. of a's	Product	Base	Index
Product of 3 a' s	*a3*	*a*	*3*
$a \times a \times a \times a$			
$a \times a \times a \times a \times a$			

Let us learn to multiply two terms.

The product of a and b, that is a × b can briefly be written as *'ab'*

Similarly, $a \times a \times b = a^2b$

$a \times a \times b \times b = a^2b^2$ and so on.

What is (i) $x \times x \times x \times y \times y =$ ....

(ii) $m \times m \times m \times n \times n \times n =$ ....

Now let us discuss some more examples.

Ex-15 Multiply *2x* by *3y*

$2x \times 3y = 2 \times x \times 3 \times y$

$= 2 \times 3 \times x \times y = 6\,xy$ *(by commutativity of multiplication)*

Ex-16 Multiply *3mn* by *-5mn*

$3mn \times (-5mn) = 3 \times m \times n \times (-5) \times m \times n$

$= 3 \times (-5) \times (m \times m) \times (n \times n)$

$= -15 \times m2 \times n2 = -15\, m^2n^2$

Ex-17 Find the product of *-5pq*, *4pqr* and *2r*

$(-5pq) \times (4pqr) \times 2r = (-5) \times p \times q \times 4 \times p \times q \times r \times 2 \times r$

$= (-5) \times 4 \times 2 \times p \times p \times q \times q \times r \times r$ (rearranging the factors)

$= -40 \times p2 \times q2 \times r2 = -40\ p2\ q2\ r2$

Try these :

Find the product

(i) $4xy \times 2x^2$

(ii) $5m \times 3n \times 7mn$

(b) ***Multiplication of a monomial with a polynomial***

For this multiplication commutative, associative and distributive properties are used as and when required.

Ex-18 Multiply : *(3x – 5)* with *2x*

Product $= (3x - 5) \times 2x$

$= 3x \times 2x - 5 \times 2x$ (distributive law)

$= 3 \times 2 \times x \times x - 5 \times 2 \times x$

$= 6x^2 - 10x$

Ex-19 Multiply $3a$ with $(5a - 2b + 4)$

Product $= 3a \times (5a - 2b + 4)$

$= 3a \times 5a + 3a \times (-2b) + 3a \times 4$ (distributive Law)

$= 3 \times a \times 5 \times a + 3 \times a \times (-2) \times b + 3 \times a \times 4$

$= 3 \times 5 \times a \times a + 3 \times (-2) \times a \times b + 3 \times 4 \times a$

$= 15a2 - 6ab + 12a$

Try these :

Find the product of

(i) $2x - 3y$ *and* $5xy$

(ii) $3mn$ *and* $(5m - 7mn + 3n)$

(c) ***Multiplication of a polynomial by a polynomial*:**

Here also we use distributive property.

Ex-20 Multiply $(a + b)$ by $(3a - 5b)$

Solution : $(a + b)(3a - 5b)$

$= a(3a - 5b) + b(3a - 5b)$ (distributive Law)

$= a \times 3a - a \times 5b + b \times 3a + b \times (-5b)$

$= 3 \times a \times a - 5 \times a \times b + 3 \times a \times b - 5 \times b \times b$

(associative and commutative property)

$= 3a^2 - 5ab + 3ab - 5b^2$

$= 3a^2 - 2ab - 5b^2$

Ex-21 Multiply $(2x + 5)$ by $(x^2 - 3x + 2)$

Solution **:** $(2x + 5) \times (x2 - 3x + 2)$

$= 2x \times (x2 - 3x + 2) + 5(x2 - 3x + 2)$ (distributive Law)

$= 2x \times x2 - 2x \times 3x + 2x \times 2 + 5 \times x2 - 5 \times 3x + 5 \times 2$

$= 2x3 - 2\times 3 \times x \times x + 2 \times 2x + 5x2 - 15x + 10$

$= 2x3 - 6x2 + 5x2 + 4x - 15x + 10$

$= 2x3 + (-6 + 5)x2 + (4 - 15)x + 10$

$= 2x3 - x2 - 11x + 10$

Try these :

Find the poduct of

(i) $(2m + 3n)$ and $(m-2n)$

(ii) $(pq-1)$ and $(2p + 3q - 5)$

9.4.4 Division

You know the procedure for division of numbers. Similar procedure is also followed while dividing an algebraic expression by another

(a) *Division of monomial by a monomial:*

Algorithm :

(i) Write the dividend as numerator and divisor as denominator.

(ii) Express the numerator and denominator both as the product of factors.

(iii) Simplify the fraction by cancelling the common factors from the numerator and denominator.

Now observe the following examples:

Ex-22

(i) Division of 15mn by 5m $= \dfrac{15mn}{5m} = \dfrac{3\times 5\times m\times n}{5\times m} = 3n$

(ii) $18x^2y^2 \div (-6xy) = 18\dfrac{x^2y^2}{-6xy} = \dfrac{3\times 6\times x\times x\times y\times y}{-6\times x\times y} = -3xy$

Try these :

Divide (i) $25xy$ by $-5y$ (ii) $30a^2b^2c$ by $6ab$

(b) *Division of polynomial by a monomial:*

Here the dividend is a polynomial and the divisor is a monomial. The working rule for division is :

Divide each term of the dividend by the divisor.

Simplify each fraction as earlier.

Let us workout the following examples:

Ex-23

(i) Divide $9x^2 – 15xy$ by $3x$

$$\text{Now, } (9x^2 – 15xy) \div (3x) = \frac{9x^2 - 15xy}{3x} = \frac{9x^2}{3x} - \frac{15xy}{3x} = 3x - 5y$$

(ii) Divide $8a^2b - 12ab^2 + 20ab$ *by* $(-4ab)$

Now, $(8a^2b – 12ab^2 + 20ab) \div (-4ab) = -2a + 3b – 5$

Try these :

Divide (i) $6m^2n – 9mn^2$ *by* $3mn$

(ii) $10x^3y - 15x^2y^2 – 5x^2y^3$ *by* $5xy$

(c) *Division of Polynomial by a Polynomial:*

In case of numbers, we know how to divide the dividend by a divisor by the long division process. we have to follow the similar procedure here.

Consider the following examples.

Ex-24: Divide $11x + 15x^2 - 12$ by $5x - 3$

Step-1

Arrange the terms of the dividend and dvisor in descending order of powers of a certain variable contained in the polynomials.

$$5x - 3\,\overline{\big)\,15x^2 + 11x - 12}$$

Step-2

Divide the 1st term of the dividend by the 1st term of divisor and get the 1st term of the quotient.

Here $\dfrac{15x^2}{5x} = 3x$

Step-3

Multiply $3x$ with each term of the divisor and write the result below the dividend. Then subtract it from the dividend.

$$\begin{array}{r|l} & \quad 3x \\ \hline 5x - 3 & 15x^2 + 11x - 12 \\ & 15x^2 - 9x \\ & - \quad\; + \\ \hline & \qquad\quad 20x - 12 \end{array}$$

Notes

Step-5

Repeat the process from step 2 to 3 considering $20x - 12$ as the new dividend.

$$\begin{array}{r|l} & \quad 3x + 4 \\ \hline 5x - 3 & 15x^2 + 11x - 12 \\ & 15x^2 - 9x \\ & - \quad + \\ \hline & \qquad\quad 20x - 12 \\ & \qquad\quad 20x - 12 \\ & \qquad\quad - \qquad + \\ \hline & \qquad\qquad\qquad 0 \end{array}$$

Here the 2nd term of the quotient $= \dfrac{20x}{5x} = 4$

Product of 2nd term of the quotient and the divisor is now to be subtracted from the new dividend. Result is '0'.

Hence the quotient = 3x + 4 and the remainder = 0

$$\therefore \quad (15x^2 + 11x - 12) \div (5x - 3) = 3x + 4$$

Thus we can divide a polynomial by another polynomial in the above mentioned long division process.

Note : The 1st term of the dividend should not be of a lower power than the 1st term of the divisor.

Now try to Check Your Progress:

E8. Simplify: $5x^2 - 6xy - y^2 - 2x^2 - 3y^2 + 2xy - 2y^2 + x^2$

E9. Subtract $3m - 5n + 7$ from the sum of $2m - 3n + 5$ and $8 + 4n$.

E10. Fill in the blank space:

$$(a^2 - 5ab - 3a + 7) + (\ldots\ldots\ldots\ldots) = 3a^2 + 2ab - 5b + 2$$

E11. Multiply

(a) $(3x - 2)$ by $(2x + 3)$

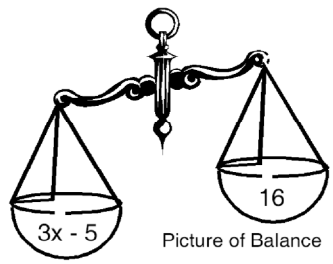
(b) $(p^2 + pq + q^2)$ by $(p - q)$

E12. Divide $3a^3 + 16a^2 + 20 + 21a$ by $(a + 4)$.

9.5 LINEAR ALGEBRAIC EQUATION AND ITS SOLUTION

An equation can be compared with a balance in equilibrium. The two sides of the balance being compared with the two sides i.e. LHS and RHS of an equation. The equality sign indicates that the scale pans are balanced. For example in the equation

$3x - 5 = 16$, the LHS $3x - 5$ is in the left pan and RHS 16 is in the right pan of the balance. Also the balance is in equilibrium as the values of the two sides of the equation are equal. x in the equation is known as the unknown.



Picture of Balance

In the sub-unit the most basic form of algebraic equations i.e. the linear equation will be extensively discussed. The process of solving these equations will also be discussed.

9.5.1 Linear Algebraic Equation

Let us discuss a simple problem, 'to find a number which, when doubled and then added to *5* gives *15*.'

To write the mathematical statement, in this case, we take the unknown as x, then double it and obtain $2x$. When 5 is added to this, the result is $2x + 5$. We know that this equals to 15. Hence the mathematical statement is $2x + 5 = 15$, which is the equation in the above case.

When a polynomial is formed using a certain symbol (taken for an unknown) and is equated to a certain number, the statement formed is an **equation**.

So, $2x + 5 = 15$ is one of the simple forms of algebraic equations. Let us observe the example and write the equation in the table.

Statement	Equation
Sum of a number **x** and 7 is 16	$x + 7 = 16$
3 subtracted from **y** is 10	
9 times of **n** is 36	
Three – fourth of **m** is 12	

The equations you write in the above table are known as a linear equation with one unknown and the power of unknown is 1. Hence we say :

An equation involving only a linear polynomial is called a **linear equation**. A linear equation in one variable generally is of the form $ax + b = 0$ where a, b are numbers and x is unknown

Linear equations are also known as **first degree** equations. We shall restrict our present discussion to such equations only.

Note :

The equation $x^2 + 7x + 6 = 0$ is not linear because the exponent of the unknown is *2*.

The equation $x + 2y = 5$ is linear but it contains 2 unknowns x & y.

9.5.2 Solution of Linear Equations

Consider the linear equation $x + 7 = 16$. Now what is the value of x ? We will go on trying the integer *1, 2, 3………* for *'x'* till we get both sides equal. It can be seen by trial that only when $x = 9$, at that point, equality of both sides of the equation occur. For no other value of x the equation is satisfied.

Thus we know that the equation is satisfied i.e. both the sides have equal value only for one value of the unknown.

For example, the equation $2y = 6$ is satisfied only for $y = 3$. Similarly the equation $m - 3 = 4$ is satisfied only for $m = 7$.

The value of the unknown which makes the equation a true statement is called the **solution** or **root** of the equation.

To solve an equation is to find the value of the unknown which satisfies the equation.

Process of Solving a Linear Equation :

(a) Process of Trial :

Let us complete the table and by inspection of the table find the solution to the equation $x + 7 = 16$.

x	1	2	3	4	5	6	7	8	9	10
x + 7										

Here we go on trying with the integers for 'x' till the equation holds good. In this way we can obtain the value of the unknown. We say this process as the **process of trial**. You have seen that after so many unsuccessful trials we got the value of the unknown which satisfies the equation. Hence, this process is tedious, time consuming and often we fail to find the solution if it is a large number.

Try these :

Solve the equation by process of trial.

(i) $m - 5 = 16$

(ii) $2y - 1 = 17$

(b) Process of Adding or Subtracting :

An equation, as already said can be compared to a balance in equilibrium. The two sides of the equation are like the two pans and the equality sign indicates that the pans are balanced. Doing an arithmetic operation on an equation is like adding weights or removing weights from the pans of the balance. If we add the same weights to both the pans the beam of the balance remains horizontal. Similarly, if we remove equal weights from the pans, the beam also remains horizontal. On the other hand, if we add or remove different weights, the balance is tilted, that is the beam of the balance does not remain horizontal. We use this principle for solving an equation.

Supposing 'x' represents the weight of packet of rice kept on the left pan and a weight 'w' kept on the right pan and the balance remains in equilibrium, then we say : x = w.

Case – I

If we add a weight '*c*' on both the pans, whether the beam remains horizontal or tilted ?

Surely it will remain horizontal.

Thus we get, $x + c = w + c$

Case – II

Had we removed weight '*c*' from both of the pans, in what state would the beam remain ?

Definitely horizontal.

Thus we get $x - c = w - c$

Case – III

Similarly, if we make the weights on both the pans '*c*' times, the beam would still remain horizontal.

The mathematical representation of this situation is

$xc = wc$

Case – IV

Also if we make the weights on both the pans '$\frac{1}{c}$', (where $c \neq o$) times then the beam would remain balanced.

Thus we get, $\frac{x}{c} = \frac{w}{c}$ (when $c \neq 0$)

From the above properties of weighing with a balance, we got four rules of equality for solving an equation. Those rules help in solving linear equations in precise manner. The rules state as follows :

The same quantity can be added to both sides of an equation and it does not change the equality.

The same quantity can be subtracted from both sides of an equation and it does not change the equality.

Both sides of an equation may be multiplied by the same number and it does not change equality.

Both sides of an equation may be divided by a non-zero number and it does not change the quality.

Let us apply the above rules in solving the linear equations :

Ex-25 Solve: $y - 5 = 11$

Solution : Solving an equation means finding the value of the unknown.

So, $y - 5 = 11$

$\Rightarrow (y - 5) + 5 = 11 + 5$ (Adding 5 on both the sides)

$\Rightarrow y + (-5 + 5) = 16$ (Associativity)

$\Rightarrow y + 0 = 16$ (Additive inverse)

$\Rightarrow y = 16$ (Additive identity)

Ex-26 Solve $z + 4 = 8$

Solution : $z + 4 = 8$

$\Rightarrow (z + 4) - 4 = 8 - 4$ (Subtracting 4 from both the sides)

$\Rightarrow z + (4 - 4) = 4$ (Associativity)

$\Rightarrow z + 0 = 4$ (Additive inverse)

$\Rightarrow z = 4$ (Additive Identity)

Ex-27 Solve : $\dfrac{x}{3} = 12$

Solution : $\dfrac{x}{3} = 12$

$\Rightarrow \dfrac{x}{3} \times 3 = 12 \times 3$ (Multiplying 3 to both sides)

$$\Rightarrow x \times \left(\frac{1}{3} \times 3\right) = 36 \qquad \text{(Associativity)}$$

$$\Rightarrow x \times 1 = 36 \qquad \text{(Multiplicative inverse)}$$

$$\Rightarrow x = 36 \qquad \text{(Multiplicative identity)}$$

Ex-28 Solve : $5x - 2 = 28$

Solution : Here the numerical term (constant) is to be eliminated first.

So, $\quad 5x - 2 = 28$

$$\Rightarrow (5x - 2) + 2 = 28 + 2 \qquad \text{(Adding 2 to both the sides)}$$

$$\Rightarrow 5x + (-2 + 2) = 30 \qquad \text{(Associativity)}$$

$$\Rightarrow 5x + 0 = 30 \qquad \text{(Additive inverse)}$$

$$\Rightarrow 5x = 30 \qquad \text{(Additive identity)}$$

$$\Rightarrow \frac{5x}{5} = \frac{30}{5} \qquad \text{(Dividing both sides by 5)}$$

$$\Rightarrow x = 6 \ \text{(Answer)}$$

Note :

In practice we shall not show the steps where the rule/properties are applied.

Try these :

Solve the following equation :

(i) $3x + 2 = 14$ *(ii)* $\frac{x}{4} - 1 = 5$

(c) Process of Transposition

While solving a linear equation, we can transpose a number from LHS to RHS or vice versa instead of adding or subtracting it from both sides of the equation. In doing so, the operation that connects the number on one side of the equation changes as it is removed to the other side. That is on transposing a term from one side to the other the operation of :

(i) Addition changes to subtraction

(ii) Subtraction changes to addition

(iii) Multiplication changes to division

(iv) Division changes to multiplication

These are called the **Rules of Transposition**.

Let us apply these rules in solving the following equations.

Ex-29 Solve: $2x - 7 = 5$

Solution: $2x - 7 = 5$

$\Rightarrow 2x = 5 + 7$ (Transposing 7 to RHS)

$\Rightarrow 2x = 12$

$\Rightarrow x = \dfrac{12}{2}$ (Transposing 2 to RHS)

$\Rightarrow x = 6$

Ex-30 Solve : $\dfrac{5y-2}{3} = 6$

Solution: $\dfrac{5y-2}{3} = 6$

$\Rightarrow 5y - 2 = 6 \times 3$

$\Rightarrow 5y - 2 = 18$

$\Rightarrow 5y = 18 + 2$

$\Rightarrow 5y = 20$

$\Rightarrow y = \dfrac{20}{5} = 4$

$\therefore y = 4$

Try these :

Solve by transposition

(i) $3p + 2 = 17$ (ii) $2(x + 4) = 12$

(d) Rule of Cross Multiplication

If the equation involves a fraction, let us learn an easier method to remove the fraction without disturbing the equality.

Suppose the equation is of the form $\dfrac{a}{b} = \dfrac{c}{d}$

$\Rightarrow a = \dfrac{c}{d} \times b$ (Transposing 'b' to RHS)

$$\Rightarrow a = \frac{c \times b}{d}$$

$\Rightarrow a \times d = c \times b$ (Transposing 'c' to LHS)

Thus we find: $\frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = c \times b$

Otherwise – $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a}{b} \times bd = \frac{c}{d} \times bd$ (Multiplying 'bd' on both sides)

$\Rightarrow ad = cb$

This is known as Rule of Cross Multiplication.

Let us observe some examples for a better understanding.

Ex-31 Solve : $\frac{3x+1}{2} = \frac{x+7}{4}$

***Solution*:** $\frac{3x+1}{2} = \frac{x+7}{4}$

$\Rightarrow (3x+1) \times 4 = (x+7) \times 2$ (By cross multiplying)

$\Rightarrow 12x + 4 = 2x + 14$ (Distribute law)

$\Rightarrow 12x - 2x = 14 - 4$ (Bringing unknown terms to LHS and Constants to RHS)

$\Rightarrow 10x = 10$

$\Rightarrow x = \frac{10}{10}$ (Transposing 10 to RHS)

$\Rightarrow x = 1$

Ex-32 Solve : $\frac{3y-1}{2y+3} = \frac{5}{7}$

Solution: $\frac{3y-1}{2y+3} = \frac{5}{7}$

$\Rightarrow (3y - 1) \times 7 = (2y + 3) \times 5$ (cross multiplication)

Notes

$$\Rightarrow 21y - 7 = 10y + 15$$

$$\Rightarrow 21y - 10y = 15 + 7$$

$$\Rightarrow 11y = 22$$

$$\Rightarrow y = \frac{22}{11} = 2$$

Thus we have discussed four processes of solving a linear equation with one unknown. Among those, processes of transposition and cross multiplication are widely used.

Now try to answer the following questions to Check Your Progress:

E13 Solve the following equations.

(i) $7x = 28$ (ii) $3y - 2 = 19$ (iii) $\frac{x}{2} - 3 = 6$

E14 Solve the following equations.

(i) $28 = 4 + 3(t + 5)$ (ii) $2(2p - 3) = 6$

E15 Solve the following equations.

(i) $\frac{8x}{3x + 6} = \frac{4}{3}$ (ii) $\frac{2x + 3}{3x + 7} = \frac{5}{8}$

9.6 SOLUTION OF WORD PROBLEMS/ APPLICATION OF ALGEBRAIC METHODS

We have learnt how to convert a mathematical statement into a simple equation. Also we have learnt how to solve simple equations. The algorithm (method) involved in solving real life problems is to :

(i) Understand the situation expressed in the word problem.

(ii) Choose a symbol and substitute it for the unknown to be determined.

(iii) Write an equation from the given relation in the problem.

(iv) Solve the equation and find the value of the unknown.

(v) Verify the correctness of the solution.

Let us discuss the following examples:

Ex- 33 Mita thinks of a number. If she takes away 7 from 4 times of the number, the result is 17. What is the number ?

Solution :

Suppose Mita thinks of the number x.

4 times of the number = 4x

After taking away 7 from 4x, Mita has 4x - 7.

According to the problem, 4x - 7 = 17

Hence we got the equation for x.

Let us now solve the equation 4x - 7 = 17

$\Rightarrow\ 4x = 17 + 7 \Rightarrow\ 4x = 24$

$\Rightarrow\ x = \frac{24}{4}$

$\Rightarrow\ x = 6$

So, the required number is 6

Checking the answer :

LHS = 4x – 7 = 4 × 6 – 7 = 24 – 7 = 17 = RHS as required

Ex-34 Vicky is 5 years older than Ricky. 15 years back Vicky's age was 2 times that of Ricky. What are their present ages ?

Solution :

Among them, Ricky is younger.

So let us take the present age of Ricky to be z

The present age of Vicky = (z + 5) years

15 years back, the age of Vicky was z + 5 – 15 = z – 10 years.

And the age of Ricky then was (z – 15) years.

According to the given relation.

Vicky's age is 2 times of Ricky's age

$\Rightarrow\ z - 10 = 2\,(z - 15)$

$\Rightarrow\ z - 10 = 2z - 30$

$\Rightarrow\ z - 2z = -30 + 10 \qquad \Rightarrow\ z = 20$

So, the present age of Ricky = 20 years and that of Vicky = 20 + 5 = 25 years.

Solve the following to Check Your Progress:

E16. The sum of two numbers is 64. One number is 14 more than the other. What are the numbers ?

E17. Solve the following problems.

(a) Sachin scores twice as many runs as Dhoni. Together their runs fell one short of a century. How many runs did each one score?

(b) People of Naraharipur planted 102 trees in the village garden. The number of non-fruit trees ware 2 more than three times the number of fruit trees. What is the number of fruit trees planted?

(c) Sania's age is half the age of her father and her father's age is half the age of her grand father. After twenty years her age will be equal to the present age of her father. What are the present ages of Sania, her father and her grand father?

E18. Solve the following equation:

$$\frac{2x+3}{3x-7} = \frac{5}{8}$$

9.7 LET US SUM UP

Algebra is generalized arithmetic where letters are used as symbols to represent numbers. Every number is a constant and every symbol can be assigned different values in different situations.

Algebraic expressions are formed using symbols and constants. We also use four fundamental operations on the symbols and constants to from expressions.

Terms are parts of an expression which are separated by '+' or '_' sign. It may be a constant, a variable or combination of both.

Any expression having one or more terms is called a Polynomial. Specifically expression having one term is called monomial, having two terms is a binomial and degree of the polynomial is the highest degree of the term among all the terms

The two sides of an equation are like the two pans of a balance

Linear equations can be solved by using any of the four methods : by trial, addition and subtraction of equal quantity to both the sides, transposition and cross multiplication.

9.8 MODEL ANSWERS TO CHECK YOUR PROGRESS

E1. p + q, 2p – q, 3p + 2q – 1 or any three similar expressions.

E2. (i) 5xy + 3 (ii) ab - (a + b)

E3. Variables = y & z, Constant = 2, 3, 5

E4. (i) -1, (ii) 2y (iii) $\frac{2}{3}$ xy

E5. $\left(2p, \frac{p}{3}\right)$, (-3pq, 5pq), $\left(\frac{1}{2}\text{pqr}, \ 3\text{pqr}\right)$

E6. (i) coefficient of xy = 5 , that of 5 = xy etc.

(ii) Coefficient of 3 = -abc, that of abc = -3 etc.

E7. (i) Terms = 3xy, 5y ; Factors of 3xy = 3, x, y ; Factors of 5y = 5, y

(ii) Terms = ab, 2a and 3y

E8. 4x2 – 4xy – 6y2

E9. - m+ 6n + 6

E10. 2a2 + 7ab + 3a - 5b – 5

E11. (a) 6x2 + 5x – 6 (b) p3 – q3

E12. (i) 3a2 + 4a + 5

E13. (i) x = 4 (ii) y = 7 (iii) x = 18

E14. (i) t = 3 (ii) p = 3

E15. (i) x = 2 (ii) x = 11

E16. 25 and 39

E17. (a) Dhoni = 33, Sachin = 66 (b) 25 (c) 20 ,40, 80

E18. x = 11

9.9 SUGGESTED READINGS AND REFERENCES

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9.10 UNIT-END EXERCISE

1. Write the expression for the followings.

    (i) One third of the sum of numbers p and q.

    (ii) Sum of the nos. a and b subtracted from their product.

    (iii) 8 added to 2 times the product of x & y.

2. Find the coefficient (i) of xy in – 5xyz, (ii) of m^2 in $2\ m^2n^2$ (iii) 9 in – 9 pqr.

3. Identify the numerical coefficient of (i) –t (ii) $\frac{2}{3}pq$ (iii) – 8 x2y2

4. Give one example of (i) Monomial (ii) Binomial (iii) Trinomial.

5. Write the like terms together in separate groups

    (i) ab^2, - 4ab, $2a^2b$, ab, - $3ab^2$, $\frac{2}{3}$ab, -$5a^2b$, a^2b^2

    (ii) 2x, - 5xy, - x, $\frac{xy}{2}$, 3y

6. Add the following expressions :

    (i) a + b – 5, b – a + 3 and a – b + 6

    (ii) 4x + 3y – 7xy, 3xy – 2x and 2xy – y

    (iii) $m^2 - n^2 - 1$, $n^2 - 1 - m^2$ and $1 - m^2 - n^2$

7. Subtract :

    (i) x (y – 3) from y (3 – x)

    (ii) $m^2 + 10m - 5$ from 5m – 10.

    (iii) $5a^2 - 7ab + 5b^2$ from $2ab - 3a^2 - 3b^2$

8. Simplify the expression :

    (i) $10x^2 - 8x + 5 + -5x - 4x^2 - 6m - 10$

    (ii) 20mn – 10n – 17m – 12n + 14m + 2

9 Multiply :

    (i) (a – b) by (a2 + ab + b2)

    (ii) (p + q – 5) by (p - q)

10. Divide:

    (i) $(8m^2 + 4m - 60)$ by $(2m - 5)$

    (ii) $(6\,a^2b^2 - 7abc - 3b^2)$ by $(3ab + c)$

11. Solve the following equations:

    (i) $2y - 5 = 9$

    (ii) $\frac{3}{5} + x = \frac{13}{5}$

    (iii) $\frac{z}{3} + \frac{z}{5} = 40$

    (iv) $\frac{8x}{6+3x} = \frac{-4}{3}$

12. Solve the following problems :

    (i) The length of a rectangular plot exceeds its breadth by 5m. The perimeter of the plot is 70m. Find the length of the plot.

    (ii) In an isosceles triangle the base angles are equal. The vertex angle is 15° more than each of the base angles. Find the measure of the angles of the triangle.