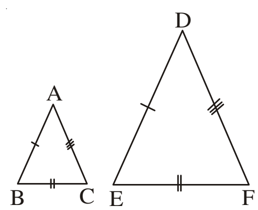
14

SIMILARITY OF TRIANGLES

- Objects which have the same shape but different sizes are called similar objects.
- Any two polygons, with corresponding angles equal and corresponding sides proportional are similar.
- Two triangles are similar if

  (i) their corresponding angles are equal and

  (ii) their corresponding sides are proportional



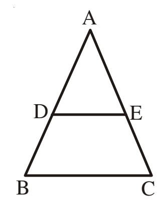
$\Delta ABC \sim \Delta DEF$ if $\angle A = \angle D$, $\angle B = \angle E$,

$\angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

The symbol ' ~ ' stands for "is similar to".

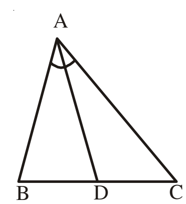
- **AAA Criterion for Similarity:** If in two triangles the corresponding angles are equal, the triangles are similar.
- **SSS Criterion for Similarity:** If the corresponding sides of two triangles are proportional, the triangles are similar.
- **SAS Criterion for Similarity:** If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.
- If a line drawn parallel to one side of a triangle intersects the other two sides at distinct points, the other two sides of the triangles are divided proportionally.

If DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$

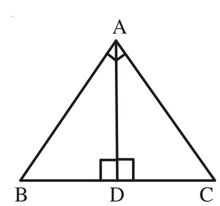


- If a line divides any two sides of a triangle in the same ratio, the line is parallel to third side of the triangle.
- The internal bisector of any angle of a triangle divides the opposite side in the ratio of sides containing the angle. If AD is internal bisector of $\angle A$ of ΔABC, then

$$\frac{BD}{DC} = \frac{AB}{AC}$$



- If a perpendicular is drawn from the vertex containing right angle of a right tringle to the hypotenuse, the triangles on each side of the perpendicular are similar to each other and to the original triangle.



$\Delta ADB \sim \Delta CDA$, $\Delta ADB \sim \Delta CAB$ and $\Delta ADC \sim \Delta BAC$..

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

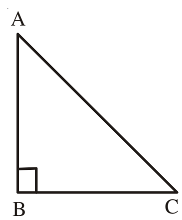
$$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

- **Baudhayan/Pythagoras Theorem**

  In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

  In triangle ABC

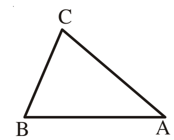
  $AC^2 = AB^2 + BC^2$



- **Converse of pythagoras Theorem**

  In a triangle, if the square on one side is equal to the sum of the squares on the other two sides, the angle opposite to the first side is a right angle.

  If in triange ABC,



$AC^2 = AB^2 + BC^2$

then $\angle B = 90^0$

CHECK YOUR PROGRESS:

1. The areas of two similar triangles are 25 sq. m and 121 sq. m. The ratio of their corresponding sides is :

   (A) 5 : 11 (B) 11 : 5 (C) $\sqrt{5} : \sqrt{11}$ (D) $\sqrt{11} : \sqrt{5}$

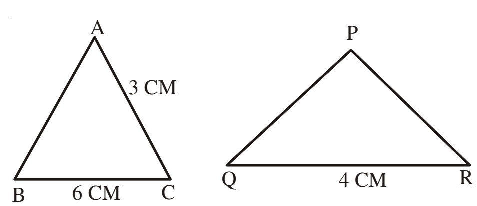
2. Two poles 6m and 11m high stands vertically on the ground If the distance between their feet is 12m, then the distance between their tops is:

   (A) 11m (B) 12 m (C) 13 m (D) 14m

3. If in two triangles DEF and PQR, $\angle D = \angle Q$, $\angle R = \angle E$, which of the following is not true?

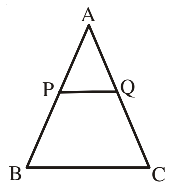
   (A) $\frac{DE}{PQ} = \frac{EF}{RP}$ (B) $\frac{EF}{PR} = \frac{DF}{PQ}$ (C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

4. In the adjoining figure, $\Delta ABC \sim \Delta PQR$, length of PR is:



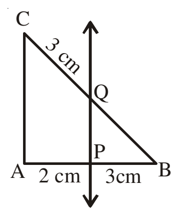
   (A) 3cm (B) 2cm (C) 4cm (D) 6cm

5. In the adjoining figure, P and Q are mid points of AB and AC respectively, If PQ = 3.4cm , then BC is :

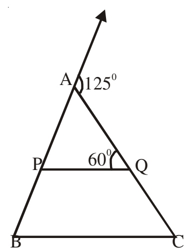


(A) 3.4cm (B) 1.7 cm (C) 6.8cm (D) 10.2cm

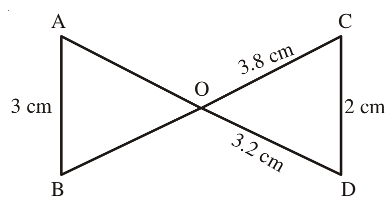
6. In the adjoining figure, QP || CA, find BC



7. In ΔABC if AB= a cm, BC= $\sqrt{3}$ a cm and AC= 2a cm, then find ∠B
8. In the adjoining figure ΔABC ~ ΔAPQ. Find ∠B



9. In the adjoining figure ΔABO ~ ΔDCO. Find OA and OB



10. In an equilateral ΔABC, AD ⊥ BC Prove that $3\,AB^2 = 4\,AD^2$.

STRETCH YOURSELF

1. Show that the altitude of an equilateral triangle with side a is $\frac{\sqrt{3}}{2}a$.
2. In ΔABC, AD ⊥ BC and $AD^2 = BD \times DC$. Prove that $\angle BAC = 90^0$.
3. Prove that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
4. If a line is drawn parallel to one side of a triangle intersecting the other two sides, then prove that the line divides the two sides in the same ratio.
5. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

ANSWERS:

CHECK YOUR PROGRESS :

1. A
2. C
3. A
4. B
5. C
6. 7.5cm
7. 90^0
8. 65^0
9. 4.8cm
10. 5.7cm