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Binomial Theorem

- **Binomial Theorem**

The Statement of Binomial expansion $(x+y)^n$, where n is the positive integer is known as Binomial theorem.

$$(x+y)^n = {}^nc_o\, x^n + {}^nc_1\, x^{n-1}\, y^1 + {}^nc_2\, x^{n-2}.$$

$$+ - - - - + \quad (x+y)^n = {}^nc_{n-1}\, xy^{n-1}$$

${}^nc_n\, y^n$, where $n \in N$ and $x, y \in R$

- General Term in a Binomial Expansion

$$T_{r+1} = {}^nc_r\, x^{n-r}.y^r$$

- Middle Terms in a Binomial Expansion

Case – 1 :- When exponent n of the binomial is even, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

Case – 2 :- When the exponent n of a binomial is an odd natural number, then the $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms are two middle terms.

Check Yourself

Q.1 Fourth term in the expansion of $\left(\frac{a}{3}+9b\right)^{10}$ is-

(A) $40\, a^7\, b^3$ (B) $40a^3b^7$

(C) $1890\, a^6b^4$ (D) $1890a^4b^6$

Q.2 Second term in the expansion of $(2x + 3y)^5$ will be -

(A) $46\, x^2y^3$ (B) $30\, x^3y^2$

(C) $240\, x^4\, y$ (D) $810\, xy^4$

Q.3 The 5^{th} term of the expansion of $(x - 2)^8$ is -

(A) ${}^8C_5x^3(-2)^5$ (B) ${}^8C_5x^3\, 2^5$

(C) ${}^8C_4x^4\,(-2)^4$ (D) ${}^8C_6x^2\,(-2)^6$

Q.4 The number of terms in expansion of $(x - 3x^2 + 3x^3)^{20}$ is-

(A) 60 (B) 61

(C) 40 (D) 41

Q.5 The term with coefficient ${}^6C_2$ in the expansion of $(1+x)^6$ is-

(A) T_1 and T_3 (B) T_2 and T_4

(C) T_3 and T_5 (D) None of these

Q.6 If n is a positive integer, then r^{th} term in the expansion of $(1-x)^n$ is-

(A) ${}^nC_r\,(-x)^r$ (B) ${}^nC_r x^r$

(C) ${}^nC_{r-1}(-x)^{r-1}$ (D) ${}^nC_{r-1}x^{r-1}$

Q.7 If the 4^{th} term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the values of a and n are-

(A) 1/2, 6 (B) 1, 3

(C) 1/2, 3 (D) can not be found

Q.8 The coefficient of $(3r)^{th}$ term and coefficient of $(r+2)^{th}$ term in the expansion of $(1+x)^{2n}$ are equal then (where $r > 1$, $n > 2$), positive integer)-

(A) $r = n/2$ (B) $r = n/3$

(C) $r = \frac{n+1}{2}$ (D) $r = \frac{n-1}{2}$

Q.9 The coefficient of a^2b^3 in $(a+b)^5$ is-

(A) 10 (B) 20

(C) 30 (D) 40

Q.10 The coefficient of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then n is equal to-

(A) 35 (B) 45

(C) 55 (D) None of these

Q.11 The coefficient of x^5 in the expansion of $(2+3x)^{12}$ is-

(A) ${}^{12}C_5 2^5, 3^7$ (B) ${}^{12}C_6 2^6.3^6$

(C) ${}^{12}C_5 2^7.3^5$ (D) None of these

Q.12 If the expansion of $\left(x^2-\frac{1}{4}\right)^n$, the coefficient of third term is 31, then the value of n is-

(A) 30 (B) 31

(C) 29 (D) 32

Q.13 If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then-

(A) A = B

(B) A ≥ B

(C) A = 0 , B for some n

(D) None of these

Q.14 If $(1 + by)^n = (1 + 8y + 24y^2 + ...)$ then the value of b and n are respectively-

(A) 4, 2 (B) 2, –4

(C) 2, 4 (D) –2, 4

Q.15 The number of terms in the expansion of

$(1 + 5\sqrt{2}\,x)^9 + (1 - 5\sqrt{2}\,x)^9$ is-

(A) 5 (B) 7 (C) 9 (D) 10

Stretch Yourself

1. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then find $\frac{(C_0+C_1)(C_1+C_2)...(C_{n-1}+C_n)}{C_1C_2...C_n}$
2. Find the 5^{th} term of the expansion of $(x - 2)^8$
3. Find the number of terms in the expansion of
   a. $(1 + 5\sqrt{2}\,x)^9 + (1 - 5\sqrt{2}\,x)^9$
4. Calculate the middle term in the expansion of $(1 - 3x + 3x^2 - x^3)^6$
5. If $(1 + x - 2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + ...+ C_{12}\,x^{12}$, then calculatethe value of $C_2 + C_4 + C_6 + ...+ C_{12}$

Answer to check yourself

1A 2C 3C 4 D 5 C

6 C 7 A 8A 9A 10 C

11C 12 D 13 A 14 C 15 A