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312en18

# MAGNETISM AND MAGNETIC EFFECT OF ELECTRIC CURRENT

In lesson 15, you learnt how charged rods attract each other or small bits of paper. You might have also played with magnets – the substances having the property of attracting small bits of iron. But did you ever think of some relation between electricity and magnetism? Such a relationship was discovered by Oersted in 1820. Now we know, for sure, how intimately magnetism and electricity are related.

In this lesson, you will learn the behaviour of magnets and their uses as also the magnetic effects of electric current. The behaviour of current carrying conductors and moving charges in a magnetic field are also discussed. On the basis of these principles, we will discuss the working of electric devices like motors and measuring devices like an ammeter, a voltmeter and a galvanometer.



## OBJECTIVES

After studying this lesson, you should be able to :

- *define magnetic field and state its SI unit;*
- *list the elements of earth's magnetic field and write the relation between them;*
- *describe the magnetic effect of electric current : Oersted's experiment;*
- *state Biot-Savart's law and explain its applications;*
- *explain Ampere's circuital law and its application;*
- *describe the motion of a charged particle in uniform electric field and magnetic field;*
- *explain the construction and working of a cyclotron;*

## MODULE - 5

### Electricity and Magnetism



Notes

## Magnetism and Magnetic Effect of Electric Current

- derive an expression for the force experienced by a current carrying conductor placed in a uniform magnetic field;
- derive an expression for the force between two infinitely long current carrying conductors placed parallel to each other; and
- explain the working principle of a galvanometer, an ammeter and a voltmeter.

### 18.1 MAGNETS AND THEIR PROPERTIES

The phenomenon of magnetism was known to Greeks as early as 600 B.C. They observed that some stones called magnetite ( $\text{Fe}_3\text{O}_4$ ) attracted iron pieces. The pieces of naturally occurring magnetite are called **natural magnets**. Natural magnets are weak, but materials like iron, nickel, cobalt may be converted into strong permanent magnets. All magnets—natural or artificial—have same properties. You must be familiar with basic properties of magnets. However, for completeness, we recapitulate these.

- Directive Property :** A small bar magnet, when suspended freely on its center of mass so as to rotate about a vertical axis, always stays in approximately geographical north-south direction.
- Attractive Property :** A magnet attracts small pieces of magnetic materials like iron, nickel and cobalt. The force of attraction is maximum at points near the ends of the magnet. These points are called **poles** of the magnet. In a freely suspended magnet, the pole which points towards the geographical north is called is **north pole** and the one which points towards the geographical south is called **south pole**. Do directive and attractive properties suggest that our earth also acts like a magnet? Yes, it does.
- Unlike poles of two magnets attract each other and like poles repel (Fig. 18.1).
- The poles of a magnet are inseparable, i.e. the simplest specimen providing magnetic field is a magnetic dipole.
- When a magnet is brought close to a piece of iron, the nearer end of the piece of iron acquires opposite polarity and the farther end acquires same polarity. This phenomenon is called **magnetic induction**.

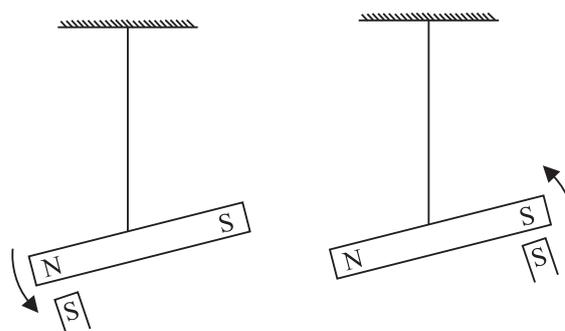


Fig. 18.1 : Unlike poles of two magnets attract each other and like poles repel.

## 18.1.1 Magnetic Field Lines

Interactions between magnets or a magnet and a piece of iron essentially represent action at a distance. This can be understood in terms of magnetic field. A very convenient method to visualize the direction and magnitude of a field is to draw the field lines :

- The direction of magnetic field vector  $\mathbf{B}$  at any point is given by the tangent to the field line at that point.
- The number of field lines that pass through unit area of a surface held perpendicular to the lines is proportional to the strength of magnetic field in that region. Thus, the magnetic field  $\mathbf{B}$  is large where the field lines are closer together and smaller where they are far apart.

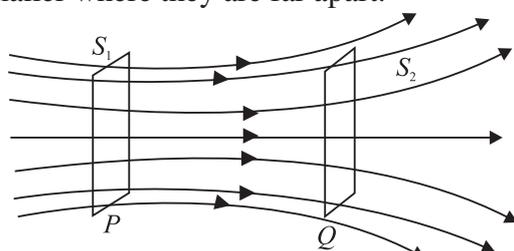


Fig 18.2: Magnetic field lines passing through two parallel surfaces

Fig 18.2 shows a certain number of field lines passing through parallel surfaces  $S_1$  and  $S_2$ . The surface area of  $S_1$  is same as that of  $S_2$  but the number of field lines passing through  $S_1$  is greater than those passing through  $S_2$ . Hence, the number of lines per unit area passing through  $S_1$  is greater than that through  $S_2$ . We can, therefore, say that the magnetic field in the region around  $P$  is stronger than that around  $Q$ .

- Outside the magnet, the field lines run from north pole to south pole and inside it, these run from south pole to north pole forming closed curves (Fig. 18.3).
- Two magnetic field lines can never cross each other.

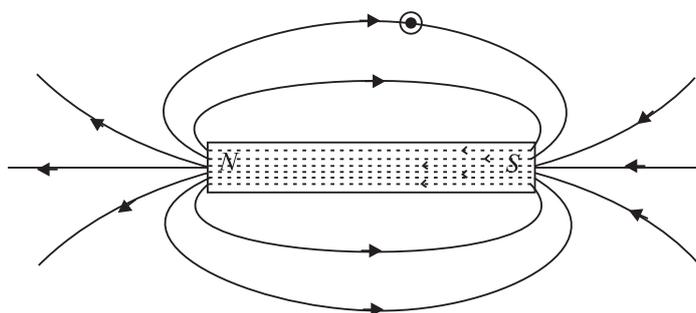


Fig. 18.3 : Magnetic field lines of a bar magnet



## INTEXT QUESTIONS 18.1

1. You are given a magnet. How will you locate its north pole?

Notes



## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

2. You are provided two identical looking iron bars. One of these is a magnet. Using just these two, how will you identify which of the two is a magnet.
3. You are given a thread and two bar magnets. Describe a method by which you can identify the polarities of the two magnets.

### Magnetic field of the Earth

The directive property of magnets could be explained by considering that the earth acts as a magnet, i.e., as if a large bar magnet is placed inside the earth. The south pole of this magnet is considered near the geographical north pole and the magnetic north pole near the geographical south pole.  $RR_1$  is the rotation axis of earth and  $MM_1$  is the magnetic axis of the earth.

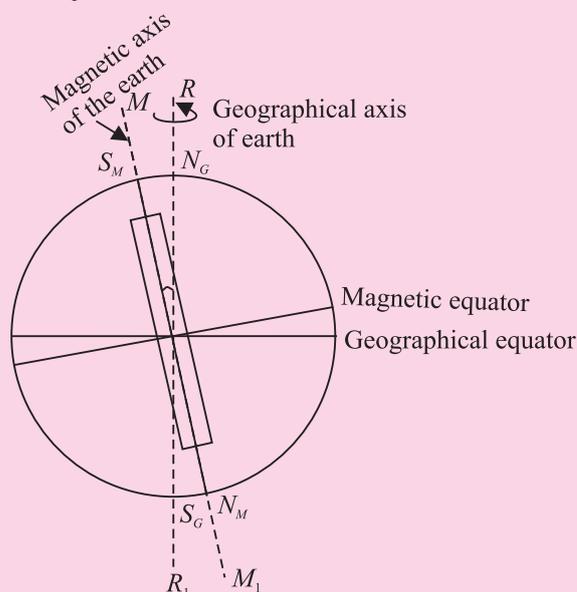


Fig. 18.4 : Magnetic field of the earth



### ACTIVITY 18.1

Let us perform an experiment with a magnetic needle. (You can actually perform the experiment with a globe containing a bar magnet along its axis of rotation with north pole of the magnet pointing south.) Suspend the needle freely in such a manner that it can rotate in horizontal as well as vertical planes. If the needle is near the equator on earth's surface, it rests in horizontal plane. Suppose this needle is taken to places in the northern hemisphere. The needle rotates in the vertical plane and the north pole dips towards the earth, as we move towards geographical north pole. Finally at a point very near to Hudson bay in Canada, the north pole of the needle will point vertically downward. This place, located at  $6^\circ$  east of north, is considered to be the south pole of the earth's magnet.

This place is about 650 km away from the earth's geographical north pole. If we take the same magnetic needle to places in the southern hemisphere, the south pole of the needle will dip downward and point vertically downward at a point 650 km west of the geographical south. This point could be considered as the *N pole of the earth's magnet*. From this we conclude that the magnetic axis of the earth does not coincide with the geographical axis.

An important aspect of earth's magnetic field is that it does not remain constant; its magnitude and direction change with time.

### Elements of the Earth's Magnetic Field

Three measurable quantities are used to describe the magnetic field of earth. These are called elements of earth's magnetic field :

- Inclination or dip ( $\delta$ );
- Declination ( $\theta$ ); and
- Horizontal component of the earth's field ( $\mathbf{B}_M$ ).

#### (a) Inclination or Dip

If you suspend a magnetic needle freely at a place, you will observe that the needle does not rest in the horizontal plane. It will point in the direction of the resultant intensity of earth's field.

Fig. 18.5 shows the plane  $PCDE$ , which is the magnetic meridian at the point  $P$  (i.e. the vertical plane passing through the north and south poles of the earth's magnet) on the surface of the earth and  $PABC$  is the geographic meridian (i.e. the vertical plane passing through the geographical north and south poles of the earth). Suppose that  $PR$  represents the magnitude and direction of the earth's magnetic field at the point  $P$ . Note that  $PR$  makes an angle  $\delta$  with the horizontal direction. This angle is known as inclination or dip at  $P$  on the surface of the earth.

The angle which the earth's magnetic field makes with the horizontal direction in the magnetic meridian is called the **dip or inclination**.

#### (b) Declination

Refer to Fig 18.5 again. The plane  $PCDE$  contains the magnetic field vector ( $\mathbf{PR}$ ) of the earth. The angle between the planes  $PCDE$  and  $PABC$  is called the declination at the point  $P$ . It is shown as angle  $\theta$ .

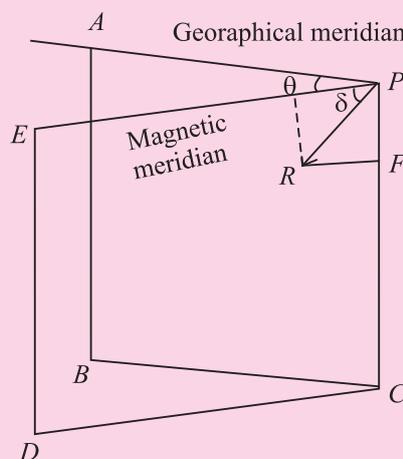


Fig. 18.5: Elements of earth's magnetic field



Notes

## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

The angle which the magnetic meridian at a place makes with the geographical meridian is called the declination at that place.

### (c) Horizontal component

Fig. 18.5 shows that  $\mathbf{PR}$  is the resultant magnetic field at the point  $P$ .  $PH$  represents the horizontal component and  $PF$  the vertical component of the earth's magnetic field in magnitude and direction. Let the magnetic field at the point  $P$  be  $\mathbf{B}$ . The horizontal component

$$B_H = B \cos \delta \quad (18.1)$$

and the vertical component

$$B_V = B \sin \delta \quad (18.2)$$

By squaring and adding Eqns. (18.1) and (18.2), we get

$$B_H^2 + B_V^2 = B^2 \cos^2 \delta + B^2 \sin^2 \delta = B^2 \quad (18.3)$$

On dividing Eqn. (18.2) by Eqn. (18.1), we have

$$\frac{B_V}{B_H} = \tan \delta \quad (18.4)$$

## 18.2 ELECTRICITY AND MAGNETISM : BASIC CONCEPTS

You now know that flow of electrons in a conductor due to a potential difference across it constitutes electric current. The current flowing in a conductor is seen to exert a force on a free magnetic needle placed in a region around it. A magnetic needle is also affected by a magnet and hence we say that a current carrying conductor has a magnetic field around it. The magnetic field  $B$  is visualized by magnetic field lines. You will learn about these and some more terms such as magnetic permeability later in this lesson.

### 18.2.1 Magnetic Field around an Electric Current

Let us do a simple experiment.



#### ACTIVITY 18.2

Take a 1.5 volt battery, a wire about 1 m in length, a compass needle and a match box. Wind 10-15 turns of the electric wire on its base. Under the windings, place a compass needle, as shown in Fig. 18.6. Place the match box on the table so as to have the wires running along the north – south direction. Connect the free ends of the wire to the battery. What happens to the needle? You will observe that

needle shows deflection. This means that there is a magnetic field in and around the coil. The deflection will reverse if you reverse the direction of current by changing the terminals of the battery. When there is no

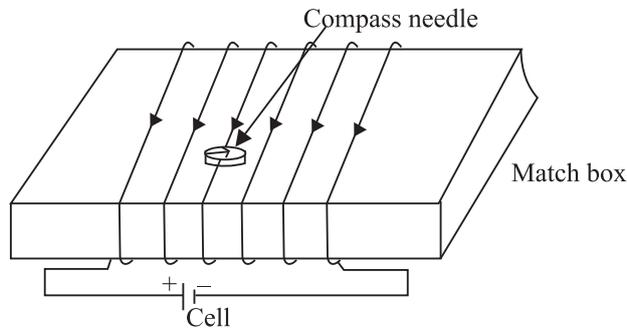
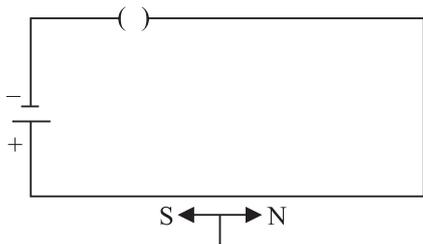
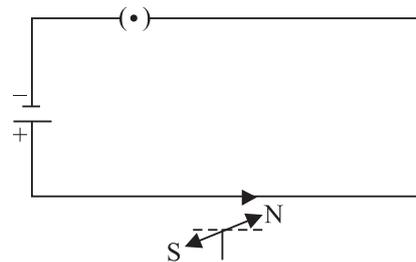


Fig. 18.6 : Demonstration of magnetic field due to electric current

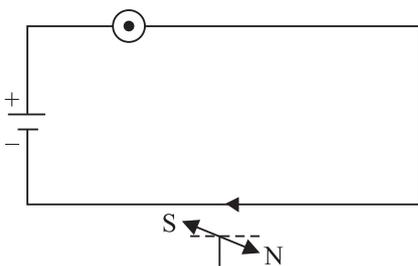
current in the wire, the compass needle points in the north – south direction (Fig. 18.7 a, b & c). When a magnetic needle is brought close to a vertical current carrying wire, the magnetic field lines are concentric circles around the wire, as shown in Fig 18.7 (d).



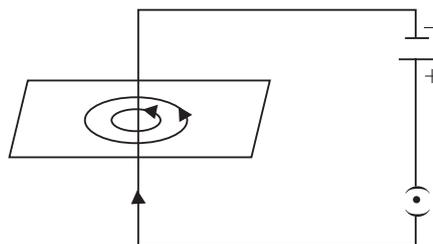
(a) No current, No deflection north



(b) Current towards north deflection of pole towards west



(c) When direction of current is reversed, direction of deflection is reversed



(d) Circular field lines around a straight current carrying conductor

Fig. 18.7 : Magnetic field around a current carrying conductor

In 1820 Hans Christian Oersted, Professor of Physics at Copenhagen in Denmark performed similar experiments and established that there is a magnetic field around a current carrying conductor.



Notes



Notes

18.3 BIOT-SAVART'S LAW

Biot-Savart's law gives a quantitative relationship between current in conductor and the resulting magnetic field at a point in the space around it. Each part of a current carrying conductor contributes to magnetic field around it. The net value of  $\mathbf{B}$  at a point is thus the combined effect of all the individual parts of the conductor. As shown in Fig. 18.8, the net magnetic field due to any current carrying conductor is the vector sum of the contributions due to the current in each infinitesimal element of length  $\Delta \ell$ .

Experiments show that the field  $B$  due to an element  $\Delta \ell$  depends on

- current flowing through the conductor,  $I$ ;
- length of the element  $\Delta \ell$ ;
- inversely proportional to the square of the distance of observation point  $P$  from the element  $\Delta \ell$ ;
- the angle between the element and the line joining the element to the observation point.

Thus, we can write

$$\begin{aligned}
 |\Delta \mathbf{B}_0| &\propto \frac{I \Delta \ell \sin \theta}{r^2} \\
 &= \frac{\mu_0}{4\pi} \frac{I d\ell \sin \theta}{r^2}
 \end{aligned}
 \tag{18.5}$$

where  $\mu_0$  is permeability of vacuum. Its value is  $4\pi \times 10^{-7} \text{ WA}^{-1}\text{m}^{-1}$ . The value of permeability of air is also nearly equal to  $\mu_0$

If the conductor is placed in a medium other than air, the value of the field is altered and is given by  $|\mathbf{B}| = \mu |\mathbf{B}_0|$ . Here  $\mu$  represents the permeability of the medium.

**Direction of  $\mathbf{B}$  :** Magnetic field at a point is a vector quantity. The direction of  $\mathbf{B}$  may be determined by applying the right hand grip rule. To apply this rule, let us consider the direction of the field produced in some simple cases. As shown in the Fig. 18.9 (a), grasp the wire in your right hand so that the thumb points in the direction of the current. Then the curled fingers of the hand will point in the direction of the magnetic field. To represent the magnetic field on paper, let us consider that current is flowing into the plane of the paper. Then according to the right hand rule, the field lines shall be in the plane of the paper (Fig.18.9 b).

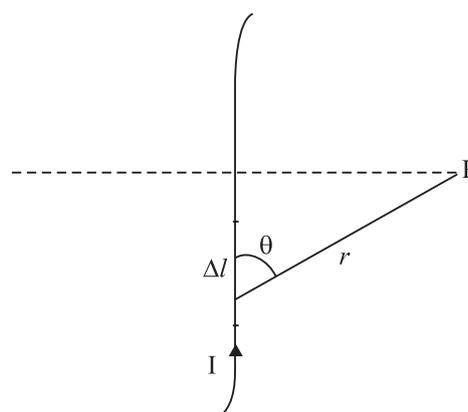
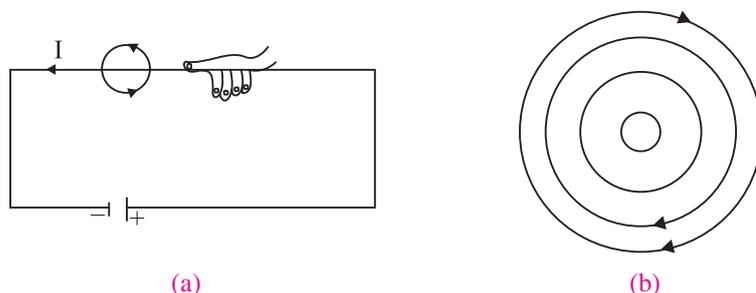


Fig. 18.8 : Magnetic field at P due to a current element  $\Delta \ell$



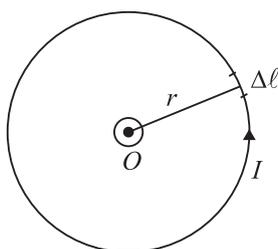
**Fig. 18.9 :** Direction of magnetic field : **a)** Right hand rule : thumb in the direction of current, field lines in the direction of curling fingers, and **b)** when current is in the plane of paper, the field lines shall be in the plane of paper, according to the right hand rule.

### 18.3.1 Applications of Biot-Savart's Law

You now know that Biot-Savart's law gives the magnitude of the magnetic field. Let us now apply it to find the field around conductors of different shapes. Note that to calculate the net field due to different segments of the conductor, we have to add up the field contributions due to each one of them. We first consider a circular coil carrying current and calculate magnetic field at its centre.

**(a) Magnetic field at the centre of a circular coil carrying current :** Refer to Fig.18.10. It shows a circular coil of radius  $r$  carrying current  $I$ . To calculate magnetic field at its centre  $O$ , we first consider a small current element  $\Delta \ell$  of the circular coil. Note that the angle between current element  $\Delta \ell$  and  $r$  is  $90^\circ$ . From Eqn. (18.5) we know that the field at the centre  $O$  due to  $\Delta \ell$  is

$$\begin{aligned} |\Delta \mathbf{B}| &= \frac{\mu_0}{4\pi} I \frac{\Delta \ell}{r^2} \sin 90^\circ \\ &= \frac{\mu_0}{4\pi} I \frac{\Delta \ell}{r^2} \quad (\text{as } \sin 90^\circ = 1) \end{aligned}$$



**Fig. 18.10:** Circular coil carrying current

The direction of  $\Delta \mathbf{B}$  is normal to the plane of the coil. Since the field due to every element of the circular coil will be in the same direction, the resultant is obtained by adding all the contributions at the centre of the loop. Therefore

$$|\mathbf{B}| = \sum |\Delta \mathbf{B}| = \frac{\mu_0 I}{4\pi r^2} \sum \Delta \ell = \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r$$

Hence, magnetic field at the centre of a coil of radius  $r$  carrying current  $I$  is given by

$$|\mathbf{B}| = \frac{\mu_0 I}{2r} \quad (18.6)$$

Notes





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In case there is more than one loop of wire (say there are  $n$  turns), the field is given by

$$|B| = \frac{\mu_0 nI}{2r}$$

You can check the direction of the net field using the rule given in Fig. 18.7. You can use right hand rule in any segment of the coil and will obtain the same result. (Another simple quick rule to identify the direction of magnetic field due to a current carrying coil is the so called End-rule, illustrated in Fig. 18.11 (a, b).

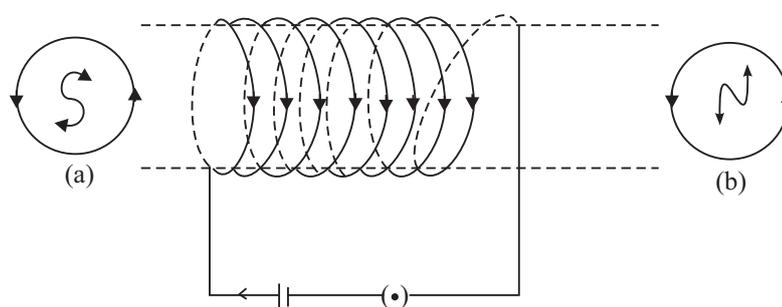


Fig 18.11: Direction of magnetic field : End-rule

When an observer looking at the circular coil at its either end finds the current to be flowing in the clockwise sense, the face of the coil behaves like the south pole of the equivalent magnet, i.e.,  $B$  is directed inwards. On the other hand, if the current is seen to flow in the anticlockwise sense, the face of the coil behaves like the north pole of the equivalent magnet or the field is directed out of that end.



INTEXT QUESTIONS 18.2

1. What can you say about the field developed by
  - (i) a stationary electron ?
  - (ii) a moving electron ?
2. Electrons in a conductor are in constant motion due to thermal energy. Why do they not show magnetism till such time that a potential difference is applied across it ?
3. A current is flowing in a long wire. It is first shaped as a circular coil of one turn, and then into a coil of two turns of smaller radius. Will the magnetic field at the centre coil change? If so, how much ?

## 18.4 AMPERE'S CIRCUITAL LAW

Ampere's circuital law provides another way of calculating magnetic field around a current carrying conductor in some simple situations.

Ampere's circuital law states that the line integral of the magnetic field  $\mathbf{B}$  around a closed loop is  $\mu_0$  times the total current,  $I$ . Mathematically, we write

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad (18.7)$$

Note that this is independent of the size or shape of the closed loop.

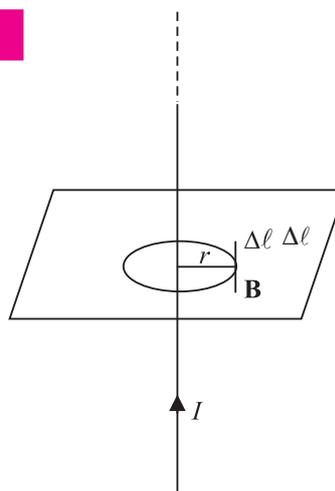


Fig. 18.12 : Ampere's circuital law



### Andre Marie Ampere (1775 – 1836)

French Physicist, mathematician and chemist, Ampere was a child prodigy. He mastered advanced mathematics at the age of 12. A mix of experimental skills and theoretical acumen, Ampere performed rigorous experiments and presented his results in the form of a theory of electrodynamics, which provides mathematical formulation of electricity and its magnetic effects. Unit of current is named in his honour. Lost in his work and ideas, he seldom cared for honours and awards. Once he forgot an invitation by emperor Napoleon to dine with him. His gravestone bears the epitaph : Tendun felix (Happy at last), which suggests that he had to face a very hard and unhappy life. But it never lowered his spirit of creativity.

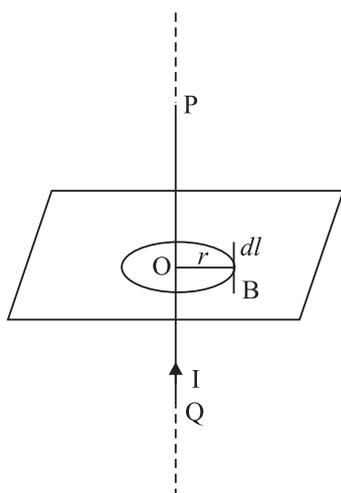


Fig. 18.13: Infinitely long current carrying conductor

#### 18.4.1 Applications of Ampere's Circuital Law

We now apply Ampere's circuital law to obtain magnetic field in two simple situations.

##### (a) Magnetic field due to an infinitely long current carrying conductor

Refer to Fig. 18.13. It shows an infinitely long current carrying conductor  $POQ$  carrying current  $I$ . Consider a circular loop of radius  $r$  around it in the plane as shown. Then

$$\Sigma \mathbf{B} \cdot d\mathbf{l} = B 2\pi r$$

By applying Ampere's circuital law, we can write

$$|\mathbf{B}| 2\pi r = \mu_0 I$$

Notes



## MODULE - 5

### Electricity and Magnetism



Notes

## Magnetism and Magnetic Effect of Electric Current

or 
$$|\mathbf{B}| = \frac{\mu_0 I}{2\pi r} \quad (18.8)$$

This gives the magnetic field around an infinitely long straight current carrying conductor.

Solenoids and toroids are widely used in motors, generators, toys, fan-windings, transformers, electromagnets etc. They are used to provide uniform magnetic field. When we need large fields, soft iron is placed inside the coil.

### (b) Magnetic field due to a solenoid

A solenoid is a straight coil having a large number of loops set in a straight line with a common axis, as shown in Fig. 18.14. We know that a current  $I$  flowing through a wire, sets up a magnetic field around it. Suppose that the length of the solenoid is  $\ell$  and it has  $N$  number of turns. To calculate the magnetic field inside the solenoid along its axis (Fig 18.14), we can treat it to be a section of a toroidal solenoid of a very large radius. Thus :

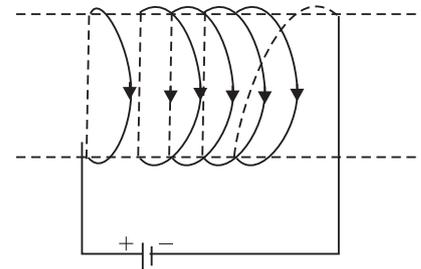


Fig. 18.14 : A solenoid

$$|\mathbf{B}| = \mu_0 nI$$

The direction of the field is along the axis of the solenoid. A straight solenoid is finite. Therefore,  $|\mathbf{B}| = \mu_0 nI$  should be correct well inside the solenoid, near its centre.

For solenoids of small radius, the magnitude of  $B$  at the ends is given by

$$|\mathbf{B}| = \frac{\mu_0 nI}{2} \quad (18.9)$$

The solenoid behaves like a bar magnet and the magnetic field is as shown in Fig. 18.15.

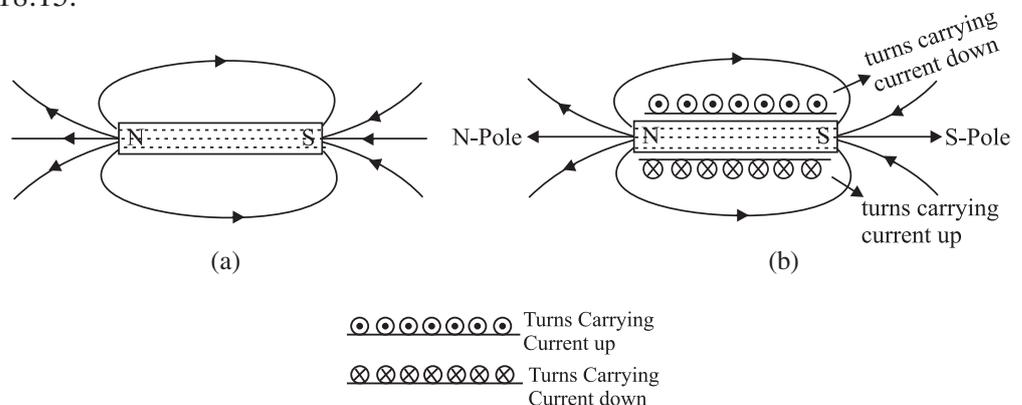


Fig. 18.15: Solenoid behaves like a bar magnet : a) Magnetic field due to a bar magnet, and b) magnetic field due to a current carrying solenoid

## 18.4.2 Application of Ampere's Circuital Law

## (b) Magnetic Field due to a Straight Solenoid

A solenoid is a straight coil having a large number of loops set in a straight line with a common axis, as shown in Fig. 18.4.2. We know that a current  $I$  flowing through a wire, sets up a magnetic field around it. Suppose that the length of the solenoid is  $\ell$  and it has  $N$  number of turns.

The magnetic field inside the solenoid, in its middle, is uniform and parallel to its axis. Outside the solenoid, however, the field is negligibly weak. These statements hold true, strictly speaking, if the length of the solenoid is very large as compared to its diameter. For a long solenoid, whose windings are very tightly and uniformly wound, the magnetic field inside it is fairly uniform everywhere and is zero outside it.

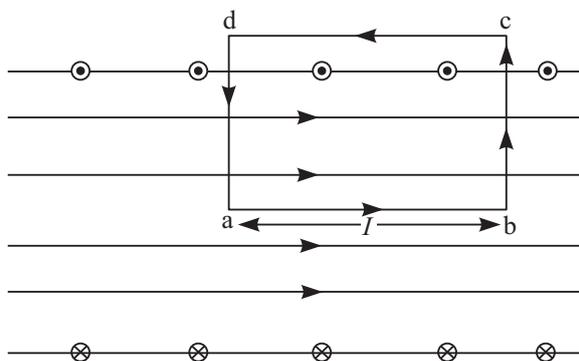


Fig. 18.4.2

Let us take a rectangular loop  $abcd$  as shown in Fig 18.4.2. Along the path  $ab$ , the magnetic field is uniform. Hence, for this path  $\mathbf{B} \cdot d\mathbf{l} = B\ell$ . Along the paths  $cd$ , as the magnetic field is weak it may be taken as zero. Hence, for this path  $\mathbf{B} \cdot d\mathbf{l} = 0$ . The two short sides  $bc$  and  $da$  also do not contribute anything to  $\mathbf{B} \cdot d\mathbf{l}$  as  $\mathbf{B}$  is either zero (outside the solenoid), or perpendicular to  $d\mathbf{l}$  (inside the solenoid).

If  $n$  be the number of turns per unit length along the length of the solenoid, then the number of turns enclosed by the rectangular loop of length  $\ell$  is  $n\ell$ . If each turn of the solenoid carries a current  $i$ , then the total current threading the loop is  $n\ell i$ . Hence, from Ampere's circuital law,

$$\sum \mathbf{B} \cdot d\mathbf{l} = \mu_0(n\ell i)$$

or  $B\ell = \mu_0 n\ell i$

or  $B = \mu_0 n i$



Notes



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**(c) Magnetic field due to a toroid**

A toroid is basically an endless solenoid which may be formed by bending a straight solenoid so as to give it a circular shape.

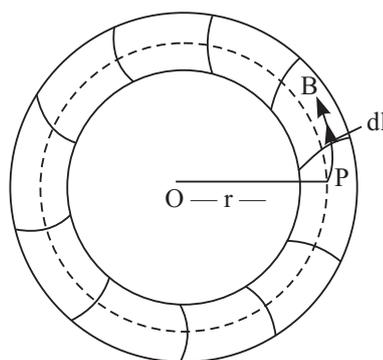


Fig. 18.4.3

Suppose, we want to find the magnetic field at a point  $P$ , inside the toroid, whose distance from the centre  $O$  is  $r$ . Draw a circle passing through the point  $P$  and concentric with the toroid. The magnetic field will everywhere be tangential to the circle, its magnitude being the same at all points of it. So, we can write:

$$\sum \mathbf{B} \cdot d\ell = \sum B d\ell = B \sum d\ell$$

But  $\sum d\ell = 2\pi r$ , the circumference of the circular path.

Therefore,

$$\sum \mathbf{B} \cdot d\ell = 2\pi r B$$

If  $N$  be the total number of turns and  $i$  the current flowing through the windings of the toroid, then the total current threaded by the circular path of radius  $r$  is  $Ni$ . Hence, from Ampere's circuital law,

$$\sum \mathbf{B} \cdot d\ell = \mu_0 Ni$$

or  $2\pi r B = \mu_0 Ni$

or  $B = \frac{\mu_0 Ni}{2\pi r}$



### 18.4.3 Electromagnets and Factors Affecting their Strength

We have seen that a current-carrying solenoid behaves as a bar magnet, with one end behaving as north pole and the other as south pole depending on the direction of flow of current. The polarity of such magnets is determined by the end rule and the strength of the magnetic field is given by

$$|\mathbf{B}| = \mu_0 nI$$

where  $\mu_0$  is the permeability of free space,  $n$  is the number of turns per unit length and  $I$  is the current flowing through the solenoid.

It is clear that the solenoid remains a magnet as long as the a current is flowing through it. Thus, a current-carrying solenoid is called an electromagnet.

Its strength depends on :

- (i) Number of turns per unit length of the solenoid, and
- (ii) The current flowing through it.

It may also be noted that the strength of the magnetic field of an electromagnet increases when a soft iron core is introduced inside it.

### 18.4.4 Concept of Displacement Current

The concept of displacement current was introduced by Maxwell. As we know, magnetic field is produced due to the conduction current. However, according to Maxwell in empty space (where no conduction current exists), the magnetic field is produced due to the displacement current which, unlike conduction current, is not associated with the motion of charges.

Consider a simple circuit consisting of a small parallel-plate capacitor being charged by a current  $I$ .

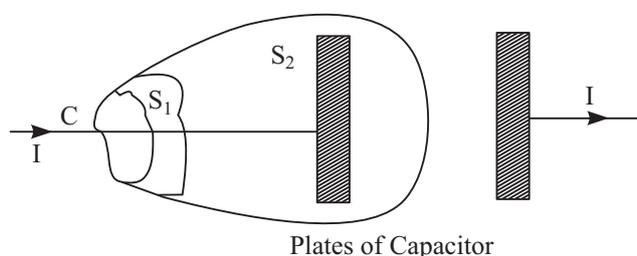


Fig. 18.4.4

Applying Ampere's circuital law to the contour  $C$  and the surface  $S_1$ , we find

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

## MODULE - 5

### Electricity and Magnetism



Notes

## Magnetism and Magnetic Effect of Electric Current

However, applying Ampere's circuital law to the contour  $C$  and the surface  $S_2$ , as there is no current through this surface, we get

$$\oint \mathbf{B} \cdot d\ell = 0$$

The above two equations are mutually contradictory. To avoid this contradiction, Maxwell assumed that a current exists between the capacitor plates. He called this current displacement current and showed that this current arises due to the variation of electric field with time.

A simple expression for the displacement current can be derived as follows. Consider a parallel plate capacitor. Let  $q$  be the charge on the capacitor plates at any instant  $t$ .

The electric field inside the capacitor is given by

$$E = \frac{q}{A\epsilon_0}$$

When  $A$  is the surface area of the plates. Therefore, the electric flux through the capacitor is

$$\phi_E = EA = \frac{q}{\epsilon_0}$$

The rate of change of the instantaneous flux can be written as

$$\frac{\Delta\phi_E}{\Delta t} = \frac{1}{\epsilon_0} \frac{\Delta q}{\Delta t} = \frac{I}{\epsilon_0}$$

So, we can write

$$\epsilon_0 \frac{\Delta\phi_E}{\Delta t} = I$$

The expression on the left hand side is equivalent to a current, which though equal to the conduction current  $I$  is actually different from it as it is not associated with the motion of free charges. It is called displacement current. Unlike the conduction current  $I$ , the displacement current arises whenever the electric field and hence the electric flux changes with time.

Adding displacement current to the conduction currents  $I$ , Maxwell modified the Ampere's circuital law in the form,

$$\sum \mathbf{B} \cdot d\ell = \mu_0 \left( I + \epsilon_0 \frac{\Delta\phi_E}{\Delta t} \right)$$

Maxwell's modification of Ampere's law tells us that, in addition to conduction current, a time-varying electric field can also produce magnetic field.

**Example 18.1 :** A 50 cm long solenoid has 3 layers of windings of 250 turns each. The radius of the lowest layer is 2cm. If the current through it is 4.0 A,

calculate the magnitude of  $\mathbf{B}$  (a) near the centre of the solenoid on and about the axis; (b) near the ends on its axis; and (c) outside the solenoid near the middle.

**Solution :**

a) At the centre or near it

$$\begin{aligned} B &= \mu_0 nI \\ &= 4\pi \times 10^{-7} \times \frac{3 \times 250}{0.5} \times 4 \\ &= 16\pi \times 1500 \times 10^{-7} \text{ T} \\ &= 24\pi \times 10^{-4} \text{ T} \end{aligned}$$

b) At the ends

$$B_{\text{ends}} = \frac{1}{2} B_{\text{centre}} = 12\pi \times 10^{-4} \text{ T}$$

c) Outside the solenoid the field is zero.

**Example 18.2:** Calculate the distance from a long straight wire carrying a current of 12A at which the magnetic field will be equal to  $3 \times 10^{-5}$  T.

**Solution :**

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow r = \frac{\mu_0 I}{2\pi B}$$

$$\therefore r = \frac{2 \times 10^{-7} \times 12}{3 \times 10^{-5}} = 0.25 \text{ m}$$



### INTEXT QUESTIONS 18.3

- A drawing of the lines of force of a magnetic field provides information on
  - direction of field only
  - magnitude of field only
  - both the direction and magnitude of the field
  - the force of the field
- What is common between Biot-Savart's law and Ampere's circuital law ?
- In the following drawing of lines of force of a non-uniform magnetic field, at which point is the field (i) uniform, (ii) weakest, (iii) strongest?

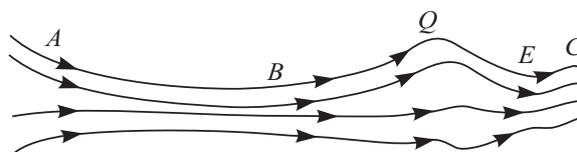


Fig. 18.16 : A typical magnetic field



Notes

## MODULE - 5

### Electricity and Magnetism



Notes

## Magnetism and Magnetic Effect of Electric Current

4. A 10 cm long solenoid is meant to have a magnetic field 0.002T inside it, when a current of 3A flows through it. Calculate the required no. of turns.
5. Derive an expression for the field due to a toroid using Ampere's circuital law.

### 18.5 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

When a charged body moves in a magnetic field, it experiences a force. Such a force experienced by a moving charge is called the *Lorentz force*. The Lorentz force on a particle with a charge  $+q$  moving with a velocity  $\mathbf{v}$  in a magnetic field  $\mathbf{B}$  is given by

$$\mathbf{F} = q (\mathbf{v} \times \mathbf{B})$$

$$\text{or } |\mathbf{F}| = q v B \sin \theta \quad (18.10)$$

where  $\theta$  is the angle between the directions of  $\mathbf{v}$  and  $\mathbf{B}$ . The direction of  $\mathbf{F}$  is given by Fleming's left hand rule.

Fleming's left hand rule states that if we stretch the fore finger, the central finger and the thumb of our left hand at right angles to each other and hold them in such a way that the fore finger points in the direction of magnetic field and the central finger points in the direction of motion of positively charged particle, then the thumb will point in the direction of the Lorentz force (Fig. 18.17).

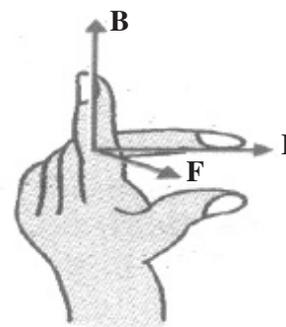


Fig. 18.17 : Fleming's left hand rule

#### Some important points to note

- $\mathbf{F}$  is a mechanical force resulting in a pull or a push.
- The direction of force is given by Fleming's left hand rule.
- In case of negative charges, the central finger should point opposite to the direction of its motion.
- If the charge stops, the force becomes zero instantly.
- Force is zero when charges move along the field  $\mathbf{B}$ .
- Force is maximum when charges move perpendicular to the field :  $F = qvB$

#### 18.5.1 Force on a Current Carrying Conductor in a Uniform Magnetic Field

The concept of Lorentz force can be easily extended to current carrying conductors placed in uniform magnetic field  $\mathbf{B}$ . Suppose that the magnetic field is parallel to the plane of paper and a conductor of length  $\Delta \ell$  carrying current  $I$  is placed normal to the field. Suppose further that the current is flowing downward with a drift velocity  $\mathbf{v}_d$  and hence each free electron constituting the current experiences a Lorentz force  $\mathbf{F} = e \mathbf{v}_d \cdot \mathbf{B}$



Notes

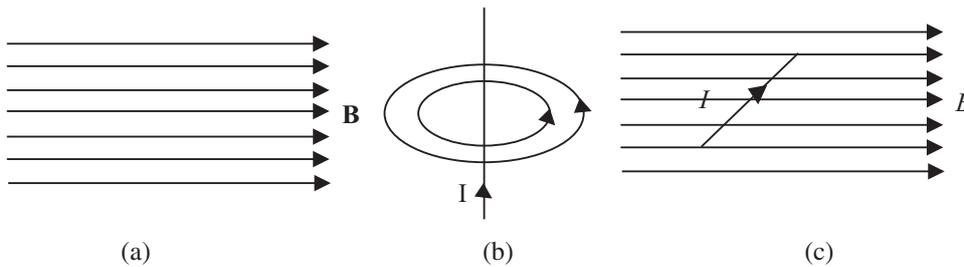
If there are  $N$  free electrons in the conductor, the net force on it is given by

$$F = N e v_d B = nA \Delta\ell e v_d B \quad (18.11)$$

where  $n$  denotes the number of free electrons per unit volume. But  $neAv_d = I$ . Hence

$$\therefore F = I \Delta\ell B \quad (18.12)$$

If conductor makes an angle  $\theta$  with  $\mathbf{B}$ , then  $|\mathbf{F}| = I \Delta\ell B \sin\theta$ .



**Fig. 18.18:** a) Uniform magnetic field, b) field due to current carrying inductor, and c) force on a current carrying conductor

The direction of the force is again given by Fleming's left hand rule.

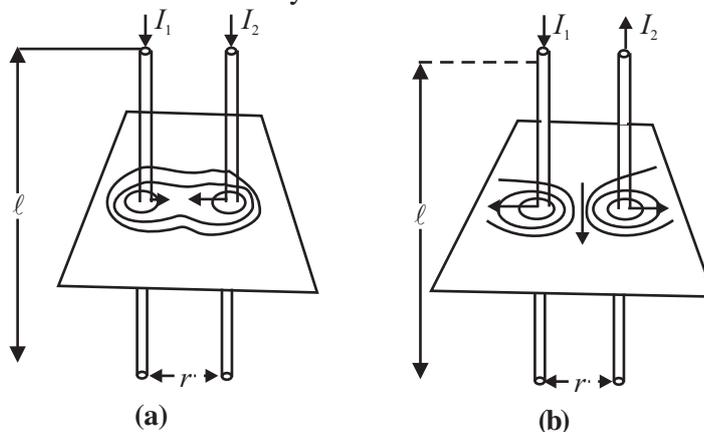
Eqn. (18.12) can be used to define the unit of magnetic field in terms of the force experienced by a current carrying conductor. By rearranging terms, we can write

$$B = \frac{F}{I\Delta\ell}$$

Since  $F$  is taken in newton,  $I$  in ampere and  $\Delta\ell$  in metre, the unit of  $B$  will be  $\text{NA}^{-1} \text{m}^{-1}$ . It is called tesla (T).

### 18.5.2 Force Between two Parallel Wires Carrying Current

You now know that every current carrying conductor is surrounded by a magnetic field. It means that it will exert force on a nearby current carrying conductor. The force between two current carrying conductors placed parallel to each other is mutual and magnetic in origin. A current carrying wire has no net electric charge, and hence cannot interact electrically with another such wire.



**Fig. 18.19:** Experimental demonstration of force between two parallel wires carrying current



Notes

Fig 18.19 shows two parallel wires separated by distance  $r$  and carrying currents  $I_1$  and  $I_2$ , respectively. The magnetic field due to one wire at a distance  $r$  from it is  $B_1 = \frac{\mu_0 I_1}{2\pi r}$ .

Similarly, the field due to second wire at a distance  $r$  from it will be  $B_2 = \frac{\mu_0 I_2}{2\pi r}$ .

These fields are perpendicular to the length of the wires and therefore the force on a length  $l$ , of the other current carrying conductor is given by

$$F = B I \ell = \frac{\mu_0 I_1}{2\pi r} I_2 \ell$$

or force per unit length

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r} \tag{18.13}$$

The forces are attractive when the currents are in the same direction and repulsive when they are in opposite directions.

Eqn (18.13) can be used to define the unit of current. If  $I_1 = I_2 = 1\text{A}$ ,  $l = 1\text{m}$  and  $r = 1\text{m}$ , then

$$F = \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \text{ N}$$

Thus, *if two parallel wires carrying equal currents and placed 1 m apart in vacuum or air experience a mutual force of  $2 \times 10^{-7} \text{ N m}^{-1}$ , the current in each wire is said to be one ampere.*

18.5.3 Motion of a Charged Particle in a Uniform Field

We can now think of various situations in which a moving charged particle or a current carrying conductor in a magnetic field experiences Lorentz force. The work done by a force on a body depends on its component in the direction of motion of the body. When the force on a charged particle in a magnetic field is perpendicular to its direction of motion, no work is said to be done. Hence the particle keeps the same speed and kinetic energy which it had while moving in the field, even though it is deflected. On the other hand, the speed and energy of a charged particle in an electrical field is always affected due to the force by the field on the particle. A charged particle moving perpendicular to a magnetic field follows a circular path (Fig. 18.20) because it experience a force at right angles to the direction of motion at every position.

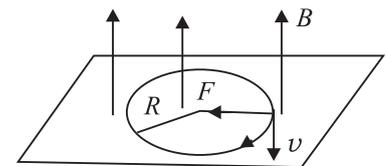


Fig. 18.20: Path of a charged particle in a uniform magnetic field

To know the radius  $R$  of the circular path of the charged particle, we note that the magnetic force  $q v B$  provides the particle with the centripetal force ( $m v^2/R$ ) that keeps it moving in a circle. So we can write

$$q v B = \frac{m v^2}{R}$$

On rearrangement, we get

$$R = \frac{m v}{q B} \quad (18.14)$$

The radius of the path traced by a charged particle in a uniform magnetic field is directly proportional to its momentum ( $mv$ ) and inversely proportional to its charge and the magnetic field. It means that greater the momentum, larger the circle, and stronger the field, the smaller the circle. The time period of rotation of the particle in a circular path is given by

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{Bq} \quad (18.14 \text{ a})$$

Note that the time period is independent of velocity of the particle and radius of the orbit. It which means that once the particle is in the magnetic field, it would go round and round in a circle of the same radius. If  $m$ ,  $B$ ,  $q$ , remain constant, the time period does not change even if  $v$  and  $R$  are changed.

Now think, what happens to  $R$  and  $T$  if a) field  $B$  is made stronger; b) field  $B$  is made weaker; c) field  $B$  ceases to exist; d) direction of  $B$  is changed; e) the particle is made to enter the magnetic field at a higher speed; f) the particle enters at an angle to  $B$ ; and g) the charged particle loses its charge.

### 18.5.4 Motion of a Charged Particle in uniform Electric Field and Magnetic Field

#### (a) Motion in Electric Field

When a charged particle  $q$  is placed in a uniform electric field  $E$ , it experiences a force,

$$\mathbf{F} = q\mathbf{E}$$

Thus, the charged particle will be accelerated under the influence of this force. The acceleration is given by

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m}$$

The acceleration will be in the direction of the force. If it is a positive charge, it will accelerate in the direction of the field and if it is a negative charge it will



Notes

## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

accelerate in a direction opposite that of the field. The velocity and displacement of charged particle can also be calculated by using the equations of motion:

$$v = u + \left(\frac{qE}{m}\right)t$$

$$s = ut + \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

where  $t$  denotes time.

### (b) Motion in magnetic field

In article 18.5 (Page 114, Book 2), it has been discussed that the force experienced by a charged particle in a magnetic field is given by

$$F = qBv\sin\theta$$

Where  $\theta$  is angle between the velocity and magnetic field.

If  $\theta = 0$ ,  $F = 0$  and charged particle will move along a straight line with constant speed.

If  $\theta = 90^\circ$ ,  $F$  will be maximum and its direction, according to Fleming's left hand rule, will be perpendicular to the plane of  $\mathbf{v}$  and  $\mathbf{B}$  and the charged particle will move along a circular path with a constant speed and frequency.

If  $\theta \neq 0^\circ \neq 90^\circ$ , then the velocity of the charged particle will be  $v\sin\theta$  perpendicular to the field and  $v\cos\theta$  parallel to the field. The particle, therefore, moves along a helical path.

What we note from the above discussion is that a magnetic field does not change the speed of a moving charge, it only changes its direction of motion.

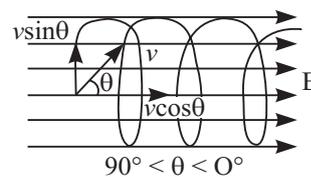
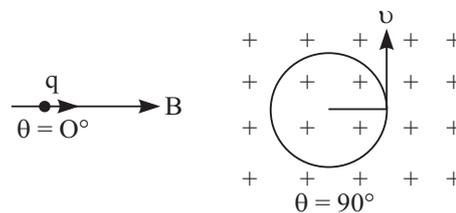
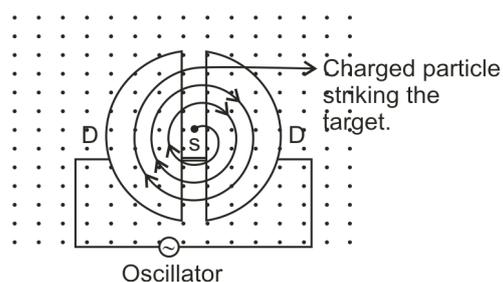


Fig. 18.5.4

**18.5.5 Cyclotron**

The cyclotron is a device invented by E.O. Lawrence in 1929, that is used for accelerating charged particles (such as protons, deuteron or  $\alpha$ -particles) to high velocities. It consists of two semi-circular hollow metallic disks  $DD$ , called dees, on account of their shape resembling the letter  $D$  of English alphabet. They are insulated from one another with a small gap between them. The dees are placed in an evacuated chamber.

**18.5.5**

A magnetic field perpendicular to the plane of the dees (out of the paper in Fig) is maintained with the help of an electromagnet having flat pole-pieces. A rapidly oscillating potential difference is applied between the dees with the help of an oscillator. This produces an oscillating electric field in the gap between the dees.

Consider a charged particle of mass  $m$  and charge  $q$  in the gap between the dees. The particle is accelerated by the electric field towards one of them. Inside the dees, it moves with constant speed in a semicircle in a clockwise direction. If the frequency of the oscillator is equal to the frequency of revolution of the charged particle, then it reaches the gap at the instant when the opposite dee becomes negative because of the reversal of the direction of electric field.

The frequency of revolution of the charged particle is given by (see Eq. 18. 14a):

$$\nu = \frac{1}{T} = \frac{v}{2\pi R} = \frac{Bq}{2\pi m}$$

where  $B$  is the magnetic field.

It is also called cyclotron frequency and denoted by  $\nu_c$ . When  $\nu_c = \nu_o$ , the frequency of the oscillator, the particle reaches the gap when the electric potential at the opposite 'D' has just reversed its sign. This condition is also known as cyclotron resonance condition. On account of this, the particle gains energy and, therefore, it moves in a circle of larger radius. This energy gain can be repeated many times.

Thus, the energy and the radius of the path of the particle keep on increasing progressively. However, the maximum radius which the path can have is limited

**Notes**



Notes

by the radius  $R$  of the dees. The high energy charged particle finally comes out through an opening in the dee.

**Example 18.3 :** Refer to Fig. 18.21 and calculate the force between wires carrying current 10A and 15A, if their length is 5m. What is the nature of this force ?

**Solution :** When currents flow in two long parallel wires in the same direction, the wires attract each other and the force of attraction is given by

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r} = \frac{2 \times 10^{-7} \times 10 \times 15}{3} = 10^{-4} \text{ N m}^{-1}$$

$$\therefore F = 5 \times 10^{-4} \text{ N}$$

**Fig. 18.21** The force is attractive in nature.

**Example 18.4 :** An electron with velocity  $3 \times 10^7 \text{ ms}^{-1}$  describes a circular path in a uniform magnetic field of 0.2T, perpendicular to it. Calculate the radius of the path.

**Solution :**

We know that 
$$R = \frac{mv}{Bq}$$

Here,  $m_e = 9 \times 10^{-31} \text{ kg}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $v = 3 \times 10^7 \text{ m s}^{-1}$  and  $B = 0.2 \text{ T}$ . Hence

$$R = \frac{9 \times 10^{-31} \times 3 \times 10^7}{0.2 \times 1.6 \times 10^{-19}}$$

$$= 0.85 \times 10^{-3} \text{ m}$$

$$= 8.5 \times 10^{-4} \text{ m}$$



**INTEXT QUESTION 18.4**

1. A stream of protons is moving parallel to a stream of electrons but in the opposite direction. What is the nature of force between them ?
2. Both electrical and magnetic fields can deflect an electron. What is the difference between them?
3. A body is suspended from a vertical spring. What shall be the effect on the position of the body when a current is made to pass through the spring.
4. How does a cyclotron accelerate charged particles?

**18.6 CURRENT LOOP AS A DIPOLE**

From Eqn. (18,6) you will recall that the field at the centre of a coil is given by

$$B = \frac{\mu_0 I}{2r}$$

On multiplying the numerator and denominator by  $2\pi r^2$ , we can rewrite it as

$$B = \frac{\mu_0 2I \cdot \pi r^2}{4\pi r^3} = \frac{\mu_0 2IA}{4\pi r^3} = \frac{\mu_0 2M}{4\pi r^3}$$

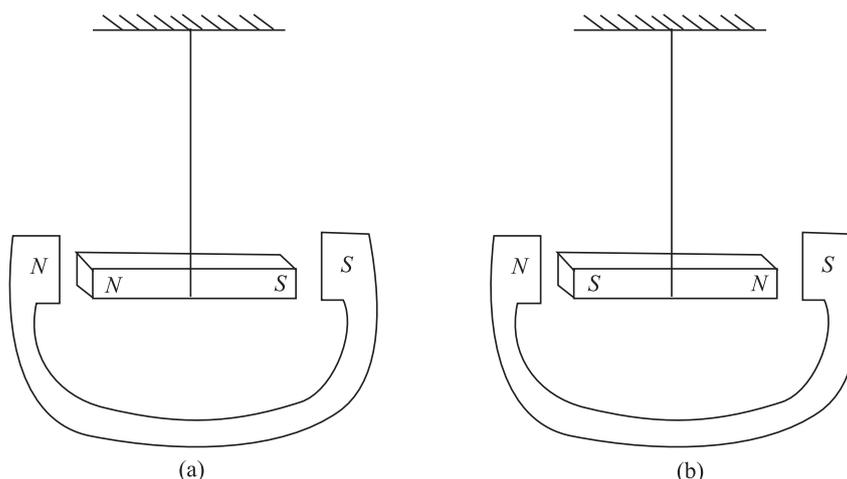
where  $A$  is area of coil and  $M$  is magnetic moment. This shows that a current carrying coil behaves like a magnetic dipole having north and south poles. One face of the loop behaves as north pole while the other behaves as south pole.

Let us now undertake a simple activity.



**ACTIVITY 18.3**

Suspend a bar magnet by a thread between pole pieces of a horse shoe magnet, as shown in Fig 18.22.



**Fig. 18.22 :** A bar magnet suspended between a horse shoe magnet

What will happen when the bar magnet shown in Fig. 18.24(a) is displaced slightly sideways? Since like poles repel, the bar magnet experiences a torque and tends to turn through  $180^\circ$  and get aligned, as shown in Fig. 18.22 (b). Since a current loop behaves as a magnet, it will align in an external field in the same way.

You have already studied the following equations in the lesson on electrostatics. The electric field of a dipole at a far point on its axis is given by

$$E = \frac{1}{4\pi\mu_0} \frac{2P}{x^3} \tag{18.15 b}$$

The magnetic field due to a current carrying coil is given by

$$B = \frac{\mu_0}{4\pi} \frac{2NIA}{x^3} = \frac{\mu_0}{4\pi} \frac{2M}{x^3} \tag{18.15 c}$$

where  $M$  is the magnetic dipole moment.



Notes

## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

A comparison between these expressions leads us to the following analogies :

- A current loop behaves as a magnetic dipole with magnetic moment

$$\mathbf{M} = NIA \quad (18.15 \text{ d})$$

- Like the poles of a magnetic dipole, the two faces of a current loop are inseparable.
- A magnetic dipole in a uniform magnetic field behaves the same way as an electric dipole in a uniform electric field.
- A magnetic dipole also has a magnetic field around it similar to the electric field around an electric dipole.

Thus magnetic field due to a magnetic dipole at an axial point is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{x^3} \quad (18.16)$$

whereas the field at an equatorial point is given by

$$\mathbf{B} = -\frac{\mu_0}{4\pi} \frac{\mathbf{M}}{x^3} \quad (18.17)$$

### Magnetism in Matter

Based on the behaviour of materials in magnetic field, we can divide them broadly into three categories : (i) **Diamagnetic** materials are feebly repelled by a magnet. (ii) **Paramagnetic** materials are feebly attracted by a magnet. (iii) **Ferromagnetic** materials are very strongly attracted by a magnet. Substances like iron, nickel and cobalt are ferromagnetic. Let us study ferromagnetic behaviour of materials in some details.

Ferromagnetic materials, when placed even in a weak magnetic field, become magnets, because their atoms act as permanent magnetic dipoles. The atomic dipoles tend to align parallel to each other in an external field. These dipoles are not independent of each other. Any dipole strongly feels the presence of a neighboring dipole. A correct explanation of this interaction can be given only on the basis of quantum mechanics. However, we can qualitatively understand the ferromagnetic character along the following lines.

A ferromagnetic substance contains small regions called **domains**. All magnetic dipoles in a domain are fully aligned. The magnetization of domains is maximum. But the domains are randomly oriented. As a result, the total magnetic moment of the sample is zero. When we apply an external magnetic field, the domains slightly rotate and align themselves in the direction of the field giving rise to resultant magnetic moment. The process can be easily understood with the help of a simple diagram shown in Fig.18.23.

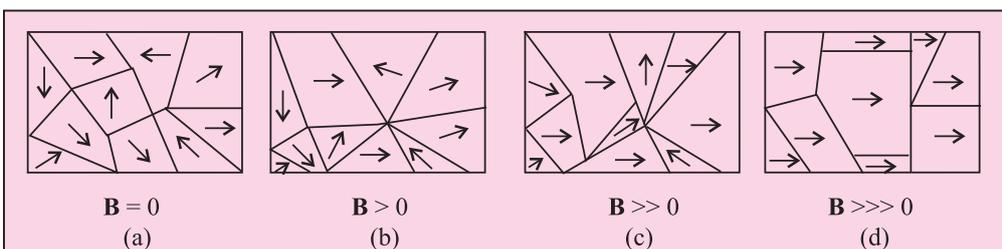


Fig. 18.23 : Domains in a ferromagnetic substance

Fig. 18.23 (a) shows ten domains. For simplicity we take a two dimensional example. All the domains are so directed that the total magnetization of the sample is zero. Fig. 18.23 (b) shows the state after the application of an external magnetic field. The boundaries of the domains (Domain Walls) reorganise in such a way that the size of the domain having magnetic moment in the direction of the field becomes larger at the cost of others. On increasing the strength of external field, the size of favorable domains increases, and the orientation of the domain changes slightly resulting in greater magnetization (Fig. 18.23 (c)). Under the action of very strong applied field, almost the entire volume behaves like a single domain giving rise to saturated magnetization. When the external field is removed, the sample retains net magnetization. The domain in ferromagnetic samples can be easily seen with the help of high power microscope.

When the temperature of a ferromagnetic substance is raised beyond a certain critical value, the substance becomes paramagnetic. This critical temperature is known as Curie temperature  $T_c$ .

## Notes

Table 18.1: Ferromagnetic substances and their curie temperatures

Substances	Curie temperature $T_c$ (K)
Iron	1043
Nickel	631
Cobalt	1394
Gadolinium	317
$Fe_2O_3$	893

**Example 18.5 :** The smallest value of magnetic moment is called the Bohr

Magneton  $\mu_B = \frac{eh}{4\pi m}$ . It is a fundamental constant. Calculate its value.

**Solution :**

$$\mu_B = \frac{eh}{4\pi m} = \frac{(1.6 \times 10^{-19} \text{ C}) \times (6.6 \times 10^{-34} \text{ Js})}{4 \times 3.14 \times (9 \times 10^{-31} \text{ kg})}$$

$$= 9.34 \times 10^{-24} \text{ J T}^{-1}$$

### 18.6.1 Torque on a Current Loop

A loop of current carrying wire placed in a uniform magnetic field ( $\mathbf{B}$ ) experiences no net force but a torque acts on it. This torque tends to rotate the loop to bring its plane perpendicular to the field direction. This is the principle that underlines the operation of all electric motors, meters etc.

Let us examine the force on each side of a rectangular current carrying loop where plane is parallel to a uniform magnetic field  $\mathbf{B}$ . (Fig. 18.24 (a).)

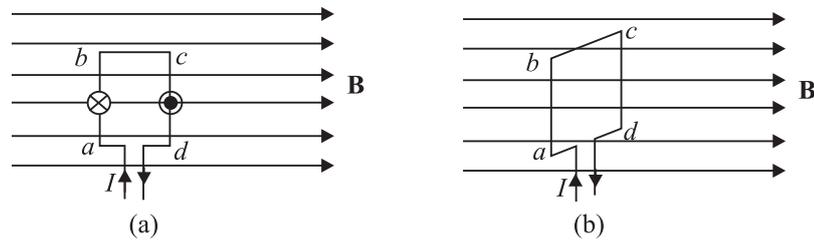
## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current



**Fig. 18.24:** Force on the sides of a rectangular loop when (a) the loop is parallel to the field, and (b) the coil is perpendicular to the field.

The sides  $ad$  and  $bc$  of the loop are parallel to  $\mathbf{B}$ . So no force will act on them. Sides  $ab$  and  $cd$  are however, perpendicular to  $\mathbf{B}$ , and these experience maximum force. We can easily find the direction of the force on  $ab$  and  $cd$ .

In fact,  $|\mathbf{F}_{ab}| = |\mathbf{F}_{cd}|$  and these act in opposite directions. Therefore, there is no net force on the loop. Since  $\mathbf{F}_{ab}$  and  $\mathbf{F}_{cd}$  do not act along the same line, they exert a torque on the loop that tends to turn it. This holds good for a current loop of any shape in a magnetic field.

In case the plane of the loop were perpendicular to the magnetic field, there would neither be a net force nor a net torque on it (see Fig 18.26 (b)).

$$\begin{aligned} \text{Torque} &= \text{force} \times \text{perpendicular distance between the force} \\ &= BIL \cdot b \sin \theta \end{aligned}$$

Refer to Fig. 18.25 which shows a loop  $PQRS$  carrying current  $I$ .  $\theta$  is the angle between the magnetic field  $\mathbf{B}$  and the normal to the plane of the coil  $n$ . The torque is then

$$\tau = NBIL b \sin \theta$$

where  $N$  is the number of turns of the coil. We can rewrite it as

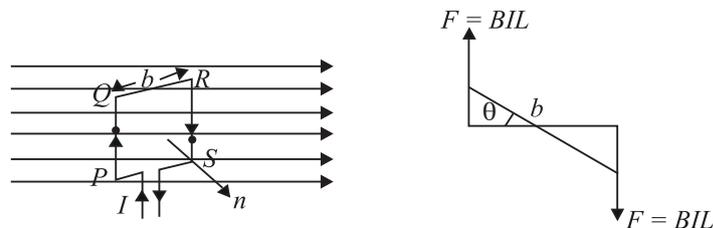
$$|\tau| = NBI A \sin \theta \quad (18.18)$$

where  $A$  is area of the coil  $= L \times b$

$$|\tau| = |\mathbf{B}| |\mathbf{M}| \sin \theta \quad (18.19)$$

where  $\mathbf{M} = NIA$  is known as the magnetic moment of the current carrying coil.

Thus, we see that the torque depends on  $B$ ,  $A$ ,  $I$ ,  $N$  and  $\theta$



**Fig. 18.25 :** Torque on the current carrying loop

If a uniform rotation of the loop is desired in a magnetic field, we need to have a constant torque. The couple would be approximately constant if the plane of the

coil were always along or parallel to the magnetic field. This is achieved by making the pole pieces of the magnet curved and placing a soft iron core at the centre so as to give a radial field.

The soft iron core placed inside the loop would also make the magnetic field stronger and uniform resulting in greater torque (Fig. 18.26).

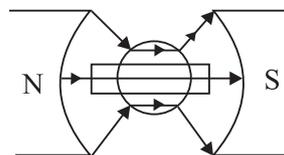


Fig. 18.26: Constant torque on a coil in a radial field



Notes

### 18.6.1 (a) Magnetic Dipole

The term magnetic dipole includes

- (i) a current-carrying circular coil of wire, and
- (ii) a small bar magnet

The magnetic field due to a magnetic dipole at a point

- (i) situated at a distance  $r$  on the axis of the dipole is given by :

$$|\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{r^3}$$

- (ii) situated at a distance  $r$  on the equatorial line is given by :

$$|\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{\mathbf{M}}{r^3}$$

This implies that the field has a cylindrical symmetry about the dipole axis.

### 18.6.1(b) The Torque on a Magnetic Dipole Placed in a uniform magnetic field

We have seen in section 18.6 that a current loop behaves as a magnetic dipole.

In 18.6.1 we have also seen that a current loop placed in a uniform magnetic field experiences a torque

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{M} \times \mathbf{B} \\ \Rightarrow |\boldsymbol{\tau}| &= |\mathbf{M}| |\mathbf{B}| \sin \theta \end{aligned}$$

The direction of  $\boldsymbol{\tau}$  is normal to the plane containing  $\mathbf{M}$  and  $\mathbf{B}$  and is determined by the right hand cork screw rule.

Note that in all these expressions  $\mathbf{M} = N\mathbf{IA}$ .

where the direction of  $\mathbf{A}$  is determined by the right hand rule.

### 18.6.2 Galvanometer

From what you have learnt so far, you can think of an instrument to detect current in any circuit. A device doing precisely this is called a galvanometer, which works on the principle that a current carrying coil, when placed in a magnetic field, experiences a torque.

## MODULE - 5

### Electricity and Magnetism

## Magnetism and Magnetic Effect of Electric Current



Notes

A galvanometer consists of a coil wound on a non-magnetic frame. A soft iron cylinder is placed inside the coil. The assembly is supported on two pivots attached to springs with a pointer. This is placed between the pole pieces of a horse shoe magnet providing radial field (see Fig. 18.27).

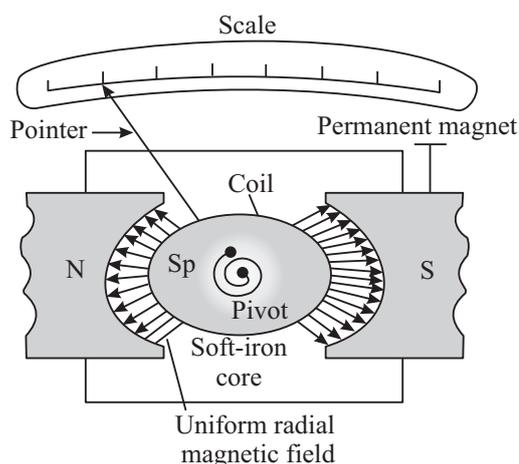


Fig. 18.27 : A moving coil galvanometer

To understand the working of a moving coil galvanometer, we recall that when a current is passed through the coil, it will rotate due to the torque acting on it. The spring sets up a restoring force and hence, a restoring torque. If  $\alpha$  is the angle of twist and  $k$  is the restoring torque per unit twist or torsional constant, we can write  $NBIA \sin\theta = k \alpha$ . For  $\theta = 90^\circ$ ,  $\sin\theta = 1$ . So, in the instant case, we can write

$$\therefore NBIA = k\alpha$$

$$\text{or } \frac{INBA}{k} = \alpha$$

$$\text{That is, } I = \frac{k\alpha}{NBA} \quad (18.20)$$

where  $\frac{k}{NBA}$  is called galvanometer constant. From this we conclude that

$$\alpha \propto I$$

That is, deflection produced in a galvanometer is proportional to the current flowing through it provided  $N$ ,  $B$ ,  $A$  and  $k$  are constant. The ratio  $\alpha/I$  is known as current sensitivity of the galvanometer. It is defined as the deflection of the coil per unit current. The more the current stronger the torque and the coil turns more. Galvanometer can be constructed to respond to very small currents (of the order of  $0.1\mu\text{A}$ ).



Notes

**Sensitivity of a galvanometer :** In order to have a more sensitive galvanometer,

- $N$  should be large;
- $B$  should be large, uniform and radial;
- area of the coil should be large; and
- torsional constant should be small.

The values of  $N$  and  $A$  cannot be increased beyond a certain limit. Large values of  $N$  and  $A$  will increase the electrical and inertial resistance and the size of the galvanometer.  $B$  can be increased using a strong horse shoe magnet and by mounting the coil on a soft iron core. The value of  $k$  can be decreased by the use of materials such as quartz or phosphor bronze.

### 18.6.3 An Ammeter and a Voltmeter

**(a) Ammeter :** An Ammeter is a suitably shunted galvanometer. Its scale is calibrated to give the value of current in the circuit. To convert a galvanometer into an ammeter, a low resistance wire is connected in parallel with the galvanometer. The resistance of the shunt depends on the range of the ammeter and can be calculated as follows :

Let  $G$  be resistance of the galvanometer and  $N$  be the number of scale divisions in the galvanometer. Let  $k$  denote figure of merit or current for one scale deflection in the galvanometer. Then current which produces full scale deflection in the galvanometer is  $I_g = Nk$

Let  $I$  be the maximum current to be measured by the galvanometer.

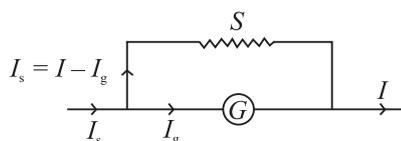
Refer to Fig. 18.28. The voltage between points  $A$  and  $B$  is given by

$$V_{AB} = I_g G = (I - I_g) S$$

so that

$$S = \frac{I_g G}{I - I_g} \quad (18.21)$$

where  $S$  is the shunt resistance.



**Fig. 18.28 :** A shunted galvanometer acts as an ammeter

## MODULE - 5

### Electricity and Magnetism

## Magnetism and Magnetic Effect of Electric Current



### Notes

As  $G$  and  $S$  are in parallel, the effective resistance  $R$  of the ammeter is given by

$$R = \frac{GS}{G+S}$$

As the shunt resistance is small, the combined resistance of the galvanometer and the shunt is very low and hence, ammeter resistance is lower than that of the galvanometer. An ideal ammeter has almost negligible resistance. That is why when it is connected in series in a circuit, all the current passes through it without any observable drop.

**(b) Voltmeter :** A voltmeter is used to measure the potential difference between two points in a circuit. We can convert a galvanometer into a voltmeter by connecting a high resistance in series with the galvanometer coil, as shown in Fig 18.29. The value of the resistance depends on the range of voltmeter and can be calculated as follows :

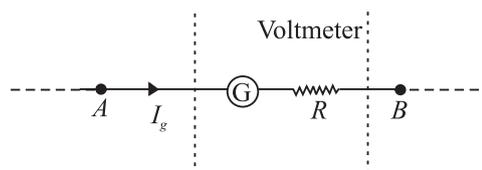


Fig. 18.29 : Galvanometer as a voltmeter

A high resistance, say  $R$  is connected in series with the galvanometer coil. If the potential difference across  $AB$  is  $V$  volt, then total resistance of the voltmeter will be  $G + R$ . From Ohm's law, we can write

$$I_g (G + R) = V$$

or 
$$G + R = \frac{V}{I_g}$$

$$\Rightarrow R = \frac{V}{I_g} - G \quad (18.22)$$

This means that if a resistance  $R$  is connected in series with the coil of the galvanometer, it works as a voltmeter of range  $0-V$  volts.

Now the same scale of the galvanometer which was recording the maximum potential  $I_g \times G$  before conversion will record the potential  $V$  after conversion into voltmeter. The scale can be calibrated accordingly. The resistance of the voltmeter is higher than the resistance of galvanometer. Effective resistance of the voltmeter, is given by

$$R_v = R + G$$

The resistance of an ideal voltmeter is infinite. It is connected in parallel to the points across which potential drop is to be measured in a circuit. It will not draw

any current. But the galvanometer coil deflects. Seems impossible! Think about it.

**Example 18.6 :** A circular coil of 30 turns and radius 8.0 cm, carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of  $90^\circ$  with the normal to the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

**Solution :** Here,  $N = 30$ ,  $I = 6.0$  A,  $B = 1.0$  T,  $\theta = 90^\circ$ ,  $r = 8.0$  cm =  $8 \times 10^{-2}$  m.

$$\text{Area (A) of the coil} = \pi r^2 = \frac{22}{7} \times (8 \times 10^{-2})^2 = 2.01 \times 10^{-2} \text{ m}^2$$

$$\begin{aligned} \therefore \quad \text{Torque} &= N I B A \sin\theta \\ &= 30 \times 6 \times 1.0 \times (2.01 \times 10^{-2}) \times \sin 90^\circ \\ &= 30 \times 6 \times (2.01 \times 10^{-2}) \\ &= 3.61 \text{ Nm} \end{aligned}$$

**Example 18.7 :** A galvanometer with a coil of resistance  $12.0 \Omega$  shows a full scale deflection for a current of 2.5 mA. How will you convert it into (a) an ammeter of range 0 – 2A, and (b) voltmeter of range 0 – 10 volt ?

**Solution :** (a) Here,  $G = 12.0 \Omega$ ,  $I_g = 2.5$  mA =  $2.5 \times 10^{-3}$  A, and  $I = 2$  A. From Eqn. (18.21), we have

$$\begin{aligned} S &= \frac{I_g G}{I - I_g} \\ &= \frac{2.5 \times 10^{-3} \times 12}{2 - 2.5 \times 10^{-3}} \\ &= 15 \times 10^{-3} \Omega \end{aligned}$$

So, for converting the galvanometer into an ammeter for reading 0 – 2V, a shunt of  $15 \times 10^{-3} \Omega$  resistance should be connected parallel to the coil.

(b) For conversion into voltmeter, let  $R$  be the resistance to be connected in series.

$$\begin{aligned} R &= \frac{V}{I_g} - G \\ &= \frac{10}{2.5 \times 10^{-3}} - 12 = 4000 - 12 \\ &= 3988 \Omega \end{aligned}$$



Notes

## MODULE - 5

### Electricity and Magnetism

## Magnetism and Magnetic Effect of Electric Current

Thus, a resistance of  $3988 \Omega$  should be connected in series to convert the galvanometer into voltmeter.



### INTEXT QUESTIONS 18.5

1. What is radial magnetic field ?
2. What is the main function of a soft iron core in a moving coil galvanometer ?
3. Which one has the lowest resistance - ammeter, voltmeter or galvanometer ? Explain.
4. A galvanometer having a coil of resistance  $20 \Omega$  needs  $20 \text{ mA}$  current for full scale deflection. In order to pass a maximum current of  $3 \text{ A}$  through the galvanometer, what resistance should be added and how ?



### WHAT YOU HAVE LEARNT

- Every magnet has two poles. These are inseparable.
- The term magnetic dipole may imply (i) a magnet with dipole moment  $\mathbf{M} = m\ell$  (ii) a current carrying coil with dipole moment  $\mathbf{M} = NIA$
- Magnetic field at the axis of a magnetic dipole is given by  $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{x^3}$  and on the equatorial line by  $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{M}}{x^3}$ .
- A magnetic dipole behaves the same way in a uniform magnetic field as an electric dipole does in a uniform electric field, i.e., it experience no net force but a torque  
$$\boldsymbol{\tau} = \mathbf{M} \times \mathbf{B}.$$
- Earth has a magnetic field which can be completely described in terms of three basic quantities called elements of earth's magnetic field :
  - angle of inclination,
  - angle of declination, and
  - horizontal component of earth's field.
- Every current carrying conductor develops a magnetic field around it. The magnetic field is given by Biot-Savart's Law :

$$|dB| = \frac{\mu_0 I}{4\pi} \frac{dl \sin \theta}{r^2}$$

Notes



- Unit of magnetic field is tesla.
  - Field at the centre of a flat coil carrying current is given by  $|\mathbf{B}| = \frac{\mu_0 I}{2r}$ .
- Ampere's circuital law gives the magnitude of the magnetic field around a conductor  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$
- The Lorentz force on a moving charge  $q$  is  $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$  and its direction is given by Fleming's left hand rule.
  - The mechanical force on a wire of length  $L$  and carrying a current of  $I$  in a magnetic field  $\mathbf{B}$  is  $\mathbf{F} = \mathbf{B} I L$ .
  - Mutual force per unit length between parallel straight conductors carrying currents  $I_1$  and  $I_2$  is given by  $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$ .
  - Magnetic field due to a toroid,  $B = \frac{\mu_0 Ni}{2\pi r}$
  - A charged particle traces a circular path of radius  $R = \frac{mv}{Bq}$ .
  - Cyclotron is a device used to accelerate charged particles to high velocities.
  - Cyclotron frequency  $\nu_c = \frac{Bq}{2m\pi}$
  - A current loop behaves like a magnetic dipole.
  - A current carrying coil placed in a magnetic field experiences a torque given by
 
$$\tau = N B I A \sin\theta$$

$$= N B I A, \text{ (if } \theta = 90^\circ)$$
  - Galvanometer is used to detect electric current in a circuit.
  - An ammeter is a shunted galvanometer and voltmeter is a galvanometer with a high resistance in series. Current is measured by an ammeter and potential difference by a voltmeter.



**TERMINAL EXERCISES**

1. A small piece of the material is brought near a magnet. Complete the following by filling up the blanks by writing Yes or No.

## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

Material	Repulsion		Attraction	
	weak	strong	weak	strong
Diamagnetic				
Paramagnetic				
Ferromagnetic				

- You have to keep two identical bar magnets packed together in a box. How will you pack and why?  
N S      OR      N      S  
N S                  S      N
- The magnetic force between two poles is 80 units. The separation between the poles is doubled. What is the force between them?
- The length of a bar magnet is 10 cm and the area of cross-section is  $1.0 \text{ cm}^2$ . The magnetization  $I = 10^2 \text{ A/m}$ . Calculate the pole strength.
- Two identical bar magnets are placed on the same line end to end with north pole facing north pole. Draw the lines of force, if no other field is present.
- The points, where the magnetic field of a magnet is equal and opposite to the horizontal component of magnetic field of the earth, are called neutral points
  - Locate the neutral points when the bar magnet is placed in magnetic meridian with north pole pointing north.
  - Locate the neutral points when a bar magnet is placed in magnetic meridian with north pole pointing south.
- If a bar magnet of length 10 cm is cut into two equal pieces each of length 5 cm then what is the pole strength of the new bar magnet compare to that of the old one.
- A 10 cm long bar magnet has a pole strength 10 A.m. Calculate the magnetic field at a point on the axis at a distance of 30 cm from the centre of the bar magnet.
- How will you show that a current carrying conductor has a magnetic field around it? How will you find its magnitude and direction at a particular place ?
- A force acts upon a charged particle moving in a magnetic field, but this force does not change the speed of the particle, Why ?
- At any instant a charged particle is moving parallel to a long, straight current carrying wire. Does it experience any force ?



12. A current of 10 ampere is flowing through a wire. It is kept perpendicular to a magnetic field of 5T. Calculate the force on its 1/10 m length.
13. A long straight wire carries a current of 12 amperes. Calculate the intensity of the magnetic field at a distance of 48 cm from it.
14. Two parallel wire, each 3m long, are situated at a distance of 0.05 m from each other. A current of 5A flows in each of the wires in the same direction. Calculate the force acting on the wires. Comment on its nature ?
15. The magnetic field at the centre of a 50cm long solenoid is  $4.0 \times 10^{-2} \text{ NA}^{-1} \text{ m}^{-1}$  when a current of 8.0A flows through it, calculate the number of turns in the solenoid.
16. Of the two identical galvanometer one is to be converted into an ammeter and the other into a milliammeter. Which of the shunts will be of a larger resistance ?
17. The resistance of a galvanometer is 20 ohms and gives a full scale deflection for 0.005A. Calculate the value of shunt required to change it into an ammeter to measure 1A. What is the resistance of the ammeter ?
18. An electron is moving in a circular orbit of radius  $5 \times 10^{-11} \text{ m}$  at the rate of  $7.0 \times 10^{15}$  revolutions per second. Calculate the magnetic field **B** at the centre of the orbit.
19. Calculate the magnetic field at the centre of a flat circular coil containing 200 turns, of radius 0.16m and carrying a current of 4.8 ampere.
20. Refer to Fig. 18.30 and calculate the magnetic field at *A*, *B* and *C*.

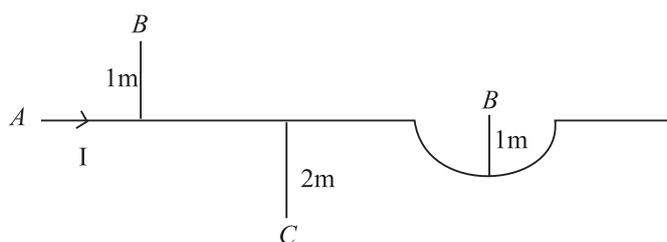


Fig. 18.30



## ANSWERS TO INTEXT QUESTIONS

## 18.1

1. Suspend the magnet with a thread at its centre of mass. Let it come to equilibrium. The end of the magnet which points towards geographical north is its north pole.

## MODULE - 5

### Electricity and Magnetism



#### Notes

## Magnetism and Magnetic Effect of Electric Current

- Bring the ends of any two bars closer together. If there is attraction between them, one of the bars is a magnet and the other is an iron bar. Now lay down one of these bars on the table and strock along its length with the other. If uniform force is experienced, the bar in hand is a magnet and that on the table is iron piece. If non-uniform force is experienced, reverse is the case.
- Suspending one of the bar magnets with thread, we can find its south pole. Then the end of the second magnet, which is repelled by the first, is its south pole.

### 18.2

- (i)electrical (ii) magnetic as well as electrical.
- A conductor in equilibrium is neutral i.e. it has no net electrical current. Due to their random motion, thermal electrons cancel the magnetic fields produced by them.
- In first case length of wire  $l_1 = 2\pi r$  In second case length of wire  $l_2 = (2\pi r_2)2$ .

$$\text{But } l_1 = l_2$$

$$\therefore 2\pi r = 4\pi r_2 \Rightarrow r_2 = \frac{r}{2}$$

$$\text{Using } |\mathbf{B}| = \frac{\mu_0 nI}{2r}$$

$$|\mathbf{B}_1| = \frac{\mu_0 I}{2r}, \quad |\mathbf{B}_2| = \frac{\mu_0 \cdot 2 \cdot I}{2 \times \frac{r}{2}} = \frac{2\mu_0 I}{r} = 4 \mathbf{B}$$

That is, the magnetic  $\mathbf{B}$  at the centre of a coil with two turns is four times stronger than the field in first case.

### 18.3

- $c$
- Both laws specify magnetic field due to current carrying conductors.
- (i)  $B$ , (ii)  $A$ , (iii)  $C$ .
- $B = \mu_0 \frac{n}{\ell} I \Rightarrow 4\pi \times \frac{10^{-7} \times n}{0.1m} \times 3A = 0.002$  or  $n = \frac{.0002 \times 10^7}{12\pi} = 50$  turns



Notes

## 18.4

1. The nature of the force will be attractive because the stream of protons is equivalent to electrons in the opposite direction.
2. The force exerted by a magnetic field on a moving charge is perpendicular to the motion of the charge and the work done by the force on the charge is zero. So the KE of the charge does not change. In an electric field, the deflection is in the direction of the field. Hence the field accelerates it in the direction of field lines.
3. The direction of current in each turn of the spring is the same. Since parallel currents in the same direction exert force of attraction, the turns will come closer and the body shall be lifted upward, whatever be the direction of the current in the spring.

## 18.5

1. Radial magnetic field is one in which plane of the coil remains parallel to it.
2. This increases the strength of magnetic field due to the crowding of magnetic lines of force through the soft iron core, which in turn increases the sensitivity of the galvanometer.
3. Ammeter has the lowest resistance whereas voltmeter has the highest resistance. In an ammeter a low resistance is connected in parallel to the galvanometer coil whereas in a voltmeter, a high resistance is connected in series with it.
4. A low resistance  $R_s$  should be connected in parallel to the coil :

$$R_s = \frac{G I_g}{I - I_g} = \frac{20 \times 20 \times 10^{-3}}{3 - 20 \times 10^{-3}} = 0.13 \Omega$$

## Answers To Problems in Terminal Exercise

- |   |  |
|---|--|
| 1. $10^{-2} \text{ T m}^{-1}$   | 7. same.   |
| 8. $2.3 \times 10^{-6} \text{ T}$   | 12. 5 N  |
| 13. 5 $\mu\text{N}$   | 14. attractive force of $10^{-4} \text{ N m}^{-1}$ |
| 15. $\frac{625}{\pi}$ turns.  | 17. $0.1 \Omega$ .                                 |
| 18. $4.48 \pi \text{ T}$  | 19. $1.2\pi \text{ mT}$                            |
| 20. $B_A = 2 \times 10^{-7} \text{ T}$ , $B_B = \pi \times 10^{-7} \text{ T}$ and $B_C = 10^{-7} \text{ T}$ . |  |