

## MODULE - 6

### Optics and Optical Instruments



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# OPTICAL INSTRUMENTS

We get most of the information about the surrounding world through our eyes. But as you know, an unaided eye has limitations; objects which are too far like stars, planets etc. appear so small that we are unable to see their details. Similarly, objects which are too small, e.g. pollen grains, bacteria, viruses etc. remain invisible to the unaided eyes. Moreover, our eyes do not keep a permanent record of what they see, except what is retained by our memory. You may therefore ask the question: How can we see very minute and very distant objects? The special devices meant for this purpose are called **optical instruments**.

In this lesson you will study about two important optical instruments, namely, a microscope and a telescope. As you must be knowing, a microscope magnifies small objects while a telescope is used to see distant objects. The design of these appliances depends on the requirement. (The knowledge of image formation by the mirrors and lenses, which you have acquired in Lesson 20, will help you understand the working of these optical instruments.) The utility of a microscope is determined by its magnifying power and resolving power. For a telescope, the keyword is *resolving power*. You must have read about Hubble's space telescope, which is being used by scientists to get details of far off galaxies and search for a life-sustaining planet beyond our solar system.



## OBJECTIVES

After studying this lesson, you should be able to:

- explain the working principle of simple and compound microscopes;
- derive an expression for the magnifying power of a microscope;
- distinguish between linear and angular magnifications;
- explain the working principle of refracting and reflecting telescopes; and
- calculate the resolving powers of an eye, a telescope and a microscope.

### 23.1 MICROSCOPE

In Lesson 20 you have learnt about image formation by mirrors and lenses. If you take a convex lens and hold it above this page, you will see images of the alphabets/ words. If you move the lens and bring it closer and closer to the page, the alphabets printed on it will start looking enlarged. This is because their enlarged, virtual and erect image is being formed by the lens. That is, it is essentially acting as a magnifying glass or simple microscope. You may have seen a doctor, examining measles on the body of a child. Watch makers and jewellers use it to magnify small components of watches and fine jewellery work. You can take a convex lens and try to focus sunlight on a small piece of paper. You will see that after some time, the piece of paper start burning. A convex lens can, therefore start a fire. That is why it is dangerous to leave empty glass bottles in the woods. The sunlight falling on the glass bottles may get focused on dry leaves in the woods and set them on fire. Sometimes, these result in wild fires, which destroy large parts of a forest and/or habitation. Such fires are quite common in Australia, Indonesia and U.S.

As a simple microscope, a convex lens is satisfactory for magnifying small nearby objects upto about twenty times their original size. For large magnification, a compound microscope is used, which is a combination of basically two lenses. In a physics laboratory, a magnifying glass is used to read vernier scales attached to a travelling microscope and a spectrometer.

While studying simple and compound microscopes, we come across scientific terms like (i) near point, (ii) least distance of distinct vision, (iii) angular magnification or magnifying power, (iv) normal adjustment etc. Let us first define these.

- (i) **Near point** is the distance from the eye for which the image of an object placed there is formed (by eye lens) on the retina. The near point varies from person to person and with the age of an individual. At a young age (say below 10 years), the near point may be as close as 7-8 cm. In the old age, the near point shifts to larger values, say 100-200 cm or even more. That is why young children tend to keep their books so close whereas the aged persons keep a book or newspaper far away from the eye.
- (ii) **Least distance of distinct vision** is the distance upto which the human eye can see the object clearly without any strain on it. For a normal human eye, this distance is generally taken to be 25 cm.
- (iii) **Angular magnification** is the ratio of the angle subtended by the image at the eye (when the microscope is used) to the angle subtended by the object at the unaided eye when the object is placed at the least distance of distinct vision. It is also called the magnifying power of the microscope.



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- (iv) **Normal Adjustment:** When the image is formed at infinity, least strain is exerted on the eye for getting it focused on the retina. This is known as normal adjustment.
- (v) **Linear magnification** is the ratio of the size of the image to the size of the object.
- (vi) **Visual angle** is the angle subtended by the object at human eye.

### 23.1.1 A Simple Microscope

When a convex lens of short focal length is used to see magnified image of a small object, it is called a simple microscope.

We know that when an object is placed between the optical center and the focus of a convex lens, its image is virtual, erect, and magnified and on the same side as the object. In practice, such a lens is held close to eye and the distance of the object is adjusted till a clear image is formed at the least distance of distinct vision. This is illustrated in Fig. 23.1, which shows an object  $AB$  placed between  $F$  and  $O$ . Its virtual image  $A'B'$  is formed on the same side as the object. The position of the object is so adjusted that the image is formed at the least distance of distinct vision ( $D$ ).

#### Magnifying power of a simple microscope

Magnifying power of an optical instrument is the ratio of the angle subtended by the image at the eye to the angle subtended by the object seen directly, when both lie at the least distance of distinct vision or the near point. It is also called angular magnification and is denoted by  $M$ . Referring to Fig. 23.1(a) and (b), the angular magnification of simple

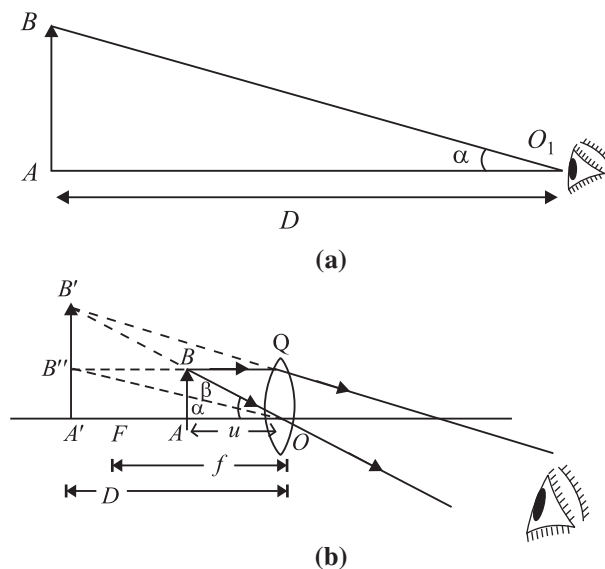


Fig.23.1 : Angular magnification of a magnifying glass

microscope is given by  $M = \frac{\angle A'OB'}{AO'B} = \frac{\beta}{\alpha}$ . In practice, the angles  $\alpha$  and  $\beta$  are small. Therefore, you can replace these by their tangents, i.e. write

$$M = \frac{\tan \beta}{\tan \alpha} \quad (23.1)$$

From  $\Delta s A'OB'$  and  $AOB$ , we can write  $\tan \beta = \frac{A'B'}{A'O} = \frac{A'B'}{D}$  and

$\tan \alpha = \frac{A'B'}{A'O} = \frac{AB}{D}$ . On putting these values of  $\tan \beta$  and  $\tan \alpha$  in Eqn. (23.1), we get

$$M = \frac{A'B'}{D} \bigg/ \frac{AB}{D} = \frac{A'B'}{AB}$$

Since  $\Delta s AOB$  and  $A'OB'$  in Fig 23.1(b) are similar, we can write

$$\frac{A'B'}{AB} = \frac{A'O}{AO} \quad (23.2)$$

Following the standard sign convention, we note that

$$A'O = -D$$

and

$$AO = -u$$

Hence, from Eqn. (23.2), we obtain

$$\frac{A'B'}{AB} = \frac{D}{u} \quad (23.3)$$

If  $f$  is the focal length of the lens acting as a simple microscope, then using the lens formula  $\left( \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \right)$  and noting that  $v = -D$ ,  $u = -u$  and  $f = f$ , we get

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

or

$$-\frac{1}{D} + \frac{1}{u} = \frac{1}{f}$$

Multiplying both the sides by  $D$ , and rearranging term, you can write

$$\frac{D}{u} = 1 + \frac{D}{f} \quad (23.4)$$



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On combining Eqns. (23.3) and (23.4), we get

$$\frac{A'B'}{AB} = 1 + \frac{D}{f}$$

or 
$$M = 1 + \frac{D}{f} \quad (23.5)$$

From this result we note that lesser the focal length of the convex lens, greater is the value of the angular magnification or magnifying power of the simple microscope.

**Normal Adjustment :** In this case, the image is formed at infinity. The magnifying power of the microscope is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the unaided eye when the object is placed at  $D$ . Fig 23.2(a) shows that the object is placed at the least distance of distinct vision  $D$ .

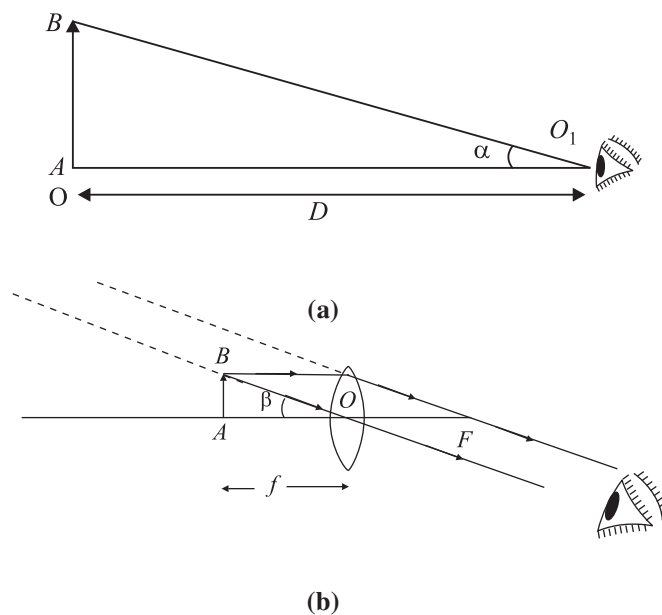


Fig.23.2 : Image formation for normal adjustment

The angles subtended by the object and the image at the unaided eye are  $\alpha$  and  $\beta$ , respectively. The magnifying power is defined as

$$M = \frac{\beta}{\alpha}$$

In practice, the angles  $\alpha$  and  $\beta$  are small, and, as before, replacing these by their tangents, we get

$$M = \frac{\tan \beta}{\tan \alpha}$$

$$\begin{aligned} \text{i.e.} \quad &= \frac{AB}{AO} / \frac{AB}{AO_1} \\ &= \frac{AO_1}{AO} = \frac{D}{f} \end{aligned}$$

$$\text{or} \quad \boxed{M = \frac{D}{f}} \quad (23.6)$$

You may note that in the normal adjustment, the viewing of the image is more comfortable. To help you fix your ideas, we now give a solved example. Read it carefully.

**Example 23.1:** Calculate the magnifying power of a simple microscope having a focal length of 2.5 cm.

**Solution :** For a simple microscope, the magnifying power is given by [Eqn. (23.5)] :

$$M = 1 + \frac{D}{f}$$

Putting  $D = 25$  cm and  $f = 2.5$  cm, we get

$$M = 1 + \frac{25}{2.5} = 1 + 10 = 11$$

### 23.1.2 A Compound Microscope

A compound microscope consists of two convex lenses. A lens of short aperture and short focal length faces the object and is called the **objective**. Another lens of short focal length but large aperture facing the eye is called the **eye piece**. The objective and eye piece are placed coaxially at the two ends of a tube.

When the object is placed between  $F$  and  $2F$  of the objective, its a real, inverted and magnified image is formed beyond  $2F$  on the other side of the objective. This image acts as an object for the eye lens, which then acts as a simple microscope. The eye lens is so adjusted that the image lies between its focus and the optical center so as to form a magnified image at the least distance of distinct vision from the eye lens.

#### Magnifying Power of a compound microscope

Magnifying power of a compound microscope is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object at unaided eye, when both are placed at the least distance of distinct vision. It is denoted by  $M$ . By referring to Fig. 23.3, we can write

$$M = \frac{\beta}{\alpha}$$



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Since the angles  $\alpha$  and  $\beta$  are small, these can be replaced by their tangents, so that

$$M = \frac{\tan \beta}{\tan \alpha}$$

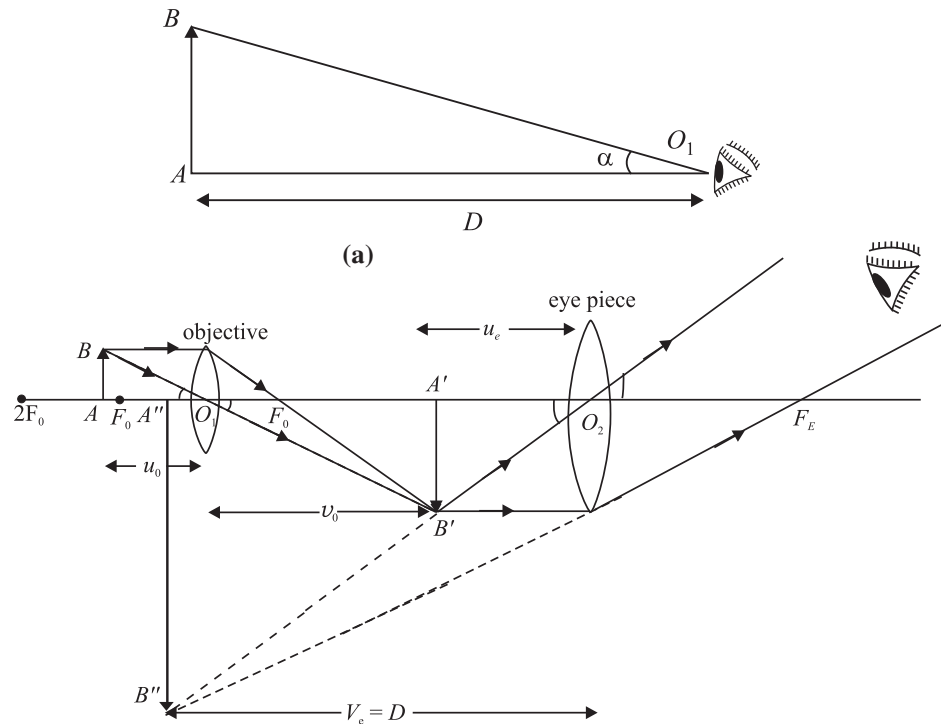


Fig.23.3 : Image formation by a compound microscope when the final image is formed at the least distance of distinct vision.

$$M = \frac{A''B''}{AB} \cdot \frac{AB}{A'B'}$$

$$\Rightarrow M = \frac{A''B''}{AB} = \frac{A''B''}{A'B'} \cdot \frac{A'B'}{AB}$$

From similar  $\Delta s A''B''O_2$  and  $A'B'O_2$ , we can write

$$\frac{A''B''}{A'B'} = \frac{A''O_2}{A'O_2} = \frac{D}{u_e}$$

Also from similar  $\Delta s A'B'O_1$  and  $ABO$ , we have

$$\frac{A'B'}{AB} = \frac{v_o}{u_o}$$

Note that  $m_e = \frac{A''B''}{A'B'}$  defines magnification produced by eye lens and  $m_o =$

$\frac{A'B'}{AB}$  denotes magnification produced by the objective lens. Hence



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$$M = \frac{D}{u_e} \cdot \frac{v_o}{u_o} = m_e \times m_o \quad (23.7)$$

From Lesson 20, you may recall the lens formula. For eye lens, we can write

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

Multiply on both sides by  $v_e$  to get

$$\frac{v_e}{v_e} - \frac{v_e}{u_e} = \frac{v_e}{f_e}$$

$$\Rightarrow \frac{v_e}{u_e} = 1 - \frac{v_e}{f_e}$$

Since  $f_e$  is positive and  $v_e = -D$  as per sign convention, we can write

$$m_e = \frac{v_e}{u_e} = 1 + \frac{D}{f_e} \quad (23.8)$$

On combining Eqns. (23.7) and (23.8), we get

$$M = \frac{v_o}{u_o} \times \left(1 + \frac{D}{f_e}\right)$$

In practice, the focal length of an objective of a microscope is very small and object  $AB$  is placed just outside the focus of objective. That is

$$\therefore u_o \approx f_o$$

Since the focal length of the eye lens is also small, the distance of the image  $A'B'$  from the object lens is nearly equal to the length of the microscope tube i.e.

$$v_o \approx L$$

Hence, the relation for the magnifying power in terms of parameters related to the microscope may be written as

$$M = \frac{L}{f_o} \left(1 + \frac{D}{f_e}\right) \quad (23.10)$$

**Magnifying power in normal adjustment :** In this case the image is formed at infinity. As discussed earlier, the magnifying power of the compound microscope may be written as

$$\begin{aligned} M &= m_o \times m_e \\ &= \frac{v_o}{u_o} \left(\frac{D}{f_e}\right) \end{aligned}$$



## MODULE - 6

### Optics and Optical Instruments



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## Optical Instruments

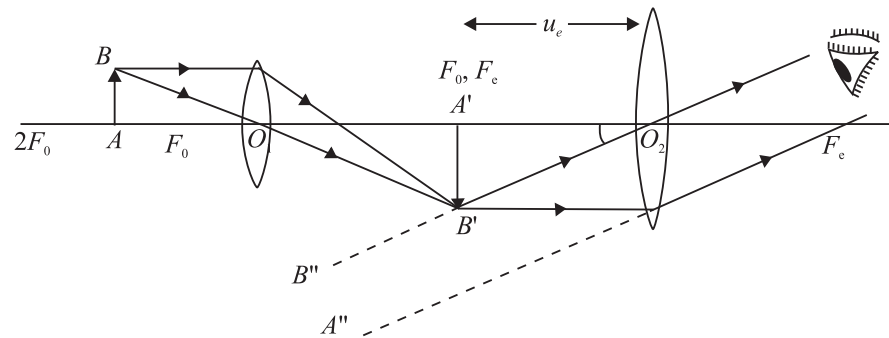


Fig. 23.4 : Compound microscope in normal adjustment

You may now like to go through a numerical example.

**Example 23.2 :** A microscope has an objective of focal length 2 cm, an eye piece of focal length 5 cm and the distance between the centers of two lens is 20 cm. If the image is formed 30 cm away from the eye piece, find the magnification of the microscope.

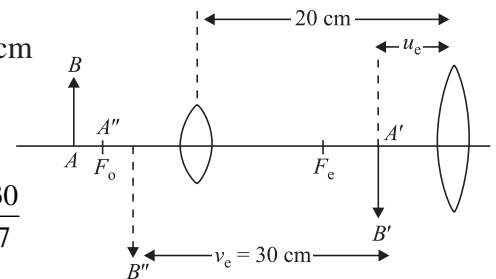
**Solution :** For the objective,  $f_o = 2$  cm and  $f_e = 5$  cm. For the eyepiece,  $v_e = -30$  cm and  $f_e = 5$  cm. We can calculate  $v_e$  using the relation

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

On solving, you will easily obtain  $u_e = -\frac{30}{7}$  cm

For the objective lens

$$\begin{aligned} v_o &= 20 - \frac{30}{7} \\ &= \frac{110}{7} \text{ cm} \end{aligned}$$



Using the formula

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

we have

$$\frac{1}{110/7} - \frac{1}{u_o} = \frac{1}{2}$$

or

$$u_o = -\frac{110}{48} \text{ cm}$$

The magnifying power of the objective

$$m_o = \frac{v_o}{u_o} = \frac{110/7}{-110/48} = -\frac{48}{7}$$

The magnification due to the eyepiece is

$$m_e = \frac{v_e}{u_e} = \frac{-30/1}{-30/7} = 7$$

Therefore, the magnification of the microscope is given by

$$\begin{aligned} M &= (m_o)(m_e) \\ &= \left(-\frac{48}{7}\right)(7) = -48 \end{aligned}$$



### INTEXT QUESTIONS 23.1

1. What is the nature of images formed by a (i) simple microscope (ii) Compound microscope?
2. Differentiate between the magnifying power and magnification?
3. The magnifying power of a simple microscope is 11. What is its focal length?
4. Suppose you have two lenses of focal lengths 100 cm and 4 cm respectively. Which one would you choose as the eyepiece of your compound microscope and why?
5. Why should both the objective and the eyepiece of a compound microscope have short focal lengths?

### 23.2 TELESCOPES

Telescopes are used to see distant objects such as celestial and terrestrial bodies. Some of these objects may not be visible to the unaided eye. The visual angle subtended by the distant objects at the eye is so small that the object cannot be perceived. The use of a telescope increases the visual angle and brings the image nearer to the eye. Mainly two types of telescopes are in common use : refracting telescope and reflecting telescope. We now discuss these.

#### 23.2.1 Refracting Telescope

The refracting telescopes are also of two types :



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- **Astronomical telescopes** are used to observe heavenly or astronomical bodies.
- **Terrestrial telescopes** are used to see distant objects on the earth. So it is necessary to see an erect image. Even Galilean telescope is used to see objects distinctly on the surface of earth.

An astronomical telescope produces a virtual and erect image. As heavenly bodies are round, the inverted image does not affect the observation. This telescope consists of a two lens system. The lens facing the object has a large aperture and large focal length ( $f_o$ ). It is called the *objective*. The other lens, which is towards the eye, is called the *eye lens*. It has a small aperture and short focal length ( $f_e$ ). The objective and eye-piece are mounted coaxially in two metallic tubes.

The objective forms a real and inverted image of the distant object in its focal plane. The position of the lens is so adjusted that the final image is formed at infinity. (This adjustment is called normal adjustment.) The position of the eyepiece can also be adjusted so that the final image is formed at the least distance of distinct vision.

(a) **When the final image is formed at infinity** (Normal adjustment), the paraxial rays coming from a heavenly object are parallel to each other and they make an angle  $\alpha$  with the principal axis. These rays after passing through the objective, form a real and inverted image in the focal plane of objective. In this case, the position of the eyepiece is so adjusted that the final image is formed at infinity.

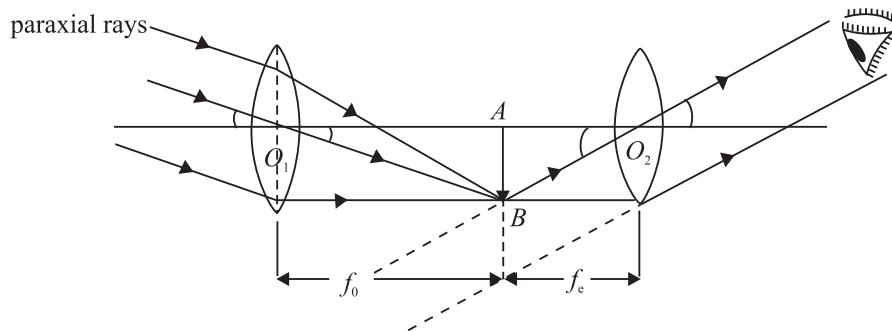


Fig 23.6 : Working principle of an astronomical telescope

**Magnifying power** of a telescope is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object at objective when both the object and the image lie at infinity. It is also called **angular magnification** and is denoted by  $M$ . By definition,

$$M = \frac{\beta}{\alpha}$$

Since  $\alpha$  and  $\beta$  are small, they can be replaced by their tangents. Therefore,

$$\begin{aligned}
 M &= \frac{\tan \beta}{\tan \alpha} \\
 &= \frac{AB/AO_2}{AB/AO_1} = \frac{AO_1}{AO_2} \\
 &= \frac{f_o}{f_e} \quad (23.11)
 \end{aligned}$$

It follows that the magnifying power of a telescope in normal adjustment will be large if the objective is of large focal length and the eyepiece is of short focal length. The length of telescope in normal adjustment is  $(f_o + f_e)$

**(b) When the final image is formed at the least distance of distinct vision,** the paraxial rays coming from a heavenly object make an angle  $\alpha$  with the principal axis. After passing through the objective, they meet on the other side of it and form a real and inverted image  $AB$ . The position of the eyepiece is so adjusted that it finally forms the image at the least distance of distinct vision.

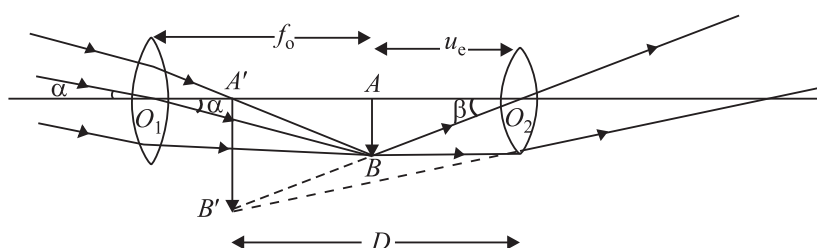


Fig 23.7 : Image formed by a telescope at  $D$

**Magnifying power:** It is defined as the ratio of the angle subtended at the eye by the image formed at  $D$  to the angle subtended by the object lying at infinity:

$$\begin{aligned}
 M &= \frac{\beta}{\alpha} \\
 &\approx \frac{\tan \beta}{\tan \alpha} \\
 &= \frac{AB/AO_2}{AB/AO_1} = \frac{AO_1}{AO_2} \\
 &= \frac{f_o}{u_e} \quad (23.12)
 \end{aligned}$$



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Since  $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$  for the eyepiece, we can write

$$\begin{aligned} \frac{1}{u_e} &= \frac{1}{v_e} - \frac{1}{f_e} \\ &= -\frac{1}{f_e} \left(1 - \frac{f_e}{v_e}\right) \end{aligned}$$

or 
$$M = \frac{f_o}{u_e} = -\frac{f_o}{f_e} \left(1 - \frac{f_e}{v_e}\right) \quad (23.13)$$

Applying the new cartesian sign convention  $f_o = +f_o$ ,  $v_e = -D$ ,  $f_e = +f_e$ , we can write

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right) \quad (23.14)$$

The negative sign of magnifying power of the telescope suggests that the final image is inverted and real. The above expression tells that the magnifying power of a telescope is larger when adjusted at the least distance of distinct vision to the telescope when focused for normal adjustment.

**Example 23.3:** The focal length of the objective of an astronomical telescope is 75 cm and that of the eyepiece is 5 cm. If the final image is formed at the least distance of distinct vision from the eye, calculate the magnifying power of the telescope.

**Solution:**

Here  $f_o = 75$  cm,  $f_e = 5$  cm,  $D = 25$  cm

$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right) = \frac{-75}{5} \left(1 + \frac{5}{25}\right) = -18$$

### 23.2.2 Reflecting telescope

A reflecting telescope is used to see distant stars and possesses large light-gathering power in order to obtain a bright image of even a faint star deep in space. The objective is made of a concave mirror, having large aperture and large focal length. This concave mirror, being parabolic in shape, is free from spherical aberration.

Before the reflected rays of light meet to form a real, inverted and diminished image of a distant star at the focal plane of concave mirror, they are intercepted and reflected by a plane Mirror  $M_1M_2$  inclined at an angle of 45° to the principal

axis of the concave mirror. This plane mirror deviates the rays and the real image is formed in front of the eye piece, which is at right angle to the principal axis of concave mirror. The function of the eye- piece is to form a magnified, virtual image of the star enabling eye to see it distinctly.

If  $f_o$  is the focal length of the concave mirror and  $f_e$  is the focal length of eye piece, the magnifying power of the reflecting telescope is given by

$$M = \frac{f_o}{f_e}$$

Further, if  $D$  is the diameter of the objective and  $d$  is the diameter of the pupil of the eye, the brightness ratio is given by

$$B = D^2/d^2$$

The other form of the reflecting telescope is shown in Fig 23.9. It was designed by **Cassegrain**. In this case the objective has a small opening at its center. The rays from the distant star, after striking the concave mirror, are made to intercept at  $A_2$  and the final image is viewed through the eyepiece.

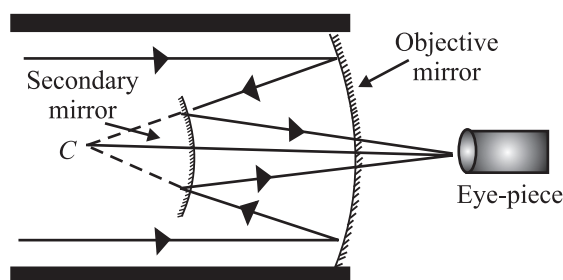


Fig 23.9 : Cassegrain reflector

There are several advantages of a reflecting telescope over a refracting telescope.

- Since the objective is not a lens, the reflecting telescopes are free from chromatic aberration. Thus rays of different colours reaching the objective from distant stars are focussed at the same point.
- Since the spherical mirrors are parabolic mirrors, free from spherical aberration, they produce a very sharp and distinct image.
- Even a very faint star can be seen through the reflecting telescope because they have large aperture and have large light-gathering power. The brightness of the image is directly proportional to the area of the objective :

$$B \propto \frac{\pi D^2}{4}$$

where  $D$  is the diameter of the objective of the telescope. If  $d$  is the diameter of the pupil of the eye then brightness of the telescope  $B$  is defined as the ratio

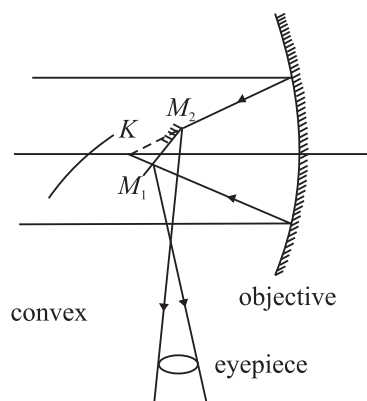


Fig 23.8 : Newtonian Reflector



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of light gathered by the telescope to that gathered by the unaided eye from the distant object

$$B = \frac{\pi D^2 / 4}{\pi d^2 / 4} = \frac{D^2}{d^2}$$

- In reflecting type of telescopes, there is negligible absorption of light.
- Large apertures of reflecting telescope enable us to see minute details of distant stars and explore deeper into space. That is why in recent years, astronomers have discovered new stars and stellar systems. You should look out for such details in science magazines and news dailies.



**INTEXT QUESTIONS 23.2**

- How would the magnification of a telescope be affected by increasing the focal length of:
  - the objective \_\_\_\_\_  
\_\_\_\_\_
  - the eye piece \_\_\_\_\_  
\_\_\_\_\_
- If the focal length of the objective of a telescope is 50 cm and that of the eyepiece is 2 cm. What is the magnification?
- State one difference between the refracting and reflecting telescope.
- What is normal adjustment?
- If the telescope is inverted, will it serve as a microscope?

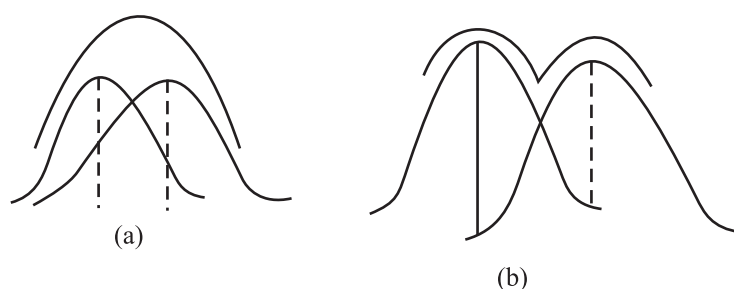
**23.3 RESOLVING POWER : THE RAYLEIGH'S CRITERION**

In earlier lessons, you have seen that the image of a point source is not a point, but has a definite size and is surrounded by a diffraction pattern. Similarly, if there are two point sources very close to each other, the two diffraction patterns formed by the two sources may overlap and hence it may be difficult to distinguish them as separate by the unaided eye. The resolving power of an optical instrument is its ability to resolve (or separate) the images of two point objects lying close to each other. Rayleigh suggested that two images can be seen as distinct when the first minimum of the diffraction pattern due to one object falls on the central maximum of the other. This is called *Rayleigh's criterion*.

If we assume that the pupil of our eye is about 2 mm in diameter, two points can be seen distinctly separate if they subtend an angle equal to about one minute of arc at the eye. **The reciprocal of this angle is known as the resolving power of the eye.**

Now let us calculate the resolving power of common optical instruments. We begin our discussion with a telescope.

### 23.3.1 Resolving Power of a Telescope



**Fig. 23.10 :** Rayleigh's criterion for resolution **a)** when the angular separation is less than  $\theta$ , the two points are seen as one, and **b)** when the angular separation is more than  $\theta$ , the two points are distinctly visible.

The resolving power of a telescope is its ability to form separate images of two distant point objects situated close to each other. It is measured in terms of the angle subtended at its objective by two close but distinct objects whose images are just seen in the telescope as separate. This angle is called the **limit of resolution** of the telescope. If the angle subtended by two distinct objects is less than this angle, the images of the objects can not be resolved by the telescope. The smaller the value of this angle, higher will be the resolving power of the telescope. Thus, the reciprocal of the limit of resolution gives the resolving power of the telescope.

If  $\lambda$  is the wavelength of light,  $D$  the diameter of the telescope objective, and  $\theta$  the angle subtended by the point object at the objective, the limit of resolution of the telescope is given by (Rayleigh's criterion)

$$\theta = \frac{1.22\lambda}{D}$$

Hence, the resolving power of the telescope.

$$(\text{R.P.})_T = \frac{1}{\theta} = \frac{D}{1.22\lambda} \quad (23.15)$$

From Eqn. (23.15) it is clear that to get a high resolving power, a telescope with large aperture objective or light of lower wavelength has to be used.

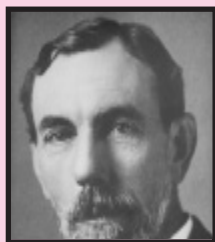


Notes





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### Lord Rayleigh (1842 – 1919)

Born to the second Baron Rayleigh of Terling place, Witham in the country of Essex, England, John strutt had a very poor health in his childhood. Due to this he had a disrupted schooling. But he had the good luck of having Edward Rath and Stokes as his teachers. As a result, he passed his tripos examination in 1865 as senior Wrangler and become the first recipient of Smiths prize.

In addition to the discovery of Argon, for which he was awarded Nobel prize (1904), Rayleigh did extensive work in the fields of hydrodynamics, thermodynamics, optics and mathematics. His travelling wave theory, which suggested that elastic waves can be guided by a surface, paved way for researches in seismology and electronic signal processing. During the later years of his life, he also showed interest in psychiatry research. Lunar feature-crater Rayleigh and planetary feature crater Rayleigh on Mars are a tribute to his contributions.

**Example 23.4:** A telescope of aperture 3 cm is focussed on a window at 80 metre distance fitted with a wiremesh of spacing 2 mm. Will the telescope be able to observe the wire mesh? Mean wavelength of light  $\lambda = 5.5 \times 10^{-7}$  m.

**Solution:** Given  $\lambda = 5.5 \times 10^{-7}$  m and  $D = 3$  cm =  $3 \times 10^{-2}$  m

Therefore, the limit of resolution

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-7} \text{ m}}{3 \times 10^{-2} \text{ m}} = 2.236 \times 10^{-5} \text{ rad}$$

The telescope will be able to resolve the wiremesh, if the angle subtended by it on the objective is equal to or greater than  $\theta$ , the limit of resolution. The angle subtended by the wiremesh on the objective

$$\begin{aligned} \alpha &= \frac{\text{spacing of wiremesh}}{\text{distance of the objective from the wiremesh}} \\ &= \frac{2 \text{ mm}}{80 \text{ m}} = \frac{2 \times 10^{-3}}{80 \text{ m}} = 2.5 \times 10^{-5} \text{ rad.} \end{aligned}$$

As the angle  $2.5 \times 10^{-5}$  radian exceeds the limit of a resolution ( $= 2.236 \times 10^{-5}$  radian), the telescope will be able to observe the wire mesh.



Notes

### 23.3.2 Resolving Power of a Microscope

The resolving power of a microscope represents its ability to form separate images of two objects situated very close to each other. The resolving power of a microscope is measured in terms of the smallest linear separation between the two objects which can just be seen through the microscope as separate. This smallest linear separation between two objects is called **the limit of resolution of the microscope**.

The smaller the value of linear separation, the higher will be the resolving power of the microscope. Thus, **the reciprocal of the limit of resolution gives the resolving power of the microscope**.

If  $\lambda$  is the wavelength of light used to illuminate the object,  $\theta$  is the half angle of the cone of light from the point object at the eye and  $n$  is the refractive index of the medium between the object and the objective, the limit of resolution of the microscope is given by

$$d = \frac{\lambda}{2n\sin\theta} \quad (23.16)$$

Thus the resolving power of microscope will be

$$(\text{R.P.})_m = \frac{2n\sin\theta}{\lambda}$$

(23.17) The expression  $2n\sin\theta$  is called numerical aperture (N.A) The highest value of N.A of the objective obtainable in practice is 1.6, and for the eye, N.A is 0.004.

It is clear from Eqn. (23.17) that the resolving power of a microscope can be increased by increasing the numerical aperture and decreasing the wavelength of the light used to illuminate the object. That is why ultraviolet microscopes and an electron microscope have a very high resolving power.

#### Applications in Astronomy

The astronomical (or optical) telescope can be used for observing stars, planets and other astronomical objects. For better resolving power, the optical telescopes are made of objectives having a large aperture (objective diameter). However, such big lenses are difficult to be made and support. Therefore, most astronomical telescopes use reflecting mirrors instead of lenses. These can be easily supported as a mirror weighs less as compared to a lens of equivalent optical quality.

The astronomical telescopes, which are ground-based, suffer from blurring of images. Also, ultraviolet, x-ray, gamma-ray etc. are absorbed by the earth's



Notes

surface. They cannot be studied by ground-based telescopes. In order to study these rays coming from astronomical objects, telescopes are mounted in satellites above the Earth's atmosphere. NASA's Hubble space telescope is an example of such telescope. Chandra X-ray observation, Compton x-ray observation and Infrared telescopes have recently been set up in space.



### INTEXT QUESTIONS 23.3

1. How can the resolving power of a telescope be improved?
2. What is the relationship between the limit of resolution and the resolving power of the eye?
3. If the wavelength of the light used to illuminate the object is increased, what will be the effect on the limit of resolution of the microscope?
4. If in a telescope objective is made of larger diameter and light of shorter wavelength is used, how would the resolving power change?



### WHAT YOU HAVE LEARNT

- The angle subtended by an object at the human eye is called as the visual angle.
- The angular magnification or magnifying power of a microscope is the ratio of the angle subtended by the image at the eye to the angle subtended by the object when both are placed at the near point.
- Linear magnification is defined as the ratio of the size of the image to the size of the object.
- The magnifying power of a simple microscope is  $M = 1 + \frac{D}{f}$ , where  $D$  is least distance of distinct vision and  $f$  is focal length of the lens.
- In a compound microscope, unlike the simple microscope, magnification takes place at two stages. There is an eye piece and an objective both having short focal lengths. But the focal length of the objective is comparatively shorter than that of the eye piece.
- The magnifying power of a compound microscope is given as

$$M = m_o \times m_e$$

But  $m_e = 1 + \frac{D}{f}$ . Therefore

$$M = \frac{v_o}{u_o} \left( 1 + \frac{D}{f_e} \right)$$

where  $v_o$  is distance between the image and the objective,  $u_o$  is object distance from the objective,  $D$  is the least distance of distinct vision ( $= 25\text{cm}$ ) and  $f_e$  is focal length of the eye-piece.

- Telescope is used to see the distant objects which subtend very small visual angle at the eye. The use of a telescope increases the visual angle at the eye. The far-off object appears to be closer to the eye and can be seen easily.
- Two types of telescopes are used (i) Refracting (ii) Reflecting.
- The objective of the refracting telescope is a converging lens. But the objective in a reflecting telescope is a spherical mirror of large focal length. There are several advantages of reflecting telescope over a refracting telescope.

The magnifying power of a telescope is

$$M = f_o/f_e$$

where  $f_o$  is focal length of the objective and  $f_e$  is focal length of the eyepiece.



### TERMINAL EXERCISE

1. What is the difference between simple and compound microscopes? Derive an expression for the magnification of a compound microscope.
2. Distinguish between the refracting and reflecting telescope. Draw a ray diagram for the Newton's telescope.
3. Derive an equation for the magnifying power of a refracting telescope.
4. What do you mean by the least distance of distinct vision? What is its value for a normal eye?
5. Can we photograph the image formed by a compound microscope? Explain your answer.
6. Define the resolving power of an optical instrument. What is the value of limit of resolution for a normal eye?
7. What are the main differences in the design of a compound microscope and a terrestrial telescope?
8. The eyepiece of a telescope has a focal length of 10 cm. The distance between the objective and eye piece is 2.1 m. What is the angular magnification of the telescope?



Notes

## MODULE - 6

### Optics and Optical Instruments



Notes

## Optical Instruments

- The image formed by a microscope objective of focal length 4 mm is 18 cm from its second focal point. The eyepiece has a focal length of 3.125 cm. What is the magnification of the microscope?
- The objective of a telescope has a diameter three times that of a second telescope. How much more amount of light is gathered by the first telescope as compared to the second?



## ANSWERS TO INTEXT QUESTIONS

### 23.1

- Image formed by a simple microscope is virtual erect and magnified. whereas the image formed by a compound microscope is real, inverted and magnified.
- Magnifying power is the ratio of the angle subtended by the image at eye piece to the angle subtended by the object placed at the near point. Magnification is the ratio of the size of image to the size of object.
- $M = 11$ ,  $m = 1 + \frac{D}{f}$ . Putting  $D = 25$  cm, we get  $f = 2.5$  cm
- If you choose the lens with 4 cm focal length, the magnifying power will be high because  $m = \frac{f_o}{f_e}$
- The magnifying power of a compound microscope is given by  $M = \frac{-L}{f_o} \left( 1 + \frac{D}{f_e} \right)$   
Obviously,  $M$  will have a large value, if both  $f_o$  and  $f_e$  are small.

### 23.2

- (a) Objective of large focal length increases the magnifying power of the telescope.  
(b) Magnification is reduced by increasing the focal length of eyepiece.
- Magnification  $m = \frac{f_o}{f_e} = \frac{50 \text{ cm}}{2 \text{ cm}} = 25$
- The objectives of a telescope is a spherical mirror of large focal length instead of converging lens as in a refracting telescope.
- A telescope is said to be in normal adjustment, if the final image is formed at infinity.
- No

## 23.3

1. By taking a large aperture or by using a light of lower wavelength.
2. The limit of resolution of an eye is inversely proportional to its resolving power. Limit of resolution will also be increased.
3. Since resolving power of telescope is given by  $R.P = \frac{D}{1.22\lambda}$ , it would increase.

**Answers To Problems in Terminal Exercise**

8. 21

9. 400

10. 9 times.

**Notes**