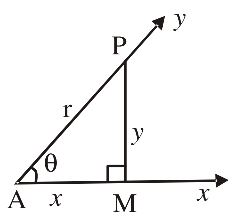
22

INTRODUCTION TO TRIGONOMETRY

- **Trigonometry** : Trigonometry is that branch of mathematics which deals with the measurement of the sides and the angles of a triangle and the problems related to angles.



- **Trigonometric Ratios :** Ratios of the sides of a triangle with respect to its acute angles are called trigonometric ratios.
  In the right angled Δ AMP
  For acute angle PAM = θ
  Base = AM = x, Perpendicular = PM = y, Hypotenuse = AP = r

Here, sine $\theta = \frac{y}{r}$, Written as sin θ

cosine $\theta = \frac{x}{r}$, Written as cos θ

tangent $\theta = {}^{y}\!/_{x}$, Written as tan θ

cosecent $\theta = {}^{r}\!/_{y}$, Written as cosec θ

secent $\theta = {}^{r}\!/_{x}$, Written as sec θ

cotangent $\theta = {}^{x}\!/_{y}$, Written as cot θ

$\Rightarrow$ sin θ, cos θ, tan θ etc. are complete symbols and can not be separated from θ.
$\Rightarrow$ Every trigonometric ratio is a real number.
$\Rightarrow$ θ is restricted to be an acute angle.
$\Rightarrow$ For convenience, we write $(\sin\theta)^2$, $(\cos\theta)^2$, $(\tan\theta)^2$ as $\sin^2\theta$, $\cos^2\theta$ and $\tan^2\theta$ respectively.

- **Relation between Trigonometric ratios :**

$\Rightarrow \sin\theta = \frac{1}{\text{cos ec}\theta}$ or $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ or $\sin\theta \times \text{cosec}\,\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta \times \sec\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta \times \cot\theta = 1$

$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

- **Trigonometric Identities :** An equation involving trigonometric ratios of an angle θ is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.
  Some special trigonometric Identities
  $\Rightarrow \sin^2\theta + \cos^2\theta = 1$ or $1 - \cos^2\theta = \sin^2\theta$ or $1 - \sin^2\theta = \cos^2\theta$.
  $\Rightarrow 1 + \tan^2\theta = \sec^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta - 1 = \tan^2\theta$
  $\Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta$ or $\text{cosec}^2\theta - \cot^2\theta = 1$ or $\text{cosec}^2\theta - 1 = \cot^2\theta$.

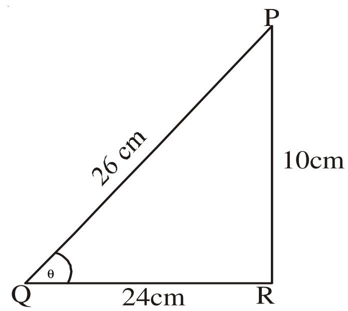
- **Trigonometric ratios of complementary angles:** If θ is an acute angle then
  $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$
  $\tan(90^\circ - \theta) = \cot\theta$ and $\cot(90^\circ - \theta) = \tan\theta$
  $\text{cosec}(90^\circ - \theta) = \sec\theta$ and $\sec(90^\circ - \theta) = \text{cosec}\,\theta$
  Here θ is an acute angle and $(90^\circ - \theta)$ is a complementary angle for θ.

- **Finding of trigonometric ratios :** $\Rightarrow$ If two sides of any right triangle are given, then all the six trigonometric ratios can be written.

$\Rightarrow$ If one trigonometric ratio is given, then other trigonometic ratios can be written by using pythagoras theorem or trigonometic identities.

CHECK YOUR PROGRESS:

1. In the given figure, which of the following is correct?



(A) $\sin\theta + \cos\theta = \frac{17}{13}$ (B) $\sin\theta - \cos\theta = \frac{17}{13}$

(C) $\sin\theta + \sec\theta = \frac{17}{13}$ (D) $\tan\theta + \sec\theta = \frac{17}{13}$

2. If $5\tan\theta - 4 = 0$, the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is :

(A) $\frac{5}{3}$ (B) $\frac{5}{6}$ (C) 0 (D) $\frac{1}{6}$

3. The value of $\left(\frac{\sin\theta\,.\cos(90^0 - \theta)}{\sin(90^0 - \theta).\cos\theta} + 1\right)$ is equal to :

(A) $\sin\theta + \cos\theta$ (B) $\cos^2\theta$ (C) $\sec^2\theta$ (D) $\text{cosec}^2\theta$

4. The value of $\frac{\sec 41^0 \text{ cos ec } 49^0 - \tan 41^0 \cot 49^0}{\sec 41^0 . \sin 49^0 + \cos 49^\circ \ . \text{cos ec} 41^0}$ is :

(A)1 (B) 0 (C) $\frac{1}{2}$ (D) 0

5. If $\sin(\theta + 36^0) = \cos\theta$ and $\theta + 36^0$ is an acute angle, then θ is equal to :
(A) 54^0 (B) 18^0 (C) 21^0 (D) 27^0

6. If $\cot\theta = \frac{12}{5}$, find the value of $\frac{\sin\theta.\cos\theta}{\sec\theta}$.

7. Prove that $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\,\text{cosec}A$.

8. If $\cos\theta = \frac{1}{2}$ and $\text{Sin}\,\theta = \frac{\sqrt{3}}{2}$, find the value of $\sec\theta$, $\text{cosec}\,\theta$ and $\tan\theta$.

STRETCH YOURSELF

1. For a right angled ΔABC, right angled at C, tanA=1, find the value of $\sin^2 B.\cos^2 B$.
2. Find the value of $\tan 1^0 . \tan 2^0 . \tan 3^0 ....\tan 89^0$.

ANSWERS

CHECK YOUR PROGRESS :

1. A 2. C 3. C

4. C 5. D 6. $\frac{720}{2197}$

8. $\sec\theta = 2$, $\text{cosec}\,\theta = \frac{2}{\sqrt{3}}$, $\tan\theta = \sqrt{3}$

STRETCH YOURSELF :

1. $\frac{1}{4}$ 2. 1