



## PROPERTIES OF FLUIDS

In the previous lesson, you have learnt that interatomic forces in solids are responsible for determining the elastic properties of solids. Does the same hold for liquids and gases? (These are collectively called fluids because of their nature to flow in suitable conditions). Have you ever visited the site of a dam on a river in your area / state/ region? If so, you would have noticed that as we go deeper, the thickness of the walls increases. Did you think of the underlying physical principle? Similarly, can you believe that you can lift a car, truck or an elephant by your own body weight standing on one platform of a hydraulic lift? Have you seen a car on the platform of a hydraulic jack at a service centre? How easily is it lifted? You might have also seen that mosquitoes can sit or walk on still water, but we cannot do so. You can explain all these observations on the basis of properties of liquids like hydrostatic pressure, Pascal's law and surface tension. You will learn about these in this lesson.

Have you experienced that you can walk faster on land than under water? If you pour water and honey in separate funnels you will observe that water comes out more easily than honey. In this lesson we will learn the properties of liquids which cause this difference in their flow.

You may have experienced that when the opening of soft plastic or rubber water pipe is pressed, the stream of water falls at larger distance. Do you know how a cricketer swings the ball? How does an aeroplane take off? These interesting observations can be explained on the basis of Bernoulli's principle. You will learn about it in this lesson.



### OBJECTIVES

After studying this lesson, you would be able to :

- calculate the hydrostatic pressure at a certain depth inside a liquid;

## MODULE - 2

### Mechanics of Solids and Fluids



#### Notes

## Properties of Fluids

- describe buoyancy and Archimedes Principle;
- state Pascal's law and explain the functioning of hydrostatic press, hydraulic lift and hydraulic brakes.;
- explain surface tension and surface energy ;
- derive an expression for the rise of water in a capillary tube;
- differentiate between streamline and turbulent motion of fluids;
- define critical velocity of flow of a liquid and calculate Reynold's number;
- define viscosity and explain some daily life phenomena based on viscosity of a liquid; and
- state Bernoulli's Principle and apply it to some daily life experiences.

### 9.1 HYDROSTATIC PRESSURE

While pinning papers, you must have experienced that it is easier to work with a sharp tipped pin than a flatter one. If area is large, you will have to apply greater force. Thus we can say that for the same force, the effect is greater for smaller area. This effect of force on unit area is called *pressure*.

Refer to Fig. 9.1. It shows the shape of the side wall of a dam. Note that it is thicker at the base. Do we use similar shape for the walls of our house. No, the walls of rooms are of uniform thickness. Do you know the basic physical characteristic which makes us to introduce this change?

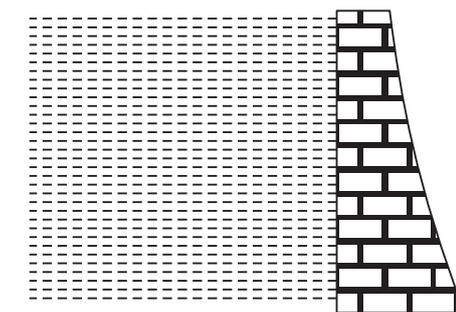


Fig. 9.1 : The structure of side wall of a dam

From the previous lesson you may recall that solids develop shearing stress when deformed by an external force, because the magnitude of inter-atomic forces is very large. But fluids do not have shearing stress and when an object is submerged in a fluid, the force due to the fluid acts normal to the surface of the object (Fig. 9.2). Also, the fluid exerts a force on the container normal to its walls at all points.



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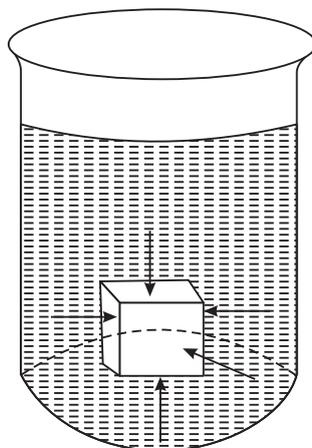


Fig. 9.2 : Force exerted by a fluid on a submerged object

The normal force or thrust per unit area exerted by a fluid is called pressure. We denote it by  $P$  :

$$P = \frac{\text{Thrust}}{\text{area}} \quad (9.1)$$

The pressure exerted by a fluid at rest is known as hydrostatic pressure

The SI Unit of pressure is  $\text{Nm}^{-2}$  and is also called pascal (Pa) in the honour of French scientist Blaise Pascal.

### 9.1.1 Hydrostatic Pressure at a point inside a liquid

Consider a liquid in a container and an imaginary right circular cylinder of cross sectional area  $A$  and height  $h$ , as shown in Fig. 9.3. Let the pressure exerted by the liquid on the bottom and top faces of the cylinder be  $P_1$ , and  $P_2$ , respectively. Therefore, the upward force exerted by the liquid on the bottom of the cylinder is  $P_1A$  and the downward force on the top of the cylinder is  $P_2A$ .

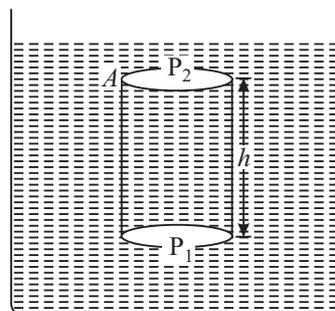


Fig. 9.3 : An imaginary cylinder of height  $h$  in a liquid.

$\therefore$  The net force in upward direction is  $(P_1A - P_2A)$ .

Now mass of the liquid in cylinder = density  $\times$  volume of the cylinder

$$= \rho \cdot A \cdot h \text{ where } \rho \text{ is the density of the liquid.}$$

$\therefore$  Weight of the liquid in the cylinder =  $\rho \cdot g \cdot h \cdot A$

Since the cylinder is in equilibrium, the resultant force acting on it must be equal to zero, i.e.



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$$P_1 A - P_2 A - \rho g h A = 0$$

$$\Rightarrow P_1 - P_2 = \rho g h \tag{9.2}$$

So, the pressure  $P$  at the bottom of a column of liquid of height  $h$  is given by

$$P = \rho g h$$

That is, hydrostatic pressure due to a fluid increases linearly with depth. It is for this reason that the thickness of the wall of a dam has to be increased with increase in the depth of the dam.

If we consider the upper face of the cylinder to be at the open surface of the liquid, as shown in Fig.(9.4), then  $P_2$  will have to be replaced by  $P_{atm}$  (Atmospheric pressure). If we denote  $P_1$  by  $P$ , the absolute pressure at a depth below the surface will be

$$P = P_{atm} + \rho g h$$

or 
$$P = P_{atm} + \rho g h \tag{9.3}$$

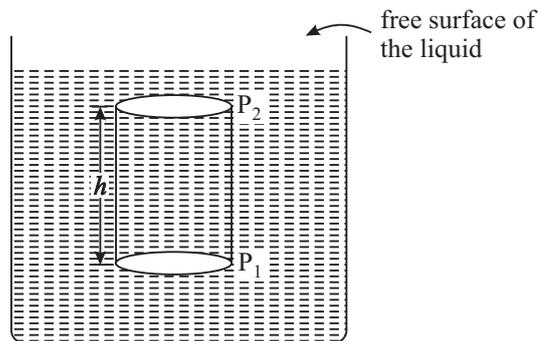


Fig. 9.4 : Cylinder in a liquid with one face at the surface of the liquid

Note that the expression given in Eqn. (9.3) does not show any term having area of the cylinder. It means that pressure in a liquid at a given depth is equal, irrespective of the shape of the vessel (Fig 9.5).

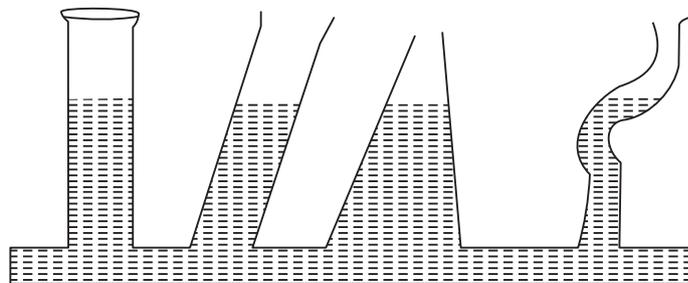


Fig. 9.5 : Pressure does not depend upon shape of the vessel.



**Example 9.1:** A cemented wall of thickness one metre can withstand a side pressure of  $10^5 \text{ Nm}^{-2}$ . What should be the thickness of the side wall at the bottom of a water dam of depth 100 m. Take density of water =  $10^3 \text{ kg m}^{-3}$  and  $g = 9.8 \text{ ms}^{-2}$ .

**Solution:** The pressure on the side wall of the dam at its bottom is given by

$$\begin{aligned} P &= h d g \\ &= 100 \times 10^3 \times 9.8 \\ &= 9.8 \times 10^5 \text{ Nm}^{-2} \end{aligned}$$

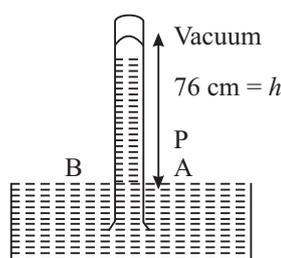
Using unitary method, we can calculate the thickness of the wall, which will withstand pressure of  $9.8 \times 10^5 \text{ Nm}^{-2}$ . Therefore thickness of the wall

$$\begin{aligned} t &= \frac{9.8 \times 10^5 \text{ Nm}^{-2}}{10^5 \text{ Nm}^{-2}} \\ &= 9.8 \text{ m} \end{aligned}$$

### 9.1.2 Atmospheric Pressure

We know that the earth is surrounded by an atmosphere upto a height of about 200 km. The pressure exerted by the atmosphere is known as the *atmospheric pressure*. A German Scientist O.V. Guericke performed an experiment to demonstrate the force exerted on bodies due to the atmospheric pressure. He took two hollow hemispheres made of copper, having diameter 20 inches and tightly joined them with each other. These could easily be separated when air was inside. When air between them was exhausted with an air pump, 8 horses were required to pull the hemispheres apart.

Toricelli used the formula for hydrostatic pressure to determine the magnitude of atmospheric pressure.



**Fig: 9.6 :** Toricelli's Barometer

He took a tube of about 1 m long filled with mercury of density  $13,600 \text{ kg m}^{-3}$  and placed it vertically inverted in a mercury tub as shown is Fig. 9.6. He observed that the column of 76 cm of mercury above the free surface remained filled in the tube.

In equilibrium, atmospheric pressure equals the pressure exerted by the mercury column. Therefore,

$$\begin{aligned} P_{atm} &= h \rho g = 0.76 \times 13600 \times 9.8 \text{ Nm}^{-2} \\ &= 1.01 \times 10^5 \text{ Nm}^{-2} \\ &= 1.01 \times 10^5 \text{ Pa} \end{aligned}$$



Notes

9.2 BUOYANCY

It is a common experience that lifting an object in water is easier than lifting it in air. It is because of the difference in the upward forces exerted by these fluids on these object. The upward force, which acts on an object when submerged in a fluid, is known as **buoyant force**. The nature of buoyant force that acts on objects placed inside a fluid was discovered by Archimedes. Based on his observations, he enunciated a law now known as Archimedes principle. It states that *when an object is submerged partially or fully in a fluid, the magnitude of the buoyant force on it is always equal to the weight of the fluid displaced by the object.*

The different conditions of an object under buoyant force is shown in Fig 9.7.

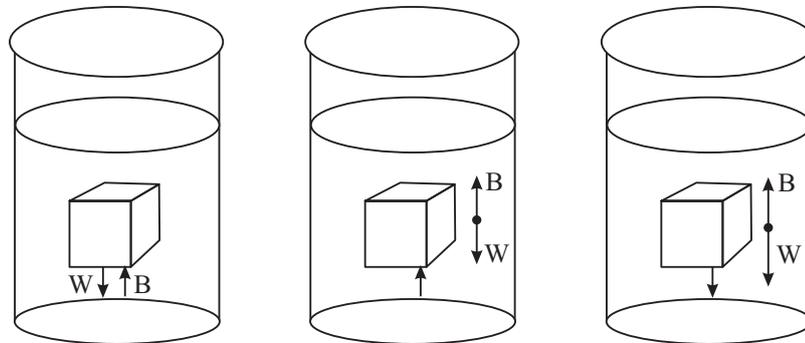


Fig. 9.7:

(a) : The magnitude of buoyant force  $B$  on the object is exactly equal to its weight in equilibrium.

(b) : A totally submerged object of density less than that of the fluid experiences a net upward force.

(c) : A totally submerged object denser than the fluid sinks.

Another example of buoyant force is provided by the motion of hot air balloon shown in Fig. 9.8. Since hot air has less density than cold air, a net upward buoyant force on the balloon makes it to float.

Floating objects

You must have observed a piece of wood floating on the surface of water. Can you identify the forces acting on it when it is in equilibrium? Obviously, one of the forces is due to gravitational force, which pulls it downwards. However, the displaced water exerts buoyant force which acts upwards. These forces balance each other in equilibrium state and the object is then said to be floating on water. It means that a floating body displaces the fluid equal to its own weight.

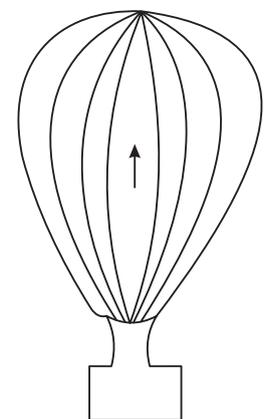
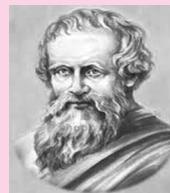


Fig. 9.8: Hot air balloon floating in air

**Archimedes**  
(287- 212 B.C)

A Greek physicist, engineer and mathematician was perhaps the greatest scientist of his time. He is well known for discovering the nature of buoyant forces acting on objects. The Archimedes screw is used even today. It is an inclined rotating coiled tube used originally to lift water from the hold of ships. He also invented the catapult and devised the system of levers and pulleys.

Once Archimedes was asked by king Hieron of his native city Syracuse to determine whether his crown was made up of pure gold or alloyed with other metals without damaging the crown. While taking bath, he got a solution, noting a partial loss of weight when submerging his arm and legs in water. He was so excited about his discovery that he ran undressed through the streets of city shouting “Eureka, Eureka”, meaning I have found it.



Notes

**9.3 PASCAL'S LAW**

While travelling by a bus, you must have observed that the driver stops the bus by applying a little force on the brakes by his foot. Have you seen the hydraulic jack or lift which can lift a car or truck up to a desired height? For this purpose you may visit a motor workshop. Packing of cotton bales is also done with the help of hydraulic press which works on the same principle.

These devices are based on Pascal's law, which states that *when pressure is applied at any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.*

This law is also known as the **law of transmission of liquid pressure.**

**9.3.1 Applications of Pascal's Law****(A) Hydraulic Press/Balance/Jack/Lift**

It is a simple device based on Pascal's law and is used to lift heavy loads by applying a small force. The basic arrangement is shown in Fig.9.9. Let a force  $F_1$  be applied to the smaller piston of area  $A_1$ . On the other side, the piston of large area  $A_2$  is attached to a platform where heavy load may be placed. The pressure on the smaller piston is transmitted to the larger piston through the liquid filled in-between the two pistons. Since the pressure is same on both the sides, we have

$$\text{Pressure on the smaller piston, } P = \frac{\text{force}}{\text{area}} = \frac{F_1}{A_1}$$



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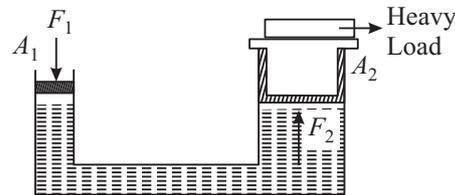


Fig. 9.9: Hydraulic lift

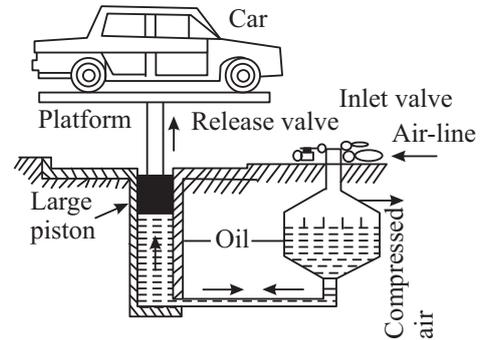


Fig. 9.10: Hydraulic jack

According to Pascal’s law, the same pressure is transmitted to the larger cylinder of area  $A_2$ .

Hence the force acting on the larger piston

$$F_2 = \text{pressure} \times \text{area} = \frac{F_1}{A_1} \times A_2 \quad (9.4)$$

It is clear from Eqn. ( 9.4) that force  $F_2 > F_1$  by an amount equal to the ratio  $(A_2/A_1)$ . With slight modifications, the same arrangement is used in hydraulic press, hydraulic balance, and hydraulic Jack, etc.

**(B) Hydraulic Jack or Car Lifts**

At automobile service stations, you would see that cars, buses and trucks are raised to the desired heights so that a mechanic can work under them (Fig 9.10). This is done by applying pressure, which is transmitted through a liquid to a large surface to produce sufficient force needed to lift the car.

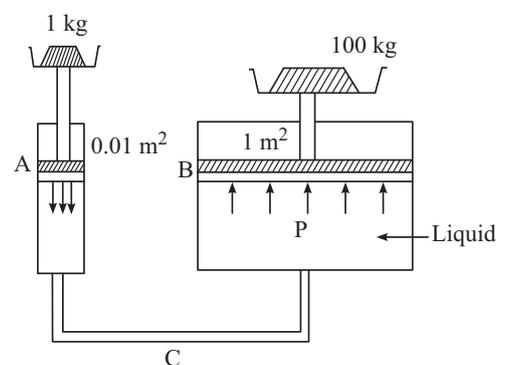


Fig. 9.11(a) : Hydraulic balance

**(C) Hydraulic Brakes**

While traveling in a bus or a car, we see how a driver applies a little force by his foot on the brake paddle to stop the vehicle. The pressure so applied gets transmitted through the brake oil to the piston of slave cylinders, which, in turn, pushes the break shoes against the break drum in all four wheels, simultaneously. The wheels stop rotating at the same time and the vehicle comes to stop instantaneously.

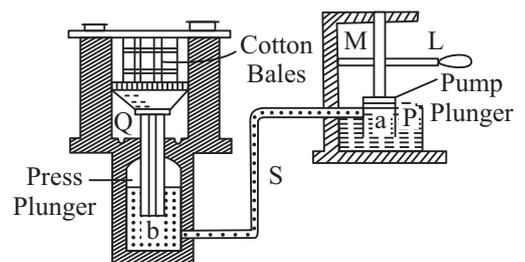


Fig. 9.11(b) : Hydraulic press



## INTEXT QUESTIONS 9.1

1. Why are the shoes used for skiing on snow made big in size?
2. Calculate the pressure at the bottom of an ocean at a depth of 1500 m. Take the density of sea water  $1.024 \times 10^3 \text{ kg m}^{-3}$ , atmospheric pressure  $= 1.01 \times 10^5 \text{ Pa}$  and  $g = 9.80 \text{ ms}^{-2}$ .
3. An elephant of weight 5000 kg f is standing on the bigger piston of area  $10 \text{ m}^2$  of a hydraulic lift. Can a boy of 25 kg wt standing on the smaller piston of area  $0.05 \text{ m}^2$  balance or lift the elephant?
4. If a pointed needle is pressed against your skin, you are hurt but if the same force is applied by a rod on your skin nothing may happen. Why?
5. A body of 50 kg f is put on the smaller piston of area  $0.1 \text{ m}^2$  of a big hydraulic lift. Calculate the maximum weight that can be balanced on the bigger piston of area  $10 \text{ m}^2$  of this hydraulic lift.



Notes

## 9.4 SURFACE TENSION

It is common experience that in the absence of external forces, drops of liquid are always spherical in shape. If you drop small amount of mercury from a small height, it spreads in small spherical globules. The water drops falling from a tap or shower are also spherical. Do you know why it is so? You may have enjoyed the soap bubble game in your childhood. But you can not make pure water bubbles with same ease? All the above experiences are due to a characteristic property of liquids, which we call **surface tension**. To appreciate this, we would like you to do the following activity.



## ACTIVITY 9.1

1. Prepare a soap solution.
2. Add a small amount of glycerin to it.
3. Take a narrow hard plastic or glass tube. Dip its one end in the soap solution so that some solution enters into it.
4. Take it out and blow air at the other end with your mouth.
5. Large soap bubble will be formed.
6. Give a jerk to the tube to detach the bubble which then floats in the air.

To understand as to how surface tension arises, let us refresh our knowledge of intermolecular forces. In the previous lesson, you have studied the variation of intermolecular forces with distance between the centres of molecules/atoms.



Notes

The intermolecular forces are of two types: **cohesive** and **adhesive**. Cohesive forces characterise attraction between the molecules of the same substance, whereas force of adhesion is the attractive force between the molecules of two different substances. It is the force of adhesion which makes it possible for us to write on this paper. Gum, Fevicol etc. show strong adhesion.

We hope that now you can explain why water wets glass while mercury does not.



ACTIVITY 9.2

To show adhesive forces between glass and water molecule.

1. Take a clean sheet of glass
2. Put a few drops of water on it
3. Hold water containing side downward.
4. Observe the water drops.

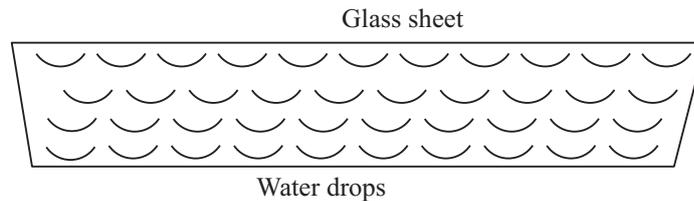


Fig. 9.12: Water drops remain stuck to the glass sheet

The Adhesive forces between glass and water molecules keep the water drops sticking on the glass sheet, as shown in Fig. 9.12.

9.4.1 Surface Energy

The surface layer of a liquid in a container exhibits a property different from the rest of the liquid. In Fig. 9.13, molecules are shown at different heights in a liquid. A molecule, say P, well inside the liquid is attracted by other molecules from all sides. However, it is not the case for the molecules at the surface.

Molecules S and R, which lie on the surface layer, experience a net resultant force downward because the number of molecules

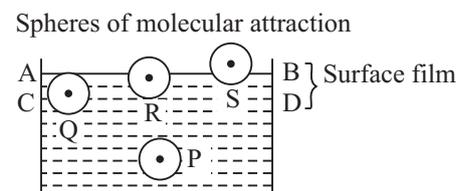


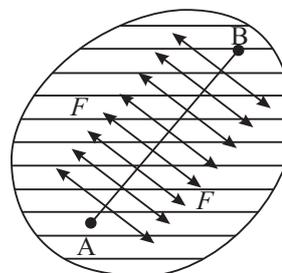
Fig 9.13 : Resultant force acting on P and Q is zero but molecules R and S experience a net vertically downward force.



in the upper half of sphere of influence attracting these molecules is less than those in the lower half. If we consider the molecules of liquid on the upper half of the surface of the liquid or liquid-air interface, even then the molecules will experience a net downward force because of less number of molecules of liquid. Therefore, if any liquid molecule is brought to the surface layer, work has to be done against the net inward force, which increases their potential energy. This means that surface layer possesses an additional energy, which is termed as *surface energy*.

For a system to be in equilibrium, its potential energy must be minimum. Therefore, the area of surface must be minimum. That is why free surface of a liquid at rest tends to attain minimum surface area. This produces a tension in the surface, called **surface tension**.

**Surface tension is a property of the liquid surface due to which it has the tendency to decrease its surface area.** As a result, the surface of a liquid acts like a stretched membrane. You can visualise its existence easily by placing a needle gently on water surface and see it float.



**Fig. 9.14 :** Direction of surface tension on a liquid surface

Let us now understand this physically. Consider an imaginary line AB drawn at the surface of a liquid at rest, as shown in Fig 9.14. The surface on either side of this line exerts a pulling force on the surface on the other side.

The **surface tension of a liquid can be defined as the force per unit length in the plane of liquid surface :**

$$T = F/L \quad (9.5)$$

where surface tension is denoted by  $T$  and  $F$  is the magnitude of total force acting in a direction normal to the imaginary line of length  $L$ , (Fig 9.14) and tangential to the liquid surface. SI unit of surface tension is  $\text{Nm}^{-1}$  and its dimensions are  $[\text{MT}^{-2}]$ .

**Let us take a rectangular frame, as shown in Fig. 9.15 having a sliding wire on one of its arms. Dip the frame in a soap solution and take out. A soap film will be formed on the frame and have two surfaces. Both the surfaces are in contact with the sliding wire, So we can say that surface tension acts on the wire due to both these surfaces.**

Let  $T$  be the surface tension of the soap solution and  $L$  be the length of the wire.



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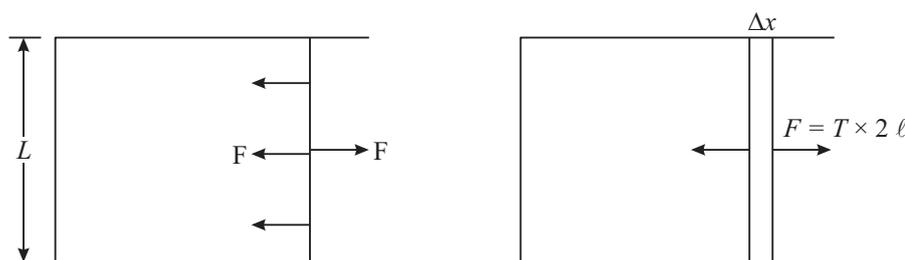


Fig. 9.15: A Film in equilibrium

The force exerted by each surface on the wire will be equal to  $T \times L$ . Therefore, the total force  $F$  on the wire =  $2TL$ .

Suppose that the surfaces tend to contract say, by  $\Delta x$ . To keep the wire in equilibrium we will have to apply an external uniform force equal to  $F$ . If we increase the surface area of the film by pulling the wire with a constant speed through a distance  $\Delta x$ , as shown in Fig. 9.15b, the work done on the film is given by

$$W = F \times \Delta x = T \times 2L \times \Delta x$$

where  $2L \times \Delta x$  is the total increase in the area of both the surfaces of the film. Let us denote it by  $A$ . Then, the expression for work done on the film simplifies to

$$W = T \times A$$

This work done by the external force is stored as the potential energy of the new surface and is called as surface energy. By rearranging terms, we get the required expression for surface tension :

$$T = W/A \tag{9.6}$$

Thus, we see that **surface tension of a liquid is equal to the work done in increasing the surface area of its free surface by one unit**. We can also say that **surface tension is equal to the surface energy per unit area**.

We may now conclude that surface tension

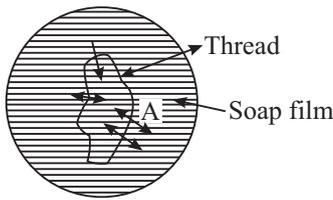
- is a property of the surface layer of the liquid or the interface between a liquid and any other substance like air;
- tends to reduce the surface area of the free surface of the liquid;
- acts perpendicular to any line at the free surface of the liquid and is tangential to its meniscus;
- has genesis in intermolecular forces, which depend on temperature; and
- decreases with temperature.

A simple experiment described below demonstrates the property of surface tension of liquid surfaces.

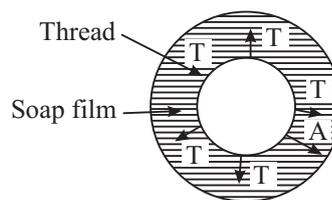


**ACTIVITY 9.3**

Take a thin circular frame of wire and dip it in a soap solution. You will find that a soap film is formed on it. Now take a small circular loop of cotton thread and put it gently on the soap film. The loop stays on the film in an irregular shape as shown in Fig. 9.16(a). Now take a needle and touch its tip to the soap film inside the loop. What do you observe?



**Fig. 9.16 (a) :** A soap film with closed loop of thread



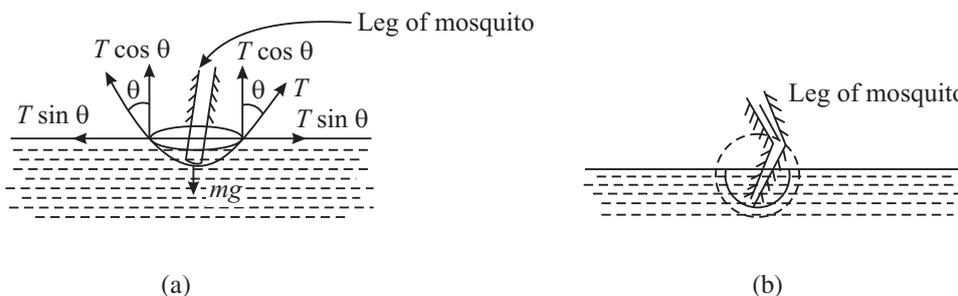
**Fig. 9.16 (b) :** The shape of the thread without inner soap film

You will find that the loop of cotton thread takes a circular shape as shown in Fig 9.16(b). Initially there was soap film on both sides of the thread. The surface on both sides pulled it and net forces of surface tension were zero. When inner side was punctured by the needle, the outside surface pulled the thread to bring it into the circular shape, so that it may acquire minimum area.

**9.4.2 Applications of Surface Tension**

**(a) Mosquitoes sitting on water**

In rainy season, we witness spread of diseases like dengue, malaria and chikungunya by mosquito breeding on fresh stagnant water. Have you seen mosquitoes sitting on water surface? They do not sink in water due to surface tension. At the points where the legs of the mosquito touch the liquid surface, the surface becomes concave due to the weight of the mosquito. The surface tension



**Fig. 9.17 :** The weight of a mosquito is balanced by the force of surface tension  $= 2\pi rT \cos \theta$  (a) Dip in the level to form concave surface, and (b) magnified image



**Notes**



Notes

acting tangentially on the free surface, therefore, acts at a certain angle to the horizontal. Its vertical component acts upwards. The total force acting vertically upwards all along the line of contact of certain length balances the weight of the mosquito acting vertically downward, as shown in Fig 9.17.

(b) Excess pressure on concave side of a spherical surface

Consider a small surface element with a line PQ of unit length on it, as shown in Fig. 9.18. If the surface is plane, i.e.  $\theta = 90^\circ$ , the surface tension on the two sides tangential to the surface balances and the resultant tangential force is zero [Fig. 9.18 (a)]. If, however, the surface is convex, [Fig. (9.18 (b))] or concave [Fig. 9.18 (c)], the forces due to surface tension acting across the sides of the line PQ will have resultant force **R** towards the center of curvature of the surface.

Thus, whenever the surface is curved, the surface tension gives rise to a pressure directed towards the center of curvature of the surface. This pressure is balanced by an equal and opposite pressure acting on the surface. Therefore, there is always an excess pressure on the concave side of the curved liquid surface [Fig. (9.18 b)].

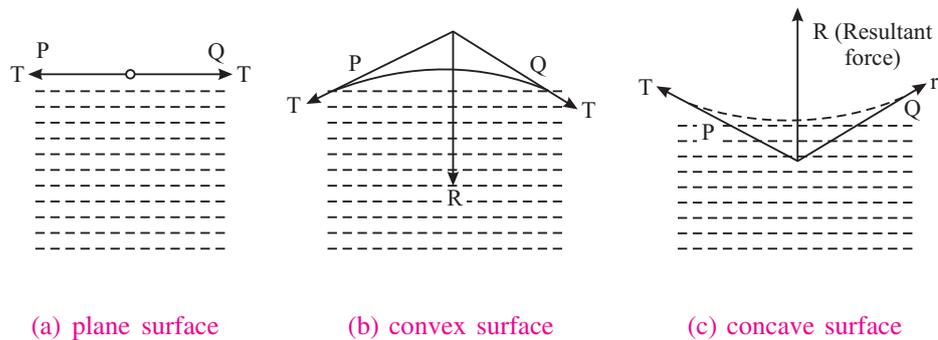


Fig. 9.18

(i) Spherical drop

A liquid drop has only one surface i.e. the outer surface. (The liquid area in contact with air is called the surface of the liquid.) Let  $r$  be the radius of a small spherical liquid drop and  $P$  be excess pressure inside the drop (which is concave on the inner side, but convex on the outside). Then

$$P = (P_i - P_0)$$

where  $P_i$  and  $P_0$  are the inside and outside pressures of the drop, respectively (Fig 9.19a)

If the radius of the drop increases by  $\Delta r$  due to this constant excess pressure  $P$ , then increase in surface area of the spherical drop is given by

$$\begin{aligned} \Delta A &= 4\pi (r + \Delta r)^2 - 4\pi r^2 \\ &= 8\pi r \Delta r \end{aligned}$$

where we have neglected the term containing second power of  $\Delta r$ .

The work done on the drop for this increase in area is given by

$$\begin{aligned} W &= \text{Extra surface energy} \\ &= T\Delta A = T \cdot 8\pi r \Delta r \end{aligned} \quad (9.7)$$

If the drop is in equilibrium, this extra surface energy is equal to the work done due to expansion under the pressure difference or excess pressure  $P$ :

$$\text{Work done} = P \Delta V = P \cdot 4\pi r^2 \Delta r \quad (9.8)$$

On combining Eqns. (9.7) and (9.8), we get

$$P \cdot 4\pi r^2 \Delta r = T \cdot 8\pi r \Delta r$$

$$\text{Or} \quad P = 2T/r \quad (9.9)$$

### (ii) Air Bubble in water

An air bubble also has a single surface, which is the inner surface (Fig. 9.19b). Hence, the excess of pressure  $P$  inside an air bubble of radius  $r$  in a liquid of surface tension  $T$  is given by

$$P = 2T/r \quad (9.10)$$

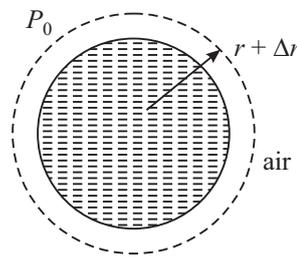


Fig. 9.19 (a) : A spherical drop

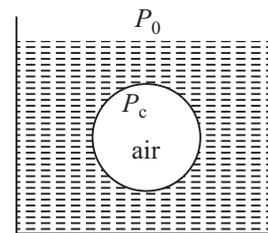


Fig. 9.19 b : Air Bubble

### (iii) Soap bubble floating in air

The soap bubble has two surfaces of equal surface area (i.e. the outer and inner), as shown in Fig. 9.19(c). Hence, excess pressure inside a soap bubble floating in air is given by

$$P = 4T/r \quad (9.11)$$

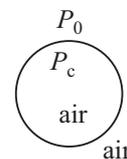


Fig. 9.19 (c)

where  $T$  is surface tension of soap solution.

This is twice that inside a spherical drop of same radius or an air bubble in water. Now you can understand why a little extra pressure is needed to form a soap bubble.

**Example 9.3:** Calculate the difference of pressure between inside and outside of a (i) spherical soap bubble in air, (ii) air bubble in water, and (iii) spherical drop of water, each of radius 1 mm. Given surface tension of water =  $7.2 \times 10^{-2} \text{ Nm}^{-1}$  and surface tension of soap solution =  $2.5 \times 10^{-2} \text{ Nm}^{-1}$ .



Notes



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**Solution:**

(i) Excess pressure inside a soap bubble of radius  $r$  is

$$\begin{aligned}
 P &= 4T/r \\
 &= \frac{4 \times 2.5 \times 10^{-2}}{1 \times 10^{-3}} \text{ Nm}^{-1} \\
 &= 100 \text{ Nm}^{-2}
 \end{aligned}$$

(ii) Excess pressure inside an air bubble in water

$$\begin{aligned}
 &= 2T'/r \\
 &= \frac{2 \times 7.2 \times 10^{-2} \text{ Nm}^{-1}}{1 \times 10^{-3} \text{ m}} \\
 &= 144 \text{ Nm}^{-2}
 \end{aligned}$$

(iii) Excess pressure inside a spherical drop of water  $= 2T'/r$

$$= 144 \text{ Nm}^{-2}$$

**(c) Detergents and surface tension**

You may have seen different advertisements highlighting that detergents can remove oil stains from clothes. Water is used as cleaning agent. Soap and detergents lower the surface tension of water. This is desirable for washing and cleaning since high surface tension of pure water does not allow it to penetrate easily between the fibers of materials, where dirt particles or oil molecules are held up.

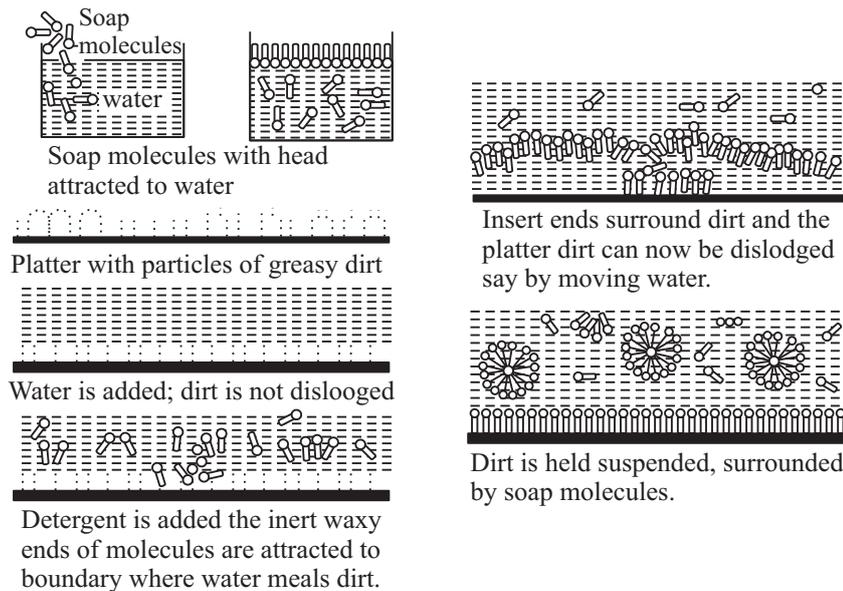


Fig: 9.20 : Detergent action



You now know that surface tension of soap solution is smaller than that of pure water but the surface tension of detergent solutions is smaller than that of soap solution. That is why detergents are more effective than soap. A detergent dissolved in water weakens the hold of dirt particles on the cloth fibers which therefore, get easily detached on squeezing the cloth.

The addition of detergent, whose molecules attract water as well as oil, drastically reduces the surface tension ( $T$ ) of water-oil. It may even become favourable to form such interfaces, i.e. globes of dirt surrounded by detergent and then by water. This kind of process using surface active detergents is important for not only cleaning the clothes but also in recovering oil, mineral ores etc.

#### (d) Wax-Duck floating on water

You have learnt that the surface tension of liquids decreases due to dissolved impurities. If you stick a tablet of camphor to the bottom of a wax-duck and float it on still water surface, you will observe that it begins to move randomly after a minute or two. This is because camphor dissolves in water and the surface tension of water just below the duck becomes smaller than the surrounding liquid. This creates a net difference of force of surface tension which makes the duck to move.

Now, it is time for you to check how much you have learnt. Therefore, answer the following questions.



#### INTEXT QUESTIONS 9.2

1. What is the difference between force of cohesion and force of adhesion?
2. Why do small liquid drops assume a spherical shape.
3. Do solids also show the property of surface tension? Why?
4. Why does mercury collect into globules when poured on plane surface?
5. Which of the following has more excess pressure?
  - (i) An air bubble in water of radius 2 cm. Surface tension of water is  $727 \times 10^{-3} \text{ Nm}^{-1}$  or
  - (ii) A soap bubble in air of radius 4 cm. Surface tension of soap solution is  $25 \times 10^{-3} \text{ Nm}^{-1}$ .

### 9.5 ANGLE OF CONTACT

You can observe that the free surface of a liquid kept in a container is curved. For example, when water is filled in a glass jar, it becomes concave but if we fill water



Notes

in a paraffin wax container, the surface of water becomes convex. Similarly, when mercury is filled in a glass jar, its surface become convex. Thus, we see that shape of the liquid surface in a container depends on the nature of the liquid, material of container and the medium above free surface of the liquid. To characterize it, we introduce the concept of angle of contact.

It is the angle that the tangential plane to the liquid surface makes with the tangential plane to the wall of the container, to the point of contact, as measured from within the liquid, is known as angle of contact.

Fig. 9.21 shows the angles of contact for water in a glass jar and paraffin jar. The angle of contact is acute for concave spherical meniscus, e.g. water with glass and obtuse (or greater than  $90^\circ$ ) for convex spherical meniscus e.g. water in paraffin or mercury in glass tube.

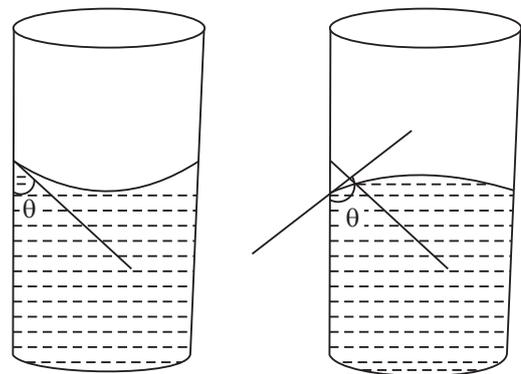


Fig 9.21 : Nature of free surface when water is filled in (a) glass jar, and (b) paraffin wax jar

Various forces act on a molecule in the surface of a liquid contained in a vessel near the boundary of the meniscus. As the liquid is present only in the lower quadrant, the resultant cohesive force acts on the molecule at P symmetrically, as shown in the Fig.9.22(a). Similarly due to symmetry, the resultant adhesive force  $F_a$  acts outwards at right angles to the walls of the container vessel. The force  $F_c$  can be resolved into two mutually perpendicular components  $F_c \cos$

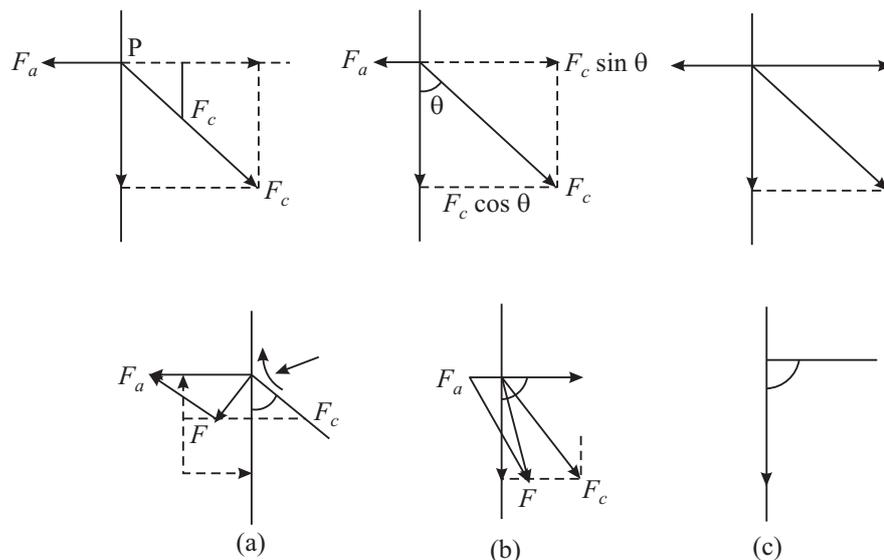


Fig. 9.22 : Different shapes of liquid meniscuses



$\theta$  acting vertically downwards and  $F_c \sin \theta$  acting at right angled to the boundary, The value of the angle of contact depends upon the relative values of  $F_c$  and  $F_a$ .

**CASE 1:** If  $F_a > F_c \sin \theta$ , the net horizontal force is outward and the resultant of  $(F_a - F_c \sin \theta)$  and  $F_c \cos \theta$  lies outside the wall. Since liquids can not sustain constant shear, the liquid surface and hence all the molecules in it near the boundary adjust themselves at right angles to  $F_c$  so that no component of  $F$  acts tangential to the liquid surface. Obviously such a surface at the boundary is concave spherical ( Since radius of a circle is perpendicular to the circumference at every point.) This is true in the case of water filled in a glass tube.

**Case 2 :** If  $F_a < F_c \sin \theta$  the resultant  $F$  of  $(F_c \sin \theta - F_a)$  acting horizontally and  $F_c \cos \theta$  acting vertically down wards is in the lower quadrant acting into the liquid. The liquid surface at the boundary, therefore, adjusts itself at right angles to this and hence becomes convex spherical. This is true for the case of mercury filled in the glass tube.

**Case 3 :** When  $F_a = F_c \sin \theta$ , the resultant force acts vertically downwards and hence the liquid surface near the boundary becomes horizontal or plane.

## 9.6 CAPILLARY ACTION

You might have used blotting paper to absorb extra ink from your notebook. The ink rises in the narrow air gaps in the blotting paper. Similarly, if the lower end of a cloth gets wet, water slowly rises upward. Also water given to the fields rises in the innumerable capillaries in the stems of plants and trees and reaches the branches and leaves. Do you know that farmers plough their fields only after rains so that the capillaries formed in the upper layers of the soil are broken. Thus, water trapped in the soil is taken up by the plants. On the other hand, we find that when a capillary tube is dipped into mercury, the level of mercury inside it is below the outside level. Such an important phenomenon of the elevation or depression of a liquid in an open tube of small cross- section (i.e., capillary tube) is basically due to surface tension and is known as capillary action.

**The phenomenon of rise or depression of liquids in capillary tubes is known as capillary action or capillarity.**

### 9.6.1 Rise of a Liquid in a Capillary Tube

Let us take a capillary tube dipped in a liquid, say water. The meniscus inside the tube will be concave, as shown in Fig. 9.23 (a). This is essentially because the forces of adhesion between glass and water are greater than cohesive forces.

Let us consider four points A, B, C and D near the liquid-air interface Fig. 9.23(a). We know that pressure just below the meniscus is less than the pressure just above it by  $2T/R$ , i.e.



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$$P_B = P_A - 2T/R \tag{9.12}$$

where  $T$  is surface tension at liquid-air interface and  $R$  is the radius of concave surface.

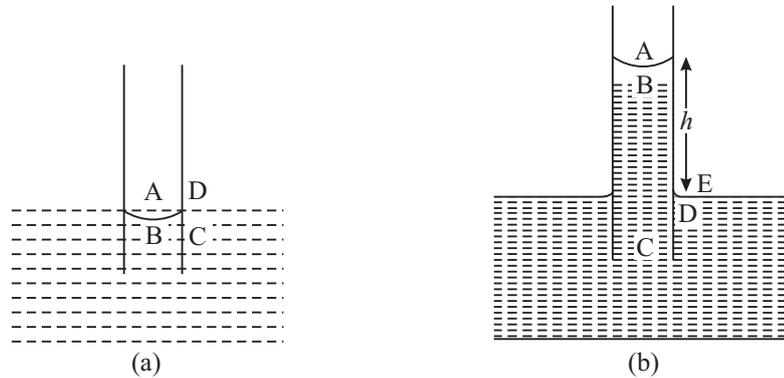


Fig. 9.23 : Capillary action

But pressure at A is equal to the pressure at D and is equal to the atmospheric pressure  $P$  (say). And pressure at D is equal to pressure at C. Therefore, pressure at B is less than pressure at D. But we know that the pressure at all points at the same level in a liquid must be same. That's why water begins to flow from the outside region into the tube to make up the deficiency of pressure at point B.

Thus liquid begins to rise in the capillary tube to a certain height  $h$  (Fig 9.23 b) till the pressure of liquid column of height  $h$  becomes equal to  $2T/R$ . Thereafter, water stops rising. In this condition

$$h \rho g = 2 T/R \tag{9.13}$$

where  $\rho$  is the density of the liquid and  $g$  is the acceleration due to gravity. If  $r$  be radius of capillary tube and  $\theta$  be the angle of contact, then from Fig. 9.24, we can write

$$R = r / \cos\theta$$

Substituting this value of  $R$  in Equation (9.13)

$$h \rho g = 2T / r / \cos \theta$$

$$\text{or } h = 2T \cos\theta / r \rho g \tag{9.14}$$

It is clear from the above expression that if the radius of tube is less (i.e. in a very fine bore capillary), liquid rise will be high.

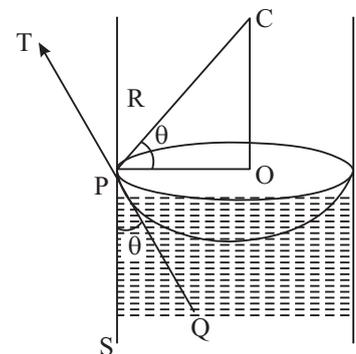


Fig. 9.24 : Angle of contact



## INTEXT QUESTIONS 9.3

1. Does the value of angle of contact depend on the surface tension of the liquid?
2. The angle of contact for a solid and liquid is less than the  $90^\circ$ . Will the liquid wet the solid? If a capillary is made of that solid, will the liquid rise or fall in it?
3. Why it is difficult to enter mercury in a capillary tube, by simply dipping it into a vessel containing mercury while designing a thermometer.
4. Calculate the radius of a capillary to have a rise of 3 cm when dipped in a vessel containing water of surface tension  $7.2 \times 10^{-2} \text{ N m}^{-1}$ . The density of water is  $1000 \text{ kg m}^{-3}$ , angle of contact is zero, and  $g = 10 \text{ m s}^{-2}$ .
5. How does kerosene oil rise in the wick of a lantern?

## 9.7 VISCOSITY

If you stir a liquid taken in a beaker with a glass rod in the middle, you will note that the motion of the liquid near the walls and in the middle is not same (Fig.9.25). Next watch the flow of two liquids (e.g. glycerin and water) through identical pipes. You will find that water flows rapidly out of the vessel whereas glycerine flows slowly. Drop a steel ball through each liquid. The ball falls more slowly in glycerin than in water. These observations indicate a characteristic property of the liquid that determines their motion. This property is known as **viscosity**. Let us now learn how it arises.

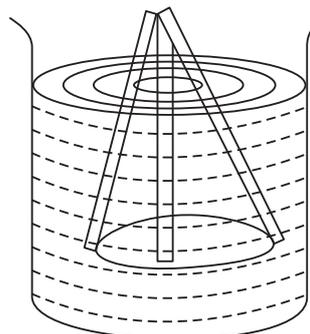


Fig. 9.25: Water being stirred with a glass rod

## 9.7.1 Viscosity

We know that when one body slides over the other, a frictional force acts between them. Similarly, whenever a fluid flows, two adjacent layers of the fluid exert a tangential force on each other; this force acts as a drag and opposes the relative motion between them. *The property of a fluid by virtue of which it opposes the relative motion in its adjacent layers is known as viscosity.*

Fig. 9.26 shows a liquid flowing through a tube. The layer of the liquid in touch with the wall of the tube can be assumed to be stationary due to friction between the solid wall and the liquid. Other layers are in motion and have different velocities. Let  $v$  be the velocity of the layer at a distance  $x$  from the surface and  $v + dv$  be the velocity at a distance  $x + dx$ .



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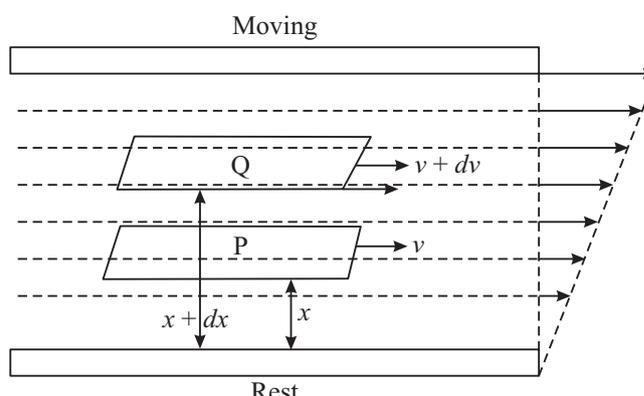


Fig. 9.26 : Flow of a liquid in a tube: Different layers move with different velocities

Thus, the velocity changes by  $dv$  in going through a distance  $dx$  perpendicular to it. The quantity  $dv/dx$  is called the **velocity gradient**.

The viscous force  $F$  between two layers of the fluid is proportional to

- area ( $A$ ) of the layer in contact :  $F \propto A$
- velocity gradient ( $dv/dx$ ) in a direction perpendicular to the flow of liquid :  $F \propto dv/dx$

On combining these, we can write

$$F \propto A \, dv/dx$$

or

$$F = -\eta A \, (dv/dx) \tag{9.15}$$

where  $\eta$  is constant of proportionality and is called **coefficient of viscosity**. The negative sign indicates that force is frictional in nature and opposes motion.

The SI unit of coefficient of viscosity is  $\text{Nsm}^{-2}$ . In cgs system, the unit of viscosity is poise.

$$1 \text{ poise} = 0.1 \text{ Nsm}^{-2}$$

Dimensions of coefficient of viscosity are  $[\text{ML}^{-1} \text{T}^{-1}]$

### 9.8 TYPES OF LIQUID FLOW

Have you ever seen a river in floods? Is it similar to the flow of water in a city water supply system? If not, how are the two different? To discover answer to such questions, let us study the flow of liquids.

Table 10.1 : Viscosity of a few typical fluids

Name of fluid	T [ $^{\circ}\text{C}$ ]	Viscosity $\eta$ (PR)
Water	20	$1.0 \times 10^{-3}$
Water	100	$0.3 \times 10^{-3}$
blood	37	$2.7 \times 10^{-3}$
Air	40	$1.9 \times 10^{-5}$

### 9.8.1 Streamline Motion

The path followed by fluid particles is called line of flow. If every particle passing through a given point of the path follows the same line of flow as that of preceding particles, the flow is said to be *streamlined*. A streamline can be represented as the curve or path whose tangent at any point gives the direction of the liquid velocity at that point. In steady flow, the streamlines coincide with the line of flow (Fig. 9.27).

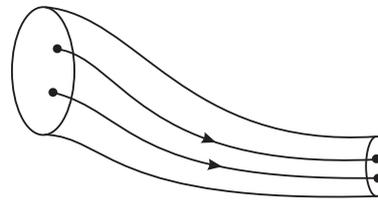


Fig. 9.27: Streamline flow

Note that streamlines do not intersect each other because two tangents can then be drawn at the point of intersection giving two directions of velocities, which is not possible.

When the velocity of flow is less than the critical velocity of a given liquid flowing through a tube, the motion is streamlined. In such a case, we can imagine the entire thickness of the stream of the liquid to be made up of a large number of plane layers (laminae) one sliding past the other, i.e. one flowing over the other. Such a flow is called *laminar flow*.

If the velocity of flow exceeds the critical velocity  $v_c$ , the mixing of streamlines takes place and the flow path becomes zig-zag. Such a motion is said to be *turbulent*.

### 9.8.2 Equation of Continuity

If an incompressible, non-viscous fluid flows through a tube of non-uniform cross section, the product of the area of cross section and the fluid speed at any point in the tube is constant for a streamline flow. Let  $A_1$  and  $A_2$  denote the areas of cross section of the tube where the fluid is entering and leaving, as shown in Fig. 9.28. If  $v_1$  and  $v_2$  are the speeds of the fluid at the ends A and B respectively, and  $\rho$  is the density of the fluid, then the liquid entering the tube at A covers a distance  $v_1$  in one second. So volume of the liquid entering per second =  $A_1 \times v_1$ . Therefore

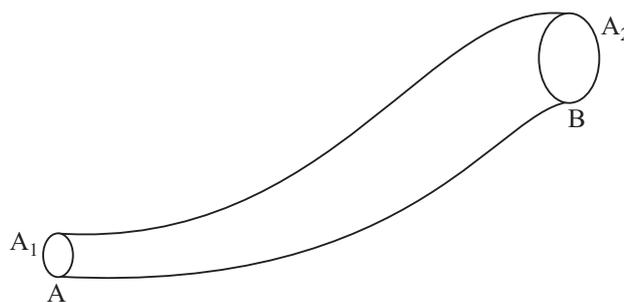


Fig. 9.28: Liquid flowing through a tube



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Mass of the liquid entering per second at point A =  $A_1 v_1 \rho$

Similarly, mass of the liquid leaving per second at point B =  $A_2 v_2 \rho$

Since there is no accumulation of fluid inside the tube, the mass of the liquid crossing any section of the tube must be same. Therefore, we get

$$A_1 v_1 \rho = A_2 v_2 \rho$$

or  $A_1 v_1 = A_2 v_2$

This expression is called **equation of continuity**.

### 9.8.3 Critical Velocity and Reynolds's Number

We now know that when the velocity of flow is less than a certain value, valled *critical velocity*, the flow remains streamlined. But when the velocity of flow exceeds the critical velocity, the flow becomes turbulent.

The value of critical velocity of any liquid depends on the

- nature of the liquid, i.e. coefficient of viscosity ( $\eta$ ) of the liquid;
- diameter of the tube ( $d$ ) through which the liquid flows; and
- density of the liquid ( $\rho$ ).

Experiments show that  $v_c \propto \eta$ ;  $v_c \propto \frac{1}{\rho}$  and  $v_c \propto \frac{1}{d}$ .

Hence, we can write

$$v_c = R \cdot \eta / \rho d \tag{9.16}$$

where  $R$  is constant of proportionality and is called Reynolds's Number. It has no dimensions. Experiments show that if  $R$  is below 1000, the flow is laminar. The flow becomes unsteady when  $R$  is between 1000 and 2000 and the flow becomes turbulent for  $R$  greater than 2000.

**Example 9.1:** The average speed of blood in the artery ( $d = 2.0$  cm) during the resting part of heart's cycle is about  $30 \text{ cm s}^{-1}$ . Is the flow laminar or turbulent? Density of blood  $1.05 \text{ g cm}^{-3}$ ; and  $\eta = 4.0 \times 10^{-2}$  poise.

**Solution:** From Eqn. (9.16) we recall that Reynold's number  $R = v_c \rho d / \eta$ . On substituting the given values, we get

$$\begin{aligned} R &= \frac{(30 \text{ cm s}^{-1}) \times 2 \text{ cm} \times (1.05 \text{ g cm}^{-3})}{(4.0 \times 10^{-2} \text{ g cm}^{-1} \text{ s}^{-1})} \\ &= 1575 \end{aligned}$$

Since  $1575 < 2000$ , the flow is unsteady.

### 9.9 STOKES' LAW

George Stokes gave an empirical law for the magnitude of the tangential backward viscous force  $F$  acting on a freely falling smooth spherical body of radius  $r$  in a highly viscous liquid of coefficient of viscosity  $\eta$  moving with velocity  $v$ . This is known as Stokes' law.

According to Stokes' law

$$F \propto \eta r v$$

or

$$F = K \eta r v$$

where  $K$  is constant of proportionality. It has been found experimentally that  $K = 6\pi$ .

Hence Stokes' law can be written as

$$F = 6\pi \eta r v \quad (9.17)$$

Stokes' Law can also be derived using the method of dimensions as follows:

According to Stokes, the viscous force depends on:

- coefficient of viscosity ( $\eta$ ) of the medium
- radius of the spherical body ( $r$ )
- velocity of the body ( $v$ )

Then

$$F \propto \eta^a r^b v^c$$

or

$$F = K \eta^a r^b v^c$$

where  $K$  is constant of proportionality

Taking dimensions on both the sides, we get

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

or

$$[MLT^{-2}] = [M^a L^{-a+b+c} T^{-a-c}]$$

Comparing the exponents on both the sides and solving the equations we get  $a = b = c = 1$ .

Hence

$$F = K \eta r v$$

#### 9.9.1 Terminal Velocity

Let us consider a spherical body of radius  $r$  and density  $\rho$  falling through a liquid of density  $\sigma$ .

The forces acting on the body will be

- Weight of the body  $\mathbf{W}$  acting downward.
- The viscous force  $\mathbf{F}$  acting vertically upward.
- The buoyant force  $\mathbf{B}$  acting upward.

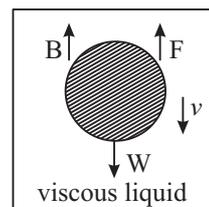


Fig. 9.29 : Force acting on a sphere falling in viscous fluid



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Under the action of these forces, at some instant the net force on the body becomes zero, (since the viscous force increases with the increase of velocity). Then, the body falls with a constant velocity known as **terminal velocity**. We know that magnitude of these forces are

$$F = 6\pi \eta r v_0$$

where  $v_0$  is the terminal velocity.

$$W = (4/3) \pi r^3 \rho g$$

and

$$B = (4/3) \pi r^3 \sigma g$$

The net force is zero when object attains terminal velocity. Hence

$$6\pi \eta r v_0 = \frac{4}{3} \pi r^3 \rho g - \frac{4}{3} \pi r^3 \sigma g$$

Hence 
$$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta} \quad (9.18)$$

### 9.9.2 Applications of Stokes' Law

#### A. Parachute

When a soldier jumps from a flying aeroplane, he falls with acceleration due to gravity  $g$  but due to viscous drag in air, the acceleration goes on decreasing till he acquires terminal velocity. The soldier then descends with constant velocity and opens his parachute close to the ground at a pre-calculated moment, so that he may land safely near his destination.

#### B. Velocity of rain drops

When raindrops fall under gravity, their motion is opposed by the viscous drag in air. When viscous force becomes equal to the force of gravity, the drop attains a terminal velocity. That is why rain drops reaching the earth do not have very high kinetic energy.

**Example 9.2:** Determine the radius of a drop of rain falling through air with terminal velocity  $0.12 \text{ ms}^{-1}$ . Given  $\eta = 1.8 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ ,  $\rho = 1.21 \text{ kg m}^{-3}$ ,  $\sigma = 1.0 \times 10^3 \text{ kg m}^{-3}$  and  $g = 9.8 \text{ m s}^{-2}$ .

**Solution:** We know that terminal velocity is given by

$$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

On rearranging terms, we can write

$$r = \sqrt{\frac{9\eta v_0}{2(\rho - \sigma)g}}$$

$$= \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 0.12}{2(1000 - 1.21) 9.8}} \text{ m}$$

$$= 10^{-5} \text{ m}$$



### INTEXT QUESTIONS 9.4

1. Differentiate between streamline flow and turbulent flow?
2. Can two streamlines cross each other in a flowing liquid?
3. Name the physical quantities on which critical velocity of a viscous liquid depends.
4. Calculate the terminal velocity of a rain drop of radius 0.01m if the coefficient of viscosity of air is  $1.8 \times 10^{-5} \text{ N s m}^{-2}$  and its density is  $1.2 \text{ kg m}^{-3}$ . Density of water =  $1000 \text{ kg m}^{-3}$ . Take  $g = 10 \text{ m s}^{-2}$ .
5. When a liquid contained in a tumbler is stirred and placed for some time, it comes to rest, Why?

#### Daniel Bernoulli (1700-1782)

Daniel Bernoulli, a Swiss Physicist and mathematician was born in a family of mathematicians on February 8, 1700. He made important contributions in hydrodynamics. His famous work, *Hydrodynamica* was published in 1738. He also explained the behavior of gases with changing pressure and temperature, which led to the development of kinetic theory of gases.



He is known as the founder of mathematical physics. Bernoulli's principle is used to produce vacuum in chemical laboratories by connecting a vessel to a tube through which water is running rapidly.

### 9.10 BERNOULLI'S PRINCIPLE

Have you ever thought how air circulates in a dog's burrow, smoke comes quickly out of a chimney or why car's convertible top bulges upward at high speed? You must have definitely experienced the bulging upwards of your umbrella on a stormy- rainy day. All these can be understood on the basis of Bernoulli's principle.

Bernoulli's Principle states that **where the velocity of a fluid is high, the pressure is low and where the velocity of the fluid is low, pressure is high.**





Notes

9.10.1 Energy of a Flowing Fluid

Flowing fluids possess three types of energy. We are familiar with the kinetic and potential energies. The third type of energy possessed by the fluid is pressure energy. It is due to the pressure of the fluid. The pressure energy can be taken as the product of pressure difference and its volume. If an element of liquid of mass  $m$ , and density  $d$  is moving under a pressure difference  $p$ , then

$$\text{Pressure energy} = p \times (m/d) \text{ joule}$$

$$\text{Pressure energy per unit mass} = (p/d) \text{ J kg}^{-1}$$

9.10.2 Bernoulli's Equation

Bernoulli developed an equation that expresses this principle quantitatively. Three important assumptions were made to develop this equation:

1. The fluid is incompressible, i.e. its density does not change when it passes from a wide bore tube to a narrow bore tube.
2. The fluid is non-viscous or the effect of viscosity is not to be taken into account.
3. The motion of the fluid is streamlined.

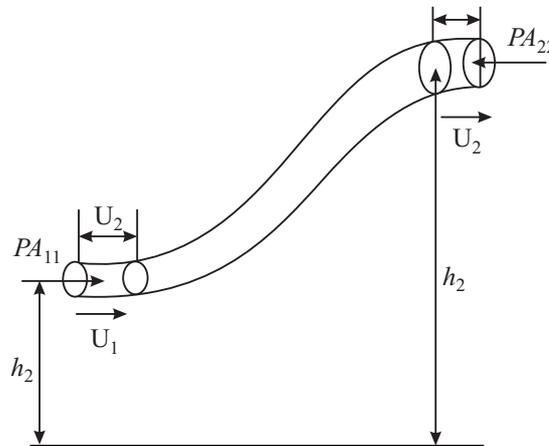


Fig. 9.30

We consider a tube of varying cross section shown in the Fig. 9.30. Suppose at point A the pressure is  $P_1$ , area of cross section  $A_1$ , velocity of flow  $v_1$ , height above the ground  $h_1$  and at B, the pressure is  $P_2$ , area of cross-section  $A_2$  velocity of flow =  $v_2$ , and height above the ground  $h_2$ .

Since points A and B can be any two points along a tube of flow, we write Bernoulli's equation

$$P + \frac{1}{2} \rho v^2 + \rho h = \text{Constant.}$$

That is, the sum of pressure energy, kinetic energy and potential energy of a fluid remains constant in streamline motion.



**ACTIVITY 9.4**

1. Take a sheet of paper in your hand.
2. Press down lightly on horizontal part of the paper as shown in Fig. 9.31 so that the paper curves down.
3. Blow on the paper along the horizontal line.

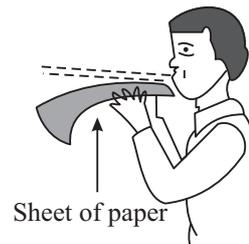


Fig. 9.31

Watch the paper. It lifts up because speed increases and pressure on the upper side of the paper decreases.

Notes



**9.10.3 Applications of Bernoulli's Theorem**

Bernoulli's theorem finds many applications in our lives. Some commonly observed phenomena can also be explained on the basis of Bernoulli's theorem.

**A. Flow meter or Venturimeter**

It is a device used to measure the rate of flow of liquids through pipes. The device is inserted in the flow pipe, as shown in the Fig. 9.32

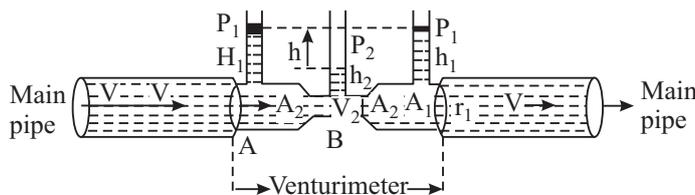


Fig. 9.32 : A Venturimeter

It consists of a manometer, whose two limbs are connected to a tube having two different cross-sectional areas say  $A_1$  and  $A_2$  at A and B, respectively. Suppose the main pipe is horizontal at a height  $h$  above the ground. Then applying Bernoulli's theorem for the steady flow of liquid through the venturimeter at A and B, we can write

$$\text{Total Energy at A} = \text{Total Energy at B}$$

$$\frac{1}{2} m v_1^2 + mgh + \frac{mp_1}{d} = \frac{1}{2} m v_2^2 + mgh + \frac{mp_2}{d}$$



Notes

On rearranging terms we can write,

$$(p_1 - p_2) = \frac{d}{2} (v_2^2 - v_1^2) = \frac{v_1^2 d}{2} \left[ \left( \frac{v_2}{v_1} \right)^2 - 1 \right] \quad (9.19)$$

It shows that points of higher velocities are the points of lower pressure (because of the sum of pressure energy and K.E. remain constant). This is called *Venturi's Principle*.

For steady flow through the venturimeter, volume of liquid entering per second at A = liquid volume leaving per second at B. Therefore

$$A_1 v_1 = A_2 v_2 \quad (9.20)$$

(The liquid is assumed incompressible i.e., velocity is more at narrow ends and vice versa.

Using this result in Eqn. (9.19), we conclude that pressure is lesser at the narrow ends;

$$\begin{aligned} p_1 - p_2 &= \frac{v_1^2 d}{2} \left[ \frac{A_1^2}{A_2^2} - 1 \right] \\ &= \frac{1}{2} d v_1^2 \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right] \\ v_1 &= \sqrt{\frac{2(p_1 - p_2)}{d \left( \frac{A_1^2}{A_2^2} \right) - 1}} \quad (9.21) \end{aligned}$$

If  $h$  denotes level difference between the two limbs of the venturimeter, then

$$p_1 - p_2 = h d g$$

and

$$v_1 = \sqrt{2hg / \left[ \left( \frac{A_1^2}{A_2^2} \right) - 1 \right]}$$

From this we note that  $v_1 \propto \sqrt{h}$  since all other parameters are constant for a given venturimeter. Thus

$$v_1 = K \sqrt{h};$$

where  $K$  is constant.

The volume of liquid flowing per second is given by

$$V = A_1 v_1 = A_1 \times K \sqrt{h}$$

or

$$V = K' h$$

where  $K' = K A_1$  is another constant.

Bernoulli's principle has many applications in the design of many useful appliances like atomizer, spray gun, Bunsen burner, carburetor, Aerofoil, etc.

**(i) Atomizer :** An atomizer is shown in Fig. 9.33. When the rubber bulb A is squeezed, air blows through the tube B and comes out of the narrow orifice with larger velocity creating a region of low pressure in its neighborhood. The liquid (scent or paint) from the vessel is, therefore, sucked into the tube to come out to the nozzles N. As the liquid reaches the nozzle, the air stream from the tube B blows it into a fine spray.

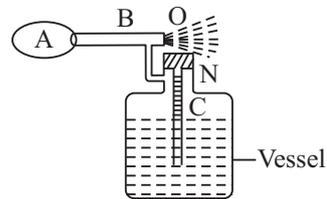


Fig. 9.33 : Atomizer

**(ii) Spray gun :** When the piston is moved in, it blows the air out of the narrow hole 'O' with large velocity creating a region of low pressure in its neighborhood. The liquid (e.g. insecticide) is sucked through the narrow tube attached to the vessel end having its opening just below 'O'. The liquid on reaching the end gets sprayed by out blown air from the piston (Fig. 9.34).

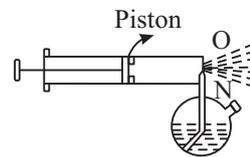


Fig. 9.34: Spray gun

**(iii) Bunsen Burner :** When the gas emerges out of the nozzle N, its velocity being high the pressure becomes low in its vicinity. The air, therefore, rushed in through the side hole A and gets mixed with the gas. The mixture then burns at the mouth when ignited, to give a hot blue flame (Fig.9.35).

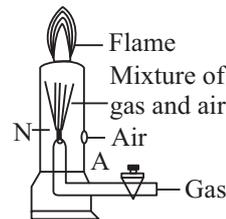


Fig. 9.35: Bunsen Burner

**(iv) Carburetor :** The carburetor shown in Fig. 9.36. is a device used in motor cars for supplying a proper mixture of air and petrol vapours to the cylinder of the engine. The energy is supplied by the explosion of this mixture inside the cylinders of the engine. Petrol is contained in the float chamber. There is a decrease in the pressure on the side A due to motion of the piston. This causes the air from outside to be sucked in with large velocity. This causes a low pressure near the nozzle B (due

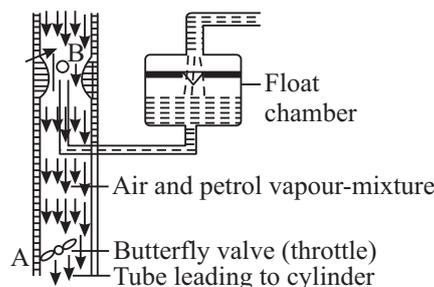


Fig. 9.36: Carburettor



Notes



Notes

to constriction, velocity of air sucked is more near B) and, therefore, petrol comes out of the nozzle B which gets mixed with the incoming Air. The mixture of vaporized petrol and air forming the fuel then enters the cylinder through the tube A.

(Sometimes when the nozzle B gets choked due to deposition of carbon or some impurities, it checks the flow of petrol and the engine not getting fuel stops working. The nozzle has therefore, to be opened and cleaned.

**(v) Aerofoil :** When a solid moves in air , streamlines are formed . The shape of the body of the aeroplane is designed specially as shown in the Fig. 9.37. When the aeroplane runs on its runway, high velocity streamlines of air are formed. Due to crowding of more streamlines on the upper side, it becomes a region of more velocity and hence of comparatively low pressure region than below it. This pressure difference gives the lift to the aeroplane.

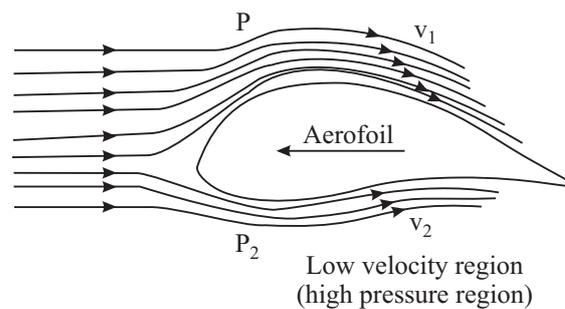


Fig. 9.37 : Crowding of streamlines on the upper side.

Based on this very principle i.e., the regions of high velocities due to crowding of steam lines are the regions of low pressure, following are interesting demonstrations.1

**(a) Attracted disc paradox :** When air is blown through a narrow tube handle into the space between two cardboard sheets [Fig. 9.38] placed one above the other and the upper disc is lifted with the handle, the lower disc is attracted to stick to the upper disc and is lifted with it. This is called attracted disc paradox,

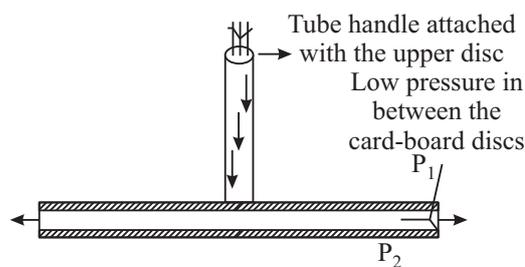


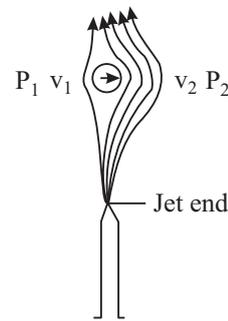
Fig. 9.38 : Attracted disc paradox



Notes

**(b) Dancing of a ping pong ball on a jet of water:**

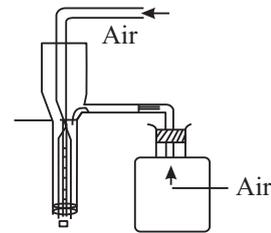
If a light hollow spherical ball (ping-pong ball or table tennis ball) is gently put on a vertical stream of water coming out of a vertically upward directed jet end of a tube, it keeps on dancing this way and that way without falling to the ground (Fig.9.39). When the ball shifts to the lefts , then most of the jet streams pass by its right side thereby creating a region of high velocity and hence low pressure on its right side in comparison to that on the left side and the ball is again pushed back to the center of the jet stream .



**Fig. 9.39: Dancing Pring Pongball**

**(c) Water vacuum pump or aspirator or filter pump :** Fig. 9.40 shows a filter

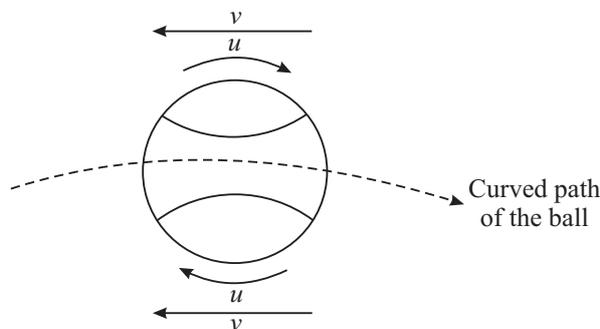
pump used for producing moderately low pressures. Water from the tap is allowed to come out of the narrow jet end of the tube A . Due to small aperture of the nozzle, the velocity becomes high and hence a low-pressure region is created around the nozzle N. Air is, therefore, sucked from the vessel to be evacuated through the tube B; gets mixed with the steam of water and goes out through the outlet. After a few minutes., the pressure of air in the vessel is decreased to about 1 cm of mercury by such a pump



**Fig. 9.40 : Filter Pump**

**(d) Swing of a cricket ball:**

When a cricketer throws a spinning ball, it moves along a curved path in the air. This is called swing of the ball. It is clear from Fig. 9.41. That when a ball is moved forward, the air occupies the space left by the ball with a velocity  $v$  (say). When the ball spins, the layer of air around it also moves with the ball, say with the velocity ' $u$ '. So the resultant velocity of air above the ball becomes  $(v - u)$  and below the ball becomes  $(v + u)$ . Hence, the pressure difference above and below the ball moves the ball in a curved path.



**Fig. 9.41 : Swing of a cricket ball**



Notes

**Example 9.3:** Water flows out of a small hole in the wall of a large tank near its bottom (Fig. 9.42). What is the speed of efflux of water when the height of water level in the tank is 2.5m?

**Solution:** Let B be the hole near the bottom. Imagine a tube of flow A to B for the water to flow from the surface point A to the hole B. We can apply the Bernoulli's theorem to the points A and B for the streamline flow of small mass  $m$ .

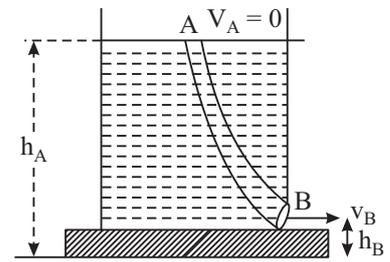


Fig. 9.42

Total energy at B = Total energy at A

At A,  $v_A = 0$ ,  $p_A = p =$  atmospheric pressure,  $h =$  height above the ground.

At B,  $v_B = v = ?$ ,  $p_B = p$ ,  $h_B =$  height of the hole above the ground.

Let  $h_A - h_B = H =$  height of the water level in the vessel = 2.5m

and  $d =$  density of the water.

Applying the Bernoulli's Principle and substituting the values we get,

$$\frac{1}{2}m v_B^2 = mg (h_A - h_B)$$

$$\begin{aligned} \text{or} \quad v_B &= \sqrt{2g(h_A - h_B)} \\ &= \sqrt{2 \times 9.8 \times 2.5} \\ &= 7 \text{ m s}^{-1} \end{aligned}$$



**INTEXT QUESTIONS 9.5**

1. The windstorm often blows off the tin roof of the houses, How does Bernoulli's equation explain the phenomenon?
2. When you press the mouth of a water pipe used for watering the plants, water goes to a longer distance, why?
3. What are the conditions necessary for the application of Bernoulli's theorem to solve the problems of flowing liquid?
4. Water flows along a horizontal pipe having non-uniform cross section. The pressure is 20 mm of mercury where the velocity is 0.20m/s. find the pressure at a point where the velocity is 1.50 m/s?
5. Why do bowlers in a cricket match shine only one side of the ball?



**WHAT YOU HAVE LEARNT**

- Hydrostatic pressure  $P$  at a depth  $h$  below the free surface of a liquid of density is given by

$$P = hdg$$

- The upward force acting on an object submerged in a fluid is known as buoyant force.
- According to Pascal's law, when pressure is applied to any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.
- The liquid molecules in the liquid surface have potential energy called surface energy.
- The surface tension of a liquid may be defined as force per unit length acting on a imaginary line drawn in the surface. It is measured in  $\text{Nm}^{-1}$ .
- Surface tension of any liquid is the property by virtue of which a liquid surface acts like a stretched membrane.
- Angle of contact is defined as the angle between the tangent to the liquid surface and the wall of the container at the point of contact as measured from within the liquid.
- The liquid surface in a capillary tube is either concave or convex. This curvature is due to surface tension. The rise in capillary is given by

$$h = \frac{2T \cos \theta}{r d g}$$

- The excess pressure  $P$  on the concave side of the liquid surface is given by

$$P = \frac{2T}{R}, \text{ where } T \text{ is surface tension of the liquid}$$

$$P = \frac{2T}{R}, \text{ for air bubble in the liquid and}$$

$$P = \frac{4T'}{r}, \text{ where } T' \text{ is surface tension of soap solution, for soap bubble in air}$$

- Detergents are considered better cleaner of clothes because they reduce the surface tension of water-oil.
- The property of a fluid by virtue of which it opposes the relative motion between its adjacent layers is known as viscosity.



Notes

## MODULE - 2

### Mechanics of Solids and Fluids



#### Notes

### Properties of Fluids

- The flow of liquid becomes turbulent when the velocity is greater than a certain value called critical velocity ( $v_c$ ) which depends upon the nature of the liquid and the diameter of the tube i.e. ( $\eta, P$  and  $d$ ).
- Coefficient of viscosity of any liquid may be defined as the magnitude of tangential backward viscous force acting between two successive layers of unit area in contact with each other moving in a region of unit velocity gradient.
- Stokes' law states that tangential backward viscous force acting on a spherical mass of radius  $r$  falling with velocity ' $v$ ' in a liquid of coefficient of viscosity  $\eta$  is given by

$$F = 6\pi \eta r v.$$

- Bernoulli's theorem states that the total energy of an element of mass ( $m$ ) of an incompressible liquid moving steadily remains constant throughout the motion. Mathematically, Bernoulli's equation as applied to any two points A and B of tube of flow

$$\frac{1}{2} m v_A^2 + m g h_A + \frac{m P_A}{d} = \frac{1}{2} m v_B^2 + m g h_B + \frac{m P_B}{d}$$



### TERMINAL EXERCISES

1. Derive an expression for hydrostatic pressure due to a liquid column.
2. State pascal's law. Explain the working of hydraulic press.
3. Define surface tension. Find its dimensional formula.
4. Describe an experiment to show that liquid surfaces behave like a stretched membrane.
5. The hydrostatic pressure due to a liquid filled in a vessel at a depth 0.9 m is  $3.0 \text{ N m}^{-2}$ . What will be the hydrostatic pressure at a hole in the side wall of the same vessel at a depth of 0.8 m.
6. In a hydraulic lift, how much weight is needed to lift a heavy stone of mass 1000 kg? Given the ratio of the areas of cross section of the two pistons is 5. Is the work output greater than the work input? Explain.
7. A liquid filled in a capillary tube has convex meniscus. If  $F_a$  is force of adhesion,  $F_c$  is force of cohesion and  $\theta =$  angle of contact, which of the following relations should hold good?  
(a)  $F_a > F_c \sin\theta$ ; (b)  $F_a < F_c \sin\theta$ ; (c)  $F_a \cos\theta = F_c$ ; (d)  $F_a \sin\theta > F_c$
8. 1000 drops of water of same radius coalesce to form a larger drop. What happens to the temperature of the water drop? Why?



9. What is capillary action? What are the factors on which the rise or fall of a liquid in a capillary tube depends?
10. Calculate the approximate rise of a liquid of density  $10^3 \text{ kg m}^{-3}$  in a capillary tube of length 0.05 m and radius  $0.2 \times 10^{-3} \text{ m}$ . Given surface tension of the liquid for the material of that capillary is  $7.27 \times 10^{-2} \text{ N m}^{-1}$ .
11. Why is it difficult to blow water bubbles in air while it is easier to blow soap bubble in air?
12. Why the detergents have replaced soaps to clean oily clothes.
13. Two identical spherical balloons have been inflated with air to different sizes and connected with the help of a thin pipe. What do you expect out of the following observations?
  - (i) The air from smaller balloon will rush into the bigger balloon till whole of its air flows into the later.
  - (ii) The air from the bigger balloon will rush into the smaller balloon till the sizes of the two become equal.

What will be your answer if the balloons are replaced by two soap bubbles of different sizes.
14. Which process involves more pressure to blow a air bubble of radius 3 cm inside a soap solution or a soap bubble in air? Why?
15. Differentiate between laminar flow and turbulent flow and hence define critical velocity.
16. Define viscosity and coefficient of viscosity. Derive the units and dimensional formula of coefficient of viscosity. Which is more viscous : water or glycerine? Why?
17. What is Reynold's number? What is its significance? Define critical velocity on the basis of Reynold's number.
18. State Bernoulli's principle. Explain its application in the design of the body of an aeroplane.
19. Explain Why :
  - (i) A spinning tennis ball curves during the flight?
  - (ii) A ping pong ball keeps on dancing on a jet of water without falling on to either side?
  - (iii) The velocity of flow increases when the aperture of water pipe is decreased by squeezing its opening.
  - (iv) A small spherical ball falling in a viscous fluid attains a constant velocity after some time.

## MODULE - 2

### Mechanics of Solids and Fluids



#### Notes

## Properties of Fluids

- (v) If mercury is poured on a flat glass plate; it breaks up into small spherical droplets.
20. Calculate the terminal velocity of an air bubble with 0.8 mm in diameter which rises in a liquid of viscosity of  $0.15 \text{ kg m}^{-1} \text{ s}^{-1}$  and density  $0.9 \text{ g m}^{-3}$ . What will be the terminal velocity of the same bubble while rising in water? For water  $\eta = 10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$ .
21. A pipe line 0.2 m in diameter, flowing full of water has a constriction of diameter 0.1 m. If the velocity in the 0.2 m pipe-line is  $2 \text{ m s}^{-1}$ . Calculate
- the velocity in the constriction, and
  - the discharge rate in cubic meters per second.
22. (i) With what velocity in a steel ball 1 mm is radius falling in a tank of glycerine at an instant when its acceleration is one-half that of a freely falling body?
- (ii) What is the terminal velocity of the ball? The density of steel and of glycerine are  $8.5 \text{ g cm}^{-3}$  and  $1.32 \text{ g cm}^{-3}$  respectively; viscosity of glycerine is 8.3 Poise.
23. Water at  $20^\circ\text{C}$  flows with a speed of  $50 \text{ cm s}^{-1}$  through a pipe of diameter of 3 mm.
- What is Reynold's number?
  - What is the nature of flow?
- Given, viscosity of water at  $20^\circ\text{C}$  as  $= 1.005 \times 10^{-2} \text{ Poise}$ ; and Density of water at  $20^\circ\text{C}$  as  $= 1 \text{ g cm}^{-3}$ .
24. Modern aeroplane design calls for a lift of about  $1000 \text{ N m}^{-2}$  of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the velocity of flow past the lower wing surface is  $100 \text{ m s}^{-1}$ , what is the required velocity over the upper surface to give a desired lift of  $1000 \text{ N m}^{-2}$ ? The density of air is  $1.3 \text{ kg m}^{-3}$ .
25. Water flows horizontally through a pipe of varying cross-section. If the pressure of water equals 5 cm of mercury at a point where the velocity of flow is  $28 \text{ cm s}^{-1}$ , then what is the pressure at another point, where the velocity of flow is  $70 \text{ cm s}^{-1}$ ? [Tube density of water  $1 \text{ g cm}^{-3}$ ].



## ANSWERS TO INTEXT QUESTIONS

### 9.1

1. Because then the weight of the person applies on a larger area hence pressure on snow decreases.

$$2. \quad P = P_a + \rho gh$$

$$P = 1.5 \times 10^7 \text{ Pa}$$

$$3. \quad \text{Pressure applied by the weight of the boy} = \frac{2.5}{0.05} = 500 \text{ N m}^{-2}.$$

$$\text{Pressure due to the weight of the elephant} = \frac{5000}{10} = 500 \text{ N m}^{-2}.$$

$\therefore$  The boy can balance the elephant.

4. Because of the larger area of the rod, pressure on the skin is small.

$$5. \quad \frac{50}{0.1} = \frac{w}{10}, \quad w = 5000 \text{ kg wt.}$$

## 9.2

1. Force of attraction between molecules of same substance is called force of cohesion and the force of attractive between molecules of different substance is called force of adhesion.
2. Surface tension leads to the minimum surface area and for a given volume, sphere has minimum surface area.
3. No, they have tightly bound molecules.
4. Due to surface tension forces.
5. For air bubble in water

$$P = \frac{2T}{r} = \frac{2 \times 727 \times 10^{-3}}{2 \times 10^{-2}} = 72.7 \text{ N m}^{-2}.$$

For soap bubble in air

$$P' = \frac{4T'}{r'} = \frac{4 \times 25 \times 10^{-3}}{4 \times 10^{-2}} = 2.5 \text{ N m}^{-2}.$$

## 9.3

1. No.
2. Yes, the liquid will rise.
3. Mercury has a convex meniscus and the angle of contact is obtuse. The fall in the level of mercury in capillary makes it difficult to enter.



## MODULE - 2

### Mechanics of Solids and Fluids



#### Notes

## Properties of Fluids

4.  $r = \frac{2T}{h\rho g} = \frac{2 \times 7.2 \times 10^{-2}}{3 \times 1000 \times 10}$   
 $= 4.8 \times 10^{-6} \text{m.}$
5. Due to capillary action.

#### 9.4

1. If every particle passing through a given point of path follows the same line of flow as that of preceding particle the flow is stream lined, if its zig-zag, the flow is turbulent.
2. No, otherwise the same flow will have two directions.
3. Critical velocity depends upon the viscous nature of the liquid, the diameter of the tube and density of the liquid.
4.  $.012 \text{ ms}^{-1}$
5. Due to viscous force.

#### 9.5

1. High velocity of air creates low pressure on the upper part.
2. Decreasing in the area creates large pressure.
3. The fluid should be incompressible and non-viscous on (very less). The motion should be streamlined.
4.  $(P_1 - P_2) = \frac{1}{2} d (v_2^2 - v_1^2)$
5. So that the stream lines with the two surfaces are different. More swing in the ball will be obtained.

#### Answers to the Terminal Exercises

5.  $2.67 \text{ N m}^{-2}$ .
6.  $200 \text{ N}$ , No.
20.  $2.1 \text{ mm s}^{-1}$ ,  $35 \text{ cm s}^{-1}$ .
21.  $8 \text{ m s}^{-1}$ ,  $6.3 \times 10^{-2} \text{ m}^3 \text{ s}^{-1}$ .
22.  $7.8 \text{ mm s}^{-1}$ ,  $0.19 \text{ m s}^{-1}$ .
23.  $1500$ , Unsteady.
24.  $2 \text{ cm}$  of mercury.