

RELATIONS BETWEEN SIDES AND ANGLES OF A TRIANGLE

Sine Formula

- The lengths of the sides are proportional to the sines of the angles opposite to the sides

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Formula

$$\text{I. } \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{II. } \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\text{III. } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Projection Formula

$$\text{I. } a = b \cos C + c \cos B$$

$$\text{II. } b = c \cos A + a \cos C$$

$$\text{III. } c = a \cos B + b \cos A$$

Check your progress

- Q1 In a triangle ABC, the sides AB = 3cm, BC = 5cm and AC = 7 cm, the greatest angle of the triangle ABC is:

- (A) $\frac{\pi}{2}$
(B) $\frac{2\pi}{3}$
(C) $\frac{\pi}{3}$
(D) π

- Q2 In a triangle ABC, if $a = 4$, $b = c$,

and $c = 8$, then the value of $2 \cos B + 5 \cos C$ is:

- (A) $\frac{7}{4}$
(B) $\frac{4}{7}$
(C) $\frac{11}{7}$
(D) $\frac{7}{11}$

- Q3 In a triangle ABC, if $a = 18$, $b = 24$ and $c = 30$, then the value of $\sin B$ is equal to:

- (A) 1
(B) $\frac{3}{5}$
(C) $\frac{2}{5}$
(D) $\frac{4}{5}$

- Q4 If $\cos A = m \cos B$, then

$\cot \frac{A+B}{2} \cot \frac{B-A}{2}$ is equal to:

- (A) $\frac{m-1}{m+1}$
(B) $\frac{m+2}{m-2}$
(C) $\frac{m-1}{m-2}$
(D) $\frac{m+2}{m+2}$

- Q5 In triangle ABC, if $a \cos A = b \cos B$, where $a \neq b$, then triangle ABC is:
- (A) Right angle triangle
 - (B) Equilateral triangle
 - (C) Isosceles triangle
 - (D) Scalene triangle

- 1 use sine formula
- 2 use sine formula
- 3 use sine and cosine formula

Stretch Yourself

Prove that

1.
$$\frac{\sin(B - C)}{\sin(B + C)} = \frac{b^2 - c^2}{a^2}$$

For any triangle ABC, prove that

2.
$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

3. In triangle ABC, prove that a^2, b^2, c^2 are in Arithmetic Progression (A.P) if and only if $\cot A, \cot B$ and $\cot C$ are in A.P
4. In any triangle ABC, if $a \cos A = b \cos B$, then the triangle ABC is isosceles or right angled.
5. In a triangle ABC, prove that $2(bc \cos A + ac \cos B + ab \cos C) = a^2 + b^2 + c^2$

Answer to Check Your Progress

- Q 1 (B)
- Q 2(D)
- Q 3 (A)
- Q 4(C)
- Q 5 (A)

Answer to Stretch Yourself