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211en08

PERCENTAGE AND ITS APPLICATIONS

You must have seen advertisements in newspapers, television and hoardings etc of the following type:

"Sale, up to 60% off".

"Voters turnout in the poll was over 70%".

"Ramesh got 93% aggregate in class XII examination".

"Banks have lowered the rate of interest on fixed deposits from 8.5% to 7%".

In all the above statements, the important word is 'percent'. The word 'percent' has been derived from the Latin word 'percentum' meaning per hundred or out of hundred.

In this lesson, we shall study percent as a fraction or a decimal and shall also study its applications in solving problems of profit and loss, discount, simple interest, compound interest, rate of growth and depreciation etc.

OBJECTIVES

After studying this lesson, you will be able to

- *illustrate the concept of percentage;*
- *calculate specified percent of a given number or a quantity;*
- *solve problems based on percentage;*
- *solve problems based on profit and loss;*
- *calculate the discount and the selling price of an article, given marked price of the article and the rate of discount;*
- *solve inverse problems pertaining to discount;*
- *calculate simple interest and the amount, when a given sum of money is invested for a specified time period on a given rate of interest;*

- *illustrate the concept of compound interest vis-a-vis simple interest;*
- *calculate compound interest, the amount and the difference between compound and simple interest on a given sum of money at a given rate and for a given time period; and*
- *solve real life problems pertaining to rate of growth and decay, using the formula of compound interest, given a uniform or variable rate.*

EXPECTED BACKGROUND KNOWLEDGE

- Four fundamental operations on whole numbers, fractions and decimals.
- Comparison of two fractions.

8.1 PERCENT

Recall that a fraction $\frac{3}{4}$ means 3 out of 4 equal parts. $\frac{7}{13}$ means 7 out of 13 equal parts and $\frac{23}{100}$ means 23 out of 100 equal parts.

A fraction whose denominator is 100 is read as percent, for example $\frac{23}{100}$ is read as twenty three percent.

The symbol '%' is used for the term percent.

A ratio whose second term is 100 is also called a percent,

So, 33 : 100 is equivalent to 33%.

Recall that while comparing two fractions, $\frac{3}{5}$ and $\frac{1}{2}$, we first convert them to equivalent fractions with common denominator (L.C.M. of the denominators).

thus $\frac{3}{5}=\frac{3}{5}\times\frac{2}{2}=\frac{6}{10}$, and

$$\frac{1}{2}=\frac{1}{2}\times\frac{5}{5}=\frac{5}{10}$$

Now, because $\frac{6}{10}>\frac{5}{10}$ $\quad\therefore\frac{3}{5}>\frac{1}{2}$

We could have changed these fractions with common denominator 100 as

$$\frac{3}{5}=\frac{3}{5}\times\frac{20}{20}=\frac{60}{100} \quad \text{or} \quad 60\%$$

$$\frac{1}{2}=\frac{1}{2}\times\frac{50}{50}=\frac{50}{100} \quad \text{or} \quad 50\%$$

and so, $\frac{3}{5}>\frac{1}{2}$ as 60% is greater than 50%.

8.2 CONVERSION OF A FRACTION INTO PERCENT AND VICE VERSA

In the above section, we have learnt that, to convert a fraction into percent, we change the fraction into an equivalent fraction with denominator 100 and then attach the symbol % with the changed numerator of the fraction. For example,

$$\frac{3}{4}=\frac{3}{4}\times\frac{25}{25}=\frac{75}{100}=75\times\frac{1}{100}=75\% \text{ and}$$

$$\frac{4}{25}=\frac{4}{25}\times\frac{4}{4}=\frac{16}{100}=16\times\frac{1}{100}=16\%$$

Note: To write a fraction as percent, we may multiply the fraction by 100, simplify it and attach % symbol. For example,

$$\frac{4}{25}=\frac{4}{25}\times 100\%=16\%$$

Conversely,

To write a percent as a fraction, we drop the % sign, multiply the number by $\frac{1}{100}$ (or divide the number by 100) and simplify it. For example,

$$47\%=47\times\frac{1}{100}=\frac{47}{100}, \qquad 17\%=17\times\frac{1}{100}=\frac{17}{100}, \qquad 3\%=\frac{3}{100}$$

$$45\%=45\times\frac{1}{100}=\frac{45}{100}=\frac{9}{20}, \quad 210\%=\frac{210}{100}=\frac{21}{10}, \quad x\%=\frac{x}{100}.$$

8.3 CONVERSION OF DECIMAL INTO A PERCENT AND VICE VERSA

Let us consider the following examples:

$$0.35 = \frac{35}{100} = 35 \times \frac{1}{100} = 35\%$$

$$4.7 = \frac{47}{10} = \frac{470}{100} = 470 \times \frac{1}{100} = 470\%$$

$$0.459 = \frac{459}{1000} = \frac{459}{10} \times \frac{1}{100} = 45.9\%$$

$$0.0063 = \frac{63}{10000} = \frac{63}{100} \times \frac{1}{100} = 0.63\%$$

Thus, **to write a decimal as a percent, we move the decimal point two places to the right and put the % sign**

Conversely,

To write a percent as a decimal, we drop the %sign and insert or move the decimal point two places to the left. For example,

43% = 0.43	75% = 0.75	12% = 0.12
9% = 0.09	115% = 1.15	327% = 3.27
0.75% = 0.0075	4.5% = 0.045	0.2% = 0.002

Let us take a few more examples:

Example 8.1: Shweta obtained 18 marks in a test of 25 marks. What was her percentage of marks?

Solution: Total marks = 25

Marks obtained = 18

$\therefore$ Fraction of marks obtained = $\frac{18}{25}$

$\therefore$ Marks obtained in percent = $\frac{18}{25} \times \frac{4}{4} = \frac{72}{100} = 72\%$

Alternatively:

$$\text{Marks obtained in percent} = \frac{18}{25} \times 100\% = 72\%$$

Example 8.2: One-fourth of the total number of shoes in a shop were on discount sale. What percent of the shoes were there on normal price?

Solution: Fraction of the total number of shoes on sale $= \frac{1}{4}$

$\therefore$ Fraction of the total number of shoes on normal price $= 1 - \frac{1}{4} = \frac{3}{4}$

$$= \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 75\% \text{ or } \frac{3}{4} \times 100\% = 75\%$$

Example 8.3: Out of 40 students in a class, 32 opted to go for a picnic. What percent of students opted for picnic?

Solution: Total number of students in a class $= 40$

Number of students, who opted for picnic $= 32$

$\therefore$ Number of students, in percent, who opted for picnic

$$= \frac{32}{40} \times 100\% = 80\%$$

Example 8.4: In the word ARITHMETIC, what percent of the letters are I's?

Solution: Total number of letters $= 10$

Number of I's $= 2$

$\therefore$ Percent of I's $= \frac{2}{10} \times 100\% = 20\%$

Example 8.5: A mixture of 80 litres, of acid and water, contains 20 litres of acid. What percent of water is in the mixture?

Solution: Total volume of the mixture $= 80$ litres

Volume of acid $= 20$ litres

$\therefore$ Volume of water $= 60$ litres

$\therefore$ Percentage of water in the mixture $= \frac{60}{80} \times 100\% = 75\%$

CHECK YOUR PROGRESS 8.1

1. Convert each of the following fractions into a percent:

(a) $\frac{12}{25}$ (b) $\frac{9}{20}$ (c) $\frac{5}{12}$ (d) $\frac{6}{15}$ (e) $\frac{125}{625}$

(f) $\frac{3}{10}$ (g) $\frac{108}{300}$ (h) $\frac{189}{150}$ (i) $\frac{72}{25}$ (j) $\frac{1231}{1250}$

2. Write each of the following percents as a fraction:

(a) 53% (b) 85% (c) $16\frac{7}{8}\%$ (d) 3.425% (e) 6.25%

(f) 70% (g) $15\frac{3}{4}\%$ (h) 0.0025% (i) 47.35% (j) 0.525%

3. Write each of the following decimals as a percent:

(a) 0.97 (b) 0.735 (c) 0.03 (d) 2.07 (e) 0.8

(f) 1.75 (g) 0.0250 (h) 3.2575 (i) 0.152 (j) 3.0015

4. Write each of the following percents as a decimal:

(a) 72% (b) 41% (c) 4% (d) 125% (e) 9%

(f) 410% (g) 350% (h) 102.5% (i) 0.025% (j) 10.25%

5. Gurpreet got half the answers correct, in an examination. What percent of her answers were correct?

6. Prakhar obtained 18 marks in a test of total 20 marks. What was his percentage of marks?

7. Harish saves ₹ 900 out of a total monthly salary of ₹ 14400. Find his percentage of saving.

8. A candidate got 47500 votes in an election and was defeated by his opponent by a margin of 5000 votes. If there were only two candidates and no votes were declared invalid, find the percentage of votes obtained by the winning candidate.

9. In the word PERCENTAGE, what percent of the letters are E's?

10. In a class of 40 students, 10 secured first division, 15 secured second division and 13 just qualified. What percent of students failed.

8.4 CALCULATION OF PERCENT OF A QUANTITY OR A NUMBER

To determine a specified percent of a number or quantity, we first change the percent to a fraction or a decimal and then multiply it with the number or the quantity. For example:

$$25\% \text{ of } 90 = \frac{25}{100} \times 90 = 22.50$$

or $25\% \text{ of } 90 = 0.25 \times 90 = 22.50$

$60\% \text{ of Rs. } 120 = 0.60 \times \text{Rs. } 120 = \text{Rs. } 72.00$

$120\% \text{ of } 80 \text{ kg} = 1.20 \times 80 \text{ kg} = 96 \text{ kg}$

Let us take some examples from daily life:

Example 8.6: In an examination, Neetu scored 62% marks. If the total marks in the examination are 600, then what are the marks obtained by Neetu?

Solution: Here we have to find 62% of 600

$\therefore$ 62% of 600 marks = 0.62 × 600 marks = 372 marks

$\therefore$ Marks obtained by Neetu = 372

Example 8.7: Naresh earns ₹ 30800 per month. He keeps 50% for household expenses, 15% for his personal expenses, 20% for expenditure on his children and the rest he saves. What amount does he save per month?

Solution: Expenditure on Household = 50%

Expenditure on self = 15%

Expenditure on children = 20%

Total expenditure = (50 + 15 + 20)% = 85%

$\therefore$ Savings (100 – 85)% = 15%

$\therefore$ 15% of ₹ 30800 = ₹ (0.15 × 30800)

= ₹ 4620

Example 8.8: What percent of 360 is 144?

Solution: Let x% of 360 = 144

$$\therefore \frac{x}{100} \times 360 = 144$$

Or $$x = \frac{144}{360} \times 100 = 40\%$$

Alternatively, 144 out of 360 is equal to the fraction $\frac{144}{360}$

$$\therefore \text{Percent} = \frac{144}{360} \times 100\% = 40\%$$

Example 8.9: If 120 is reduced to 96, what is the reduction percent?

Solution: Here, reduction = 120 – 96 = 24

$$\therefore \text{Reduction percent} = \frac{24}{120} \times 100\% = 20\%$$

Example 8.10: The cost of an article has increased from ₹ 450 to ₹ 495. By what percent did the cost increased?

Solution: The increase in Cost Price = ₹ (495 – 450)

= ₹ 45

$$\text{Increase percent} = \frac{45}{450} \times 100 = 10\%$$

Example 8.11: 60% of the students in a school are girls. If the total number of girls in the school is 690, find the total number of students in the school. Also, find the number of boys in the school.

Solution: Let the total number of students in the school be x

Then, 60% of $x = 690$

$$\therefore \frac{60}{100} \times x = 690 \text{ or } x = \frac{690 \times 100}{60} = 1150$$

$\therefore$ Total number of students in the school = 1150

$\therefore$ Hence number of boys = 1150 – 690 = 460

Example 8.12: A's income is 25% more than that of B. B's income is 8% more than that of C. If A's income is ₹ 20250, then find the income of C.

Solution: Let income of C be ₹ x

Income of B $= x + 8\%$ of x

$$= x + \frac{8x}{100} = \frac{108}{100} \times x$$

Income of A $= \frac{108x}{100} + 25\%$ of $\frac{108x}{100}$

$$= \frac{108x}{100} \times \frac{125}{100}$$

$$\therefore \frac{108}{100} \times x \times \frac{125}{100} = 20250$$

$$\text{or } x = 20250 \times \frac{100}{108} \times \frac{100}{125} = 15000$$

$\therefore$ Income of C is ₹ 15000.

Example 8.13: A reduction of 10% in the price of tea enables a dealer to purchase 25 kg more tea for ₹ 22500. What is the reduced price per kg of tea? Also, find the original price per kg.

Solution: 10% of ₹ $22500 = \frac{10}{100} \times 22500 =$ ₹ 2250

$\therefore$ Reduced price of 25 kg tea = ₹ 2250

$\therefore$ Reduced price per kg = ₹ $\frac{2250}{25}$ = ₹ 90 per kg.

Since, the reduction was 10% so the original price = ₹ 100 per kg.

Example 8.14: A student got 45% marks in the first paper and 70% in the second paper. How much percent should he get in the third paper so as to get 60% as overall score?

Solution: Let each paper be of 100 marks.

$\therefore$ Marks obtained in first paper = 45% of 100 = 45

Marks obtained in second paper = 70% of 100 = 70

Total marks (in three papers) he wants to obtain = 60% of 300

$$= \frac{60}{100} \times 300 = 180$$

$\therefore$ Marks to be obtained in third paper $= 180 - (45 + 70)$

$$= 180 - 115 = 65$$

Example 8.15: Find the sum which when increased by 15% becomes ₹ 19320.

Solution: Let the sum be ₹ x

$\therefore$ x + 15% of x = 19320

$$x + \frac{15x}{100} = 19320 \text{ or } \frac{115x}{100} = 19320$$

$$\therefore x = \frac{19320 \times 100}{115} = 16800$$

Hence, the required sum = ₹ 16800.

CHECK YOUR PROGRESS 8.2

1. Find: (i) 16% of 1250 (ii) 47% of 1200
2. A family spends 35% of its monthly budget of ₹ 7500 on food. How much does the family spend on food?

3. In a garden, there are 500 plants of which 35% are trees, 20% are shrubs, 25% are herbs and the rest are creepers. Find out the number of each type of plants.
4. 60 is reduced to 45. What percent is the reduction?
5. If 80 is increased to 125, what is the increase percent?
6. Raman has to score a minimum 40% marks for passing the examination. He gets 178 marks and fails by 22 marks. Find the maximum marks.
7. It takes me 45 minutes to go to school and I spend 80% of the time travelling by bus. How long does the bus journey last?
8. In an election, between 2 candidates 25% voters did not cast their votes. A candidate scored 40% of the votes polled and was defeated by 900 votes. Find the total number of voters.
9. A rise of 25% in the price of sugar compels a person to buy 1.5 kg of sugar less for ₹ 240. Find the increased price as well as the original price per kg of sugar.
10. A number is first increased by 20% and then decreased by 20%. What is the net increase or decrease percent?
11. 'A' scored 12 marks, while B scored 10 marks, in the first terminal examination. If in the second terminal examination (with same total number of marks) 'A' scored 14 marks and 'B' scored 12 marks, which student showed more improvement?
12. 30,000 students appeared in a contest. Of them 40% were girls and the remaining boys. If 10% boys and 12% girls won the contest with prizes, find the percentage of students who won prizes.
13. Sunil earns 10% more than Shailesh and Shailesh earns 20% more than Swami. If Swami earns ₹ 3200 less than Sunil, find the earnings of each.

8.5 APPLICATION OF PERCENTAGE

In our day to day life, we come across a number of situations wherein we use the concept of percent. In the following section, we discuss the application of percentage in different fields, like problems in profit and loss, discount, simple interest, compound interest, rate of growth and depreciation.

8.5.1 Profit and Loss

Let us recall the terms and formulae related to profit and loss.

Cost Price (C.P.): The Price at which an article is purchased, is called its cost price.

Selling Price (S.P.): The Price at which an article is sold, is called its selling price.

Profit (Gain): When S.P. > C.P., then there is profit, and

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

Loss: When C.P. > S.P., then there is loss, and

$$\text{Loss} = \text{C.P.} - \text{S.P.}$$

Formulae $\text{Profit \%} = \left(\frac{\text{Profit}}{\text{C.P.}} \times 100\right)\%$, $\text{Loss\%} = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$

$$\text{S.P.} = \frac{(\text{C.P.}) \times (100 + \text{Profit\%})}{100} = \frac{(\text{C.P.})(100 - \text{Loss\%})}{100}$$

$$\text{C.P.} = \frac{\text{S.P.} \times 100}{(100 + \text{Profit\%})} = \frac{(\text{S.P.}) \times 100}{(100 - \text{Loss\%})}$$

Note: Gain % or loss % is always calculated on C.P.

Let us take some examples to illustrate the applications of these formulae in solving problems related to profit and loss:

Example 8.16: A shopkeeper buys an article for Rs. 360 and sells it for Rs. 270. Find his gain or loss percent.

Solution: Here C.P. = Rs. 360, and S.P. = Rs. 270

Since C.P. > S.P., $\therefore$ there is a loss.

Loss = C.P. – S.P. = Rs (360 – 270) = Rs. 90

$$\text{Loss \%} = \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$$

$$= \frac{90}{360} \times 100 = 25\%$$

Example 8.17: Sudha purchased a house for ₹ 4,52,000 and spent ₹ 28,000 on its repairs. She had to sell it for ₹ 4,92,000. Find her gain or loss percent.

Solution: Here C.P. = Cost price + Overhead charges

= ₹ (452000 + 28000) = ₹ 4,80,000

S.P. = ₹ 4,92,000

Since, S.P. > C.P., $\therefore$ Gain = ₹ (492000 – 480000) = ₹ 12000

$$\text{Gain \%} = \frac{12000 \times 100}{480000} = \frac{5}{2}\% = 2.5\%$$

Example 8.18: By selling a book for ₹ 258, a publisher gains 20%. For how much should he sell it to gain 30%?

Solution: S.P. = Rs. 258

Profit = 20%

$$\text{C.P.} = \frac{\text{S.P.} \times 100}{100 + \text{Profit}\%} = ₹\ \frac{258 \times 100}{120} = ₹\ 215$$

Now, if Profit = 30% and C.P. = Rs. 215, then,

$$\text{S.P.} = \frac{\text{C.P.} \times (100 + \text{Profit}\%)}{100} = ₹\ \frac{215 \times 130}{100} = ₹\ 279.50$$

Example 8.19: A man bought oranges at 25 for ₹ 100 and sold them at 20 for ₹ 100. Find his gain or loss percent.

Solution: C.P. of 25 oranges = ₹ 100

$$\therefore \text{C.P. of 1 orange} = ₹\ \frac{100}{25} = ₹\ 4$$

$$\text{and S.P. of 1 orange} = ₹\ \frac{100}{20} = ₹\ 5$$

$$\therefore \text{Profit on 1 orange} = ₹\ (5 - 4) = ₹\ 1$$

$$\text{Profit \%} = \frac{1}{4} \times 100 = 25\%$$

Example 8.20: A man sold two horses for ₹ 29700 each. On one he lost 10% while he gained 10% on the other. Find his total gain or loss percent in the transaction.

Solution: S.P. of first horse = ₹ 29700

Loss = 10%

$$\therefore \text{C.P.} = ₹\ \frac{29700 \times 100}{90} = ₹\ 33{,}000$$

S.P. of 2nd horse = ₹ 29700,

Profit = 10%

$$\text{C.P.} = ₹\ \frac{29700 \times 100}{110} = ₹\ 27{,}000$$

Total CP = ₹ (33000 + 27000) = ₹ 60,000

Total SP = ₹ (2 × 29700) = ₹ 59400

Net Loss = ₹ (60000 – 59400) = ₹ 600

$$\text{Loss } \% = \frac{600}{60000} \times 100 = 1\%$$

Example 8.21: The cost price of 15 articles is equal to the selling price of 12 articles. Find the gain percent.

Solution: Let the C.P. of 15 articles be ₹ 15

then S.P. of 12 articles = ₹ 15

$$\text{S.P. of 15 articles} = ₹\ \frac{15}{12} \times 15 = ₹\ \frac{75}{4}$$

$$\text{Gain} = ₹\left(\frac{75}{4} - 15\right) = ₹\ \frac{15}{4}$$

$$\text{Gain } \% = \frac{15/4}{15} \times 100 = 25\%$$

Example 8.22: A watch was sold at a profit of 12%. Had it been sold for ₹ 33 more, the profit would have been 14%. Find the cost price of the watch.

Solution: Let the cost price of the watch be ₹ x

$$\therefore \text{S.P.} = \frac{x \times 112}{100} = \frac{112x}{100}$$

$$\text{If the watch is sold for Rs. 33 more then S.P.} = \left(\frac{112x}{100} + 33\right)$$

New profit = 14%

$$\therefore \text{C.P.} = x = \frac{\left(\frac{112x}{100} + 33\right) \times 100}{114}$$

or $114x = 112x + 3300$ or $2x = 3300$

$x = 1650$ $\therefore$ C.P. = ₹ 1650

CHECK YOUR PROGRESS 8.3

1. A shopkeeper bought an almirah from a wholesale dealer for ₹ 4500 and sold it for ₹ 6000. Find his profit or loss percent.

2. A retailer buys a cooler for ₹ 3800 but had to spend ₹ 200 on its transport and repair. If he sells the cooler for ₹ 4400, determine, his profit percent.
3. A vendor buys lemons at the rate of 5 for ₹ 7 and sells them at ₹ 1.75 per lemon. Find his gain percent.
4. A man purchased a certain number of oranges at the rate of 2 for ₹ 5 and sold them at the rate of 3 for ₹ 8. In the process, he gained ₹ 20. Find the number of oranges he bought.
5. By selling a bi-cycle for ₹ 2024, the shopkeeper loses 12%. If he wishes to make a gain of 12% what should be the selling price of the bi-cycle?
6. By selling 45 oranges for ₹ 160, a woman loses 20%. How many oranges should she sell for ₹ 112 to gain 20% on the whole?
7. A dealer sold two machines at ₹ 2400 each. On selling one machine, he gained 20% and on selling the other, he lost 20%. Find the dealer's net gain or loss percent.
8. Harish bought a table for ₹ 960 and sold it to Raman at a profit of 5%. Raman sold it to Mukul at a profit of 10%. Find the money paid by Mukul for the table.
9. A man buys bananas at 6 for ₹ 5 and an equal number at ₹ 15 per dozen. He mixes the two lots and sells them at ₹ 14 per dozen. Find his gain or loss percent, in the transaction.
10. If the selling price of 20 articles is equal to the cost price of 23 articles, find the loss or gain percent.

8.5.2 Discount

You must have seen advertisements of the following types, especially during the festival season.

SALE	}	DIWALI BONANZA
discount upto 50%		20% discount on all items.

A discount is a reduction in the marked (or list) price of an article. "20% discount"means a reduction of 20% in the marked price of an article. For example, if the marked price of an article is ₹ 100, it is sold for ₹ 80, i.e. ₹ 20 less than the marked price. Let us define the terms, we shall use:

Marked Price (or List price): The marked price (M.P.) of an article is the price at which the article is listed for sale. Since this price is written (marked) on the article, so it is called the marked price.

Discount: The discount is the reduction from the marked price of the article.

Net selling price (S.P.): In case of discount selling, the price of the article obtained by subtracting discount from the marked price is called the Net Selling price or Selling price (S.P.). Let us take the following examples, to illustrate:

Example 8.23: A coat is marked at ₹ 2400. Find its selling price if a discount of 12% is offered.

Solution: Here, Marked Price (M.P.) of the coat = ₹ 2400

Discount = 12%

Net selling price (S.P.) = M.P. – Discount

= ₹ 2400 – 12% of ₹ 2400

$= ₹\ 2400 - ₹\left(\frac{12}{100}\times 2400\right)$

= ₹ (2400 – 288)

= ₹ 2112

Thus, the net selling price of coat is ₹ 2112.

Example 8.24: A machine listed at ₹ 8400 is available for ₹ 6300. Find the rate of discount offered.

Solution: Here, Marked Price (M.P.) = ₹ 8400

Net selling price (S.P.) = ₹ 6300

Discount offered = ₹ (8400 – 6300)

= ₹ 2100

$$\text{Discount \%} = \frac{2100}{8400}\times 100\% = 25\%$$

Note: Discount is always calculated on Marked Price.

Example 8.25: A wholesaler's list price of a fan is ₹ 1250 and is available to a retailer at a discount of 20%. For how much should the retailer sell it, to earn a profit of 15%.

Solution: M.P. = ₹ 1250

Discount = 20% of ₹ 1250

$= ₹\ \frac{20}{100}\times 1250 = ₹\ 250$

∴ Cost Price of the retailer = ₹ (1250 – 250)

= ₹ 1000

Profit = 15%

$$\therefore \text{S.P.} = \frac{\text{C.P.}(100+\text{Profit\%})}{100} = ₹\ \frac{1000\times 115}{100}$$

= ₹ 1150

Example 8.26: A shopkeeper marks his goods 25% more than their cost price and allows a discount of 10%. Find his gain or loss percent.

Solution: Let the C.P. of an article = ₹ 100

∴ Marked Price (M.P.) = ₹ 100 + 25% of ₹ 100

= ₹ 125

Discount offered = 10%

∴ Net selling Price = ₹ 125 – 10% of ₹ 125

$= ₹\ 125 - ₹\left(\frac{10}{100}\times 125\right)$

= ₹ (125 – 12.50) = ₹ 112.50

∴ Gain = ₹ (112.50 – 100) = ₹ 12.50

$\text{Gain \%} = \frac{12.50}{100}\times 100 = 12.5\%$

Example 8.27: An article listed at ₹ 5400 is offered at a discount of 15%. Due to festival season, the shopkeeper allows a further discount of 5%. Find the selling price of the article.

Solution: M.P. = ₹ 5400, Discount = 15%

∴ SP = ₹ 5400 – 15% of ₹ 5400

$= ₹\ 5400 - ₹\ \frac{15}{100}\times 5400$

= ₹ (5400 – 810) = ₹ 4590

Festival discount = 5%

∴ Net selling Price = ₹ 4590 – 5% of ₹ 4590

$= ₹\ 4590 - ₹\ \frac{5}{100}\times 4590$

= ₹ (4590 – 229.50)

= ₹ 4360.50

∴ Net selling price of article = ₹ 4360.50.

Example 8.28: A retailer buys books from a wholesaler at the rate of ₹ 300 per book and marked them at ₹ 400 each. He allows some discount and gets a profit of 30% on the cost price. What percent discount does he allow to his customers?

Solution: C.P. of one book = ₹ 300

M.P. = ₹ 400

Profit = 30%

$$\therefore \text{S.P.} = \frac{\text{C.P.}(100 + \text{Profit}\%)}{100} = ₹\ \frac{300 \times 130}{100} = ₹\ 390$$

$\therefore$ Discount offered = ₹ (400 – 390) = ₹ 10

$$\text{Discount } \% = \frac{10}{400} \times 100 = 2.5\%$$

CHECK YOUR PROGRESS 8.4

1. A shirt with marked price ₹ 375/- is sold at a discount of 15%. Find its net selling price.
2. A pair of socks marked at ₹ 60 is being offered for ₹ 48. Find the discount percent being offered.
3. A washing machine is sold at a discount of 10% on its marked price. A further discount of 5% is offered for cash payment. Find the selling price of the washing machine if its marked price is ₹ 18000.
4. A man pays ₹ 2100 for a machine listed at ₹ 2800. Find the rate of discount offered.
5. The list price of a table fan is ₹ 840 and it is available to a retailer at a discount of 25%. For how much should the retailer sell it to earn a profit of 15%.
6. A shopkeeper marks his goods 50% more than their cost price and allows a discount of 40%, find his gain or loss percent.
7. A dealer buys a table listed at ₹ 2500 and gets a discount of 28%. He spends ₹ 100 on transportation and sells it at a profit of 15%. Find the selling price of the table.
8. A retailer buys shirts from a manufacturer at the rate of ₹ 175 per shirt and marked them at ₹ 250 each. He allows some discount and earns a profit of 28% on the cost price. What percent discount does he allow to his customers?

8.5.3 Simple Interest

When a person has to borrow some money as a loan from his friends. relatives, bank etc. he promises to return it after a specified time period along with some extra money for using the money of the lender.

The money borrowed is called the **Principal**, usually denoted by P, and the extra money paid is called the **Interest**, usually denoted by I.

The total money paid back, that is, the sum of Principal and the Interest is called the **Amount**, and is usually denoted by A.

Thus, $A = P + I$

The interest is mostly expresed as a rate percent per year (per annum).

Interest depends on, how much money (P) has been borrowed and the duraton of time (T) for which it is used. Interest is calculated according to a mutually agreed rate percent, per annum (R). [i.e. $R = r\% = \frac{r}{100}$]

Thus, Interest = (Principal) × (Rate % per annum) × time

$\therefore I = P \times R \times T$

Interest calculated as above, is called simple interest. Let us take some examples, involving simple interest.

Example 8.29: Find the simple interest in each of the following cases

	P	R	T
(a)	₹ 8000	5%	2 yrs
(b)	₹ 20,000	15%	$1\frac{1}{2}$ yrs

Solution: (a) $I = P.\ R.\ T.$

$$= ₹\left[8000 \times \frac{5}{100} \times 2\right] = ₹\ 800$$

(b) $$I = ₹\left[20000 \times \frac{15}{100} \times \frac{3}{2}\right] = ₹\ 4500$$

Example 8.30: Find at what rate of simple interest per annum will ₹ 5000 amount to ₹ 6050 in 3 years.

Solution: Here A = ₹ 6050, P = ₹ 5000, T = 3 yrs

$\therefore I = ₹\ (6050 - 5000) = ₹\ 1050$

$I = P \times R \times T$ or $r\% = \frac{I}{P \times T}$ $\therefore r = \frac{I \times 100}{P \times T}$

$$r = \frac{1050 \times 100}{5000 \times 3} = 7 \qquad \therefore R = 7\%$$

Example 8.31: A sum amounts to ₹ 4875 at $12\frac{1}{2}$ % simple interest per annum after 4 years. Find the sum.

Solution: Here A = ₹ 4875, $R = 12\frac{1}{2}\% = \frac{25}{2}\%$, T = 4 yrs

$I = P \times R \times T$

$I = ₹ \left(P \times \frac{25}{200} \times 4\right) = ₹ \frac{P}{2}$

$\therefore A = ₹ \left(P + \frac{P}{2}\right) = ₹ \frac{3P}{2}$

Thus, $\frac{3P}{2} = ₹ 4875$ or $3P = ₹ 9750$ or $P = ₹ 3250$

Example 8.32: In how many years will a sum of ₹ 2000 yield an interest (Simple) of ₹ 560 at the rate of 14% per annum?

Solution: Here P = ₹ 2000, I = ₹ 560 R = 14%

$I = P \times R \times T$ or $560 = 2000 \times \frac{14}{100} \times T$

$\therefore T = \frac{560 \times 100}{2000 \times 14} = 2$ years

Thus, in 2 years, a sum of ₹ 2000 will yield an interest of ₹ 560 at 14% per annum.

Example 8.33: A certain sum of money at simple interest amounts to ₹ 1300 in 4 years and to ₹ 1525 in 7 years. Find the sum and rate percent.

Solution: Here $1300 = \frac{P \times R \times 4}{100} + P$ ...(i)

and $1525 = \frac{P \times R \times 7}{100} + P$ ...(ii)

Subtracting (i) from (ii) $225 = \frac{P \times R \times 3}{100}$ or $\frac{P \times R}{100} = 75$

Putting in (i) we get

$1300 = 75 \times 4 + P$ or $P = ₹ (1300 - 300) = ₹ 1000$

Again, we have $\frac{P \times R}{100} = 75$ or $R = \frac{75 \times 100}{P} = \frac{75 \times 100}{1000} = 7.5\%$

$\therefore$ Principal = ₹ 1000 and rate = 7.5%

Alternatively:

Amount after 4 years = ₹ 1300

Amount after 7 years = ₹ 1525

$\therefore$ Interest for 3 years = ₹ [1525 – 1300] = ₹ 225

$\therefore$ Interest for 1 year = ₹ $\frac{225}{3}$ = ₹ 75

$\therefore$ 1300 = P + Interest for 4 yrs = P + 4 × 75 or P = ₹ (1300 – 300) = ₹ 1000

$$R = \frac{75 \times 100}{1000 \times 1} = 7.5\%$$

Example 8.34: A certain sum of money doubles itself in 10 years. In how many years will it become $2\frac{1}{2}$ times at the same rate of simple interest.

Solution: Let P = ₹ 100, T = 10 yrs, A = ₹ 200, $\therefore$ I = ₹ 100

$$\therefore 100 = \frac{100 \times R \times 10}{100} \text{ or } R = 10\%$$

Now P = ₹ 100, R = 10% and A = ₹ 250 $\therefore$ I = ₹ 150

$$\therefore 150 = 100 \times \frac{10}{100} \times T \text{ or } T = 15 \text{ yrs}$$

Thus, in 15 yrs, the sum will become $2\frac{1}{2}$ times.

Example 8.35: Out of ₹ 70,000 to invest for one year, a man invests ₹ 30,000 at 4% and ₹ 20,000 at 3% per annum simple interest. At what rate percent, should he lend the remaining money, so that he gets 5% interest on the total amount he has?

Solution: Interest on total amount at 5% for one year

$$= ₹\ 70{,}000 \times \frac{5}{100} \times 1 = ₹\ 3500$$

Interest on ₹ 30,000 at 4% for 1 year $= ₹\ 30000 \times \frac{4}{100} \times 1$

$= ₹\ 1200$

Interest on ₹ 20,000 at 3% for 1 year $= ₹\ 20000 \times \frac{3}{100} \times 1$

$= ₹\ 600$

$\therefore$ Interest on remaining ₹ 20,000 for 1 yr $= ₹\ [3500 - 1200 - 600]$

$= ₹\ 1700$

$\therefore 1700 = 20000 \times \frac{R}{100} \times 1$ or $R = \frac{1700 \times 100}{20000} = 8.5\%$

$\therefore$ The remaining amount should be invested at 8.5% per annum.

CHECK YOUR PROGRESS 8.5

1. Rama borrowed ₹ 14000 from her friend at 8% per annum simple interest. She returned the money after 2 years. How much did she pay back altogether?
2. Ramesh deposited ₹ 15600 in a financial company, which pays simple interest at 8% per annum. Find the interest he will receive at the end of 3 years.
3. Naveen lent ₹ 25000 to his two friends. He gave ₹ 10,000 at 10% per annum to one of his friend and the remaining to other at 12% per annum. How much interest did he receive after 2 years.
4. Shalini deposited ₹ 29000 in a finance company for 3 years and received ₹ 38570 in all. What was the rate of simple interest per annum?
5. In how much time will simple interest on a sum of money be $\frac{2}{5}$ th of the sum, at the rate of 10% per annum.
6. At what rate of interest will simple interest be half the principal in 5 years.
7. A sum of money amounts to ₹ 1265 in 3 years and to ₹ 1430 in 6 years, at simple interest. Find the sum and the rate percent.
8. Out of ₹ 75000 to invest for one year, a man invested ₹ 30000 at 5% per annum and ₹ 24000 at 4% per annum. At what percent per annum, should he invest the remaining money to get 6% interest on the whole money.
9. A certain sum of money doubles itself in 8 years. In how much time will it become 4 times of itself at the same rate of interest?
10. In which case, is the interest earned more:
    (a) ₹ 5000 deposited for 5 years at 4% per annum, or
    (b) ₹ 4000 deposited for 6 years at 5% per annum?

8.5.4 Compound Interest

In the previous section, you have studied about simple interest. When the interest is calculated on the Principal for the entire period of loan, the interest is called simple interest and is given by

$$I = P \times R \times T$$

But if this interest is due (not paid) after the decided time period, then it becomes a part of the principal and so is added to the principal for the next time preiod, and the interest is calculated for the next time period on this new principal. Interest calculated, this way is called compound interest.

The time period after which the interest is added to the principal for the next time period is called the **Conversion Period.**

The conversion period may be one year, six months or three months and the interest is said to compounded, annually, semi-annually or quarterly, respectively. Let us take an example:

Example 8.36: Find the compound interest on a sum of Rs. 2000, for two years when the interest is compounded annually at 10% per annum.

Solution: Here P = ₹ 2000 and R = 10%

∴ Interest for the first conversion time period (i.e. first year)

$$= ₹\ 2000 \times \frac{10}{100} \times 1 = ₹\ 200$$

∴ Principal for the second year (or 2nd conversion period)

$$= ₹\ (2000 + 200) = ₹\ 2200$$

∴ Interest for the 2nd time period $= ₹\ 2200 \times \frac{10}{100} \times 1 = ₹\ 220$

∴ Amount payable at the end of two years $= ₹\ (2200 + 220)$

$$= ₹\ 2420$$

∴ Total interest paid at the end of two years $= ₹\ (2420 - 2000)$

$$= ₹\ 420$$

or [₹ (200 + 220) = ₹ 420]

∴ Compound interest = ₹ 420

Thus, for calculating the compound interest, the interest due after every coversion period is added to the principal and then interest is calculated for the next period.

8.5.4.1 Formula for Compound Interest

Let a sum P be borrowed for n years at the rate of r% per annum, then

$$\text{Interest for the first year} = P \times \frac{r}{100} \times 1 = \frac{Pr}{100}$$

Amount after one year = Principal for 2nd year = $P + \frac{Pr}{100}$

$= P\left(1+\frac{r}{100}\right)$

Interest for 2nd year = $P\left(1+\frac{r}{100}\right) \times \frac{r}{100} \times 1 = \frac{Pr}{100}\left(1+\frac{r}{100}\right)$

Amount after 2 years = $P\left(1+\frac{r}{100}\right)+\frac{Pr}{100}\left(1+\frac{r}{100}\right) = P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$

$= P\left(1+\frac{r}{100}\right)^2$

Similarly, amount after 3 years = $P\left(1+\frac{r}{100}\right)^3$ and so on.

Amount after n years = $P\left(1+\frac{r}{100}\right)^n$

Thus, if A represents the amount and R represents r% or $\frac{r}{100}$, then

$A = P(1 + R)^n = P\left(1+\frac{r}{100}\right)^n$

and compound interest $= A - P = P(1 + R)^n - P$

$= P[(1 + R)^n - 1]$ or $P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

Note: Simple interest and compound interest are equal for first year (first conversion period)

Example 8.37: Calculate the compound interest on ₹ 20,000 for 3 years at 5% per annum, when the interest is compounded annually.

Solution: Here P = ₹ 20,000, R = 5% and n = 3

$\therefore$ CI $= P[(1 + R)^n - 1]$

$= ₹\ 20000\left[\left(1+\frac{5}{100}\right)^3 - 1\right]$

$= ₹\left[\left(\frac{21}{20}\right)^3 - 1\right] = ₹\ 20000 \times \left[\frac{9261-8000}{8000}\right]$

$= ₹\ 3152.50$

Example 8.38: Calculate the compound interest on ₹ 20,000 for $1\frac{1}{2}$ years at the rate of 10% per annum, when the interest is compounded semi-annually.

Solution: Here P = ₹ 20,000, R = 10% per annum

= 5% per half year

and n = $1\frac{1}{2}$ yrs = 3 half years

$$\therefore \text{CI} = P[(1+R)^n - 1] = ₹\ 20{,}000\left[\left(1+\frac{5}{100}\right)^3 - 1\right]$$

$$= ₹\ 20{,}000 \times \left[\frac{9261}{8000} - 1\right] = ₹\ 3152.50$$

Example 8.39: Calculate the compound interest on ₹ 20,000 for 9 months at the rate of 4% per annum, when the interest is compounded quarterly.

Solution: Here P = ₹ 20,000, R = 4% per annum

= 1% per quarter of year

and n = 3/4 yrs = 3 quarters

$$\therefore \text{CI} = P[(1+R)^n - 1] = ₹\ 20{,}000\left[\left(1+\frac{1}{100}\right)^3 - 1\right]$$

$$= ₹\ 20{,}000 \times \left[\left(\frac{101}{100}\right)^3 - 1\right] = ₹\ \frac{20000 \times 30301}{100 \times 100 \times 100}$$

$$= ₹\ 606.02$$

Example 8.40: calculate the amount and compound interest on ₹ 12000 for $1\frac{1}{2}$ years at the rate of 10% per annum compounded annually.

Solution: Here P = ₹ 12000, R = 10% and n = $1\frac{1}{2}$ years

Since interest is compounded, annually, so, amount at the end of 1 year is given by

$$A = P\left(1+\frac{R}{100}\right)^1 = ₹\ 12000\times\left(1+\frac{10}{100}\right)^1$$

$$= ₹\ 12000\times\frac{11}{10} = ₹\ 13200$$

$\therefore$ Principal for next 6 months = ₹ 13200

and Rate $R = \frac{10}{2}\% = 5\%$

$$\therefore\ A = ₹\ 13200\left(1+\frac{5}{100}\right)^1 = ₹\ 13200\times\frac{21}{20}$$

$$= ₹\ 13860$$

$\therefore$ Amount after $1\frac{1}{2}$ years = ₹ 13860

Compound interest = ₹ [13860 – 12000]

= ₹ 1860

Note: We can calculate the amount for $1\frac{1}{2}$ yrs as

$$A = ₹\ 12000\left(1+\frac{10}{100}\right)^1\left(1+\frac{5}{100}\right)^1$$

Example 8.41: At what rate percent per annum, will a sum of ₹ 15,625 become ₹ 17576 in three years, when the interest is compounded annually?

Solution: Here A = ₹ 17576, P = ₹ 15,625 and n = 3

Let R = r% per annum

$$\therefore 17576 = 15625\left(1+\frac{r}{100}\right)^3$$

$$\therefore\left(1+\frac{r}{100}\right)^3 = \frac{17576}{15625} = \left(\frac{26}{25}\right)^3$$

$$\therefore\left(1+\frac{r}{100}\right) = \frac{26}{25} \text{ or } \frac{r}{100} = \frac{26}{25} - 1 = \frac{1}{25}$$

or $r = \frac{100}{25} = 4$

$\therefore$ Rate = 4% per annum.

Example 8.42: In how much time will a sum of ₹ 8000 amount to ₹ 9261 at 10% per annum, compounded semi-annually?

Solution: Here A= ₹ 9261, P = ₹ 8000 and n = x semi yrs

R = 10% per annum = 5% semi annually

$\therefore 9261 = 8000\left(1+\frac{5}{100}\right)^{x}$

or $\frac{9261}{8000} = \left(\frac{21}{20}\right)^{x}$ or $\left(\frac{21}{20}\right)^{3} = \left(\frac{21}{20}\right)^{x}$ $\therefore x = 3$

$\therefore$ Time = 3 half years = $1\frac{1}{2}$ years

Example 8.43: Find the difference between simple interest and compound interest for $1\frac{1}{2}$ years at 4% per annum, for a sum of ₹ 24000, when the interest is compounded semi-annually..

Solution: Here P = ₹ 24000, R = 4% per annum

$T = \frac{3}{2}$ years R = 2% per semi-annually

$n = 1\frac{1}{2}$ years $= \frac{3}{2}$ years = 3 semi years

Simple Interest = P × R × T $= ₹\ 24000 \times \frac{4}{100} \times \frac{3}{2}$

$= ₹\ 1440.$

For compound interest, $A = P\left[\left(1+\frac{R}{100}\right)^{n}\right]$

$A = ₹\ 24000\left[\left(1+\frac{2}{100}\right)^{3}\right]$

$$A = ₹\ 24000\left[\left(\frac{51}{50}\right)^3\right] = ₹\ 24000\left[\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\right]$$

$$= ₹\ \frac{24\times51\times51\times51}{125} = ₹\ 25468.99 \text{ or } ₹\ 25469$$

$\therefore$ CI = ₹ [25469 – 24000] = ₹ 1469

Difference = CI – SI = ₹ [1469 – 1440]

= ₹ 29

Example 8.44: A sum of money is invested at compound interest for $1\frac{1}{2}$ year at 4% compounded annually. If the interests were compounded semi-annually, it would have fetched ₹ 20.40 more than in the previous case. Find the sum.

Solution: Let the sum be ₹ x.

Here R = 4% annually, or 2% semi-annually

$$T = 1\frac{1}{2} \text{ yrs or 3 semi years}$$

In first case

$$A = ₹\ x\left[1+\frac{4}{100}\right]^1\left[1+\frac{2}{100}\right]^1$$

$$= ₹\ x\left(\frac{26}{25}\right)\left(\frac{51}{50}\right) = ₹\ \frac{1326\,x}{1250}$$

In 2nd case

$$A = ₹\ x\left(1+\frac{2}{100}\right)^3 = ₹\ x\left(\frac{51}{50}\right)^3$$

$$= ₹\ \frac{132651}{125000}$$

$$\therefore \text{ Difference} = ₹\ \left[\frac{132651}{125000}x - \frac{1326}{1250}x\right]$$

$$= ₹\ \frac{51x}{125000}$$

$$\therefore \frac{51x}{125000} = \frac{2040}{100} \text{ or } x = ₹\ \frac{2040}{100} \times \frac{125000}{51} = ₹\ 5000$$

$$\therefore \text{ Sum} = ₹\ 50{,}000$$

CHECK YOUR PROGRESS 8.6

1. Calculate the compound interest on ₹ 15625 for 3 years at 4% per annum, compounded annually.
2. Calculate the compound interest on ₹ 15625 for $1\frac{1}{2}$ years at 8% per annum, compounded semi-annually.
3. Calculate the compound interest on ₹ 16000 for 9 months at 20% per annum, compounded quarterly
4. Find the sum of money which will amount to ₹ 27783 in 3 years at 5% per annum, the interest being compounded annually.
5. Find the difference between simple interest and compound interest for 3 years at 10% per annum, when the interest is compounded annually on ₹ 30,000.
6. The difference between simple interest and compound interest for a certain sum of money at 8% per annum for $1\frac{1}{2}$ years, when the interest is compounded half-yearly is ₹ 228. Find the sum.
7. A sum of money is invested at compound interest for 9 months at 20% per annum, when the interest is compounded half yearly. If the interest were compounded quarterly, it would have fetched ₹ 210 more than in the previous case. Find the sum.
8. A sum of ₹ 15625 amounts to ₹ 17576 at 8% per annum, compounded semi-annually. Find the time.
9. Find the rate at which ₹ 4000 will give ₹ 630.50 as compound interest in 9 months, interest being compounded quarterly.
10. A sum of money becomes ₹ 17640 in two years and ₹ 18522 in 3 years at the same rate of interest, compounded annually. Find the sum and the rate of interest per annum.

8.5.5 Rate of Growth and Depreciation

In our daily life, we come across the terms like growth of population, plants, viruses etc and depreciation in the value of articles like machinery, crops, motor cycles etc.

The problems of growth and depreciation can be solved using the formula of compound interest derived in the previous section.

If V_o is the value of an article in the beginning and V_n is its value after 'n' conversion periods and the rate of growth/depreciation for the period be denoted by r%, then we can write

$$V_n = V_o\left(1+\frac{r}{100}\right)^n \text{ in case of growth, and}$$

$$V_n = V_o\left(1-\frac{r}{100}\right)^n \text{ in case of depreciation.}$$

If the rate of growth/depreciation varies for each conversion period, then

$$V_n = V_o\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\cdots \text{ for growth, and}$$

$$V_n = V_o\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1-\frac{r_3}{100}\right)\cdots \text{ for depreciation.}$$

Let us take some examples to illustrate the above concepts.

Example 8.45: The population of a city is 9765625. What will be its population after 3 years, if the rate of growth of population is 4% per year?

Solution: Here $V_o = 9765625$, $r = 4\%$ and $n = 3$

$$\therefore\ V_3 = 9765625\left[1+\frac{4}{100}\right]^3$$

$$= 9765625\times\left(\frac{26}{25}\right)^3$$

$$= 10985000.$$

Hence, the population of that city after 3 years will be = 10985000.

Example 8.46: The cost of a car was ₹ 3,50,000 in January 2005. If the rate of depreciation is 15% for the first year and 10% for the subsequent years, find its value after 3 years.

Solution: Here $V_o = ₹\ 3{,}50{,}000$

$r_1 = 15\%$, $r_2 = 10\%$ and $r_3 = 10\%$

$$\therefore V_3 = V_o\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1-\frac{r_3}{100}\right)$$

$$= ₹\ 350000\left(1-\frac{15}{100}\right)\left(1-\frac{10}{100}\right)\left(1-\frac{10}{100}\right)$$

$$= ₹\ 350000\times\frac{17}{20}\times\frac{9}{10}\times\frac{9}{10} = ₹\ 2,40,975/-$$

$\therefore$ The value of car after 3 years = ₹ 240975.

Example 8.47: A plant gains its height at the rate of 2% per month of what was its height in the beginning of the month. If its height was 1.2 m in the beginning of January 2008, find its height in the beginning of April 2008, correct upto 3 places of decimal.

Solution: Here $V_o = 1.2$ m, $r = 2\%$, $n = 3$

$$\therefore V_3 = V_o\left(1+\frac{r}{100}\right)^n$$

$$= 1.2\left(1+\frac{2}{100}\right)^3 = 1.2\left(\frac{51}{50}\right)^3 = 1.2734 \text{ m}$$

$$= 1.273 \text{ m}$$

Hence, height of plant in the beginning of April = 1.273 m.

Example 8.48: The virus of a culture decreases at the rate of 5% per hour due to a medicine. If the virus count in the culture at 11.00 AM was 2.3×10^7, find the virus count at 1.00 PM on the same day.

Solution: $V_o = 2.3 \times 10^7$, $r = 5\%$, $n = 2$

$$V_2 = 2.3\times10^7\left(1-\frac{5}{100}\right)^2 = 2.3\times10^7\times(0.95)^2$$

$$= 2.076 \times 10^7$$

Hence, the virus count at 1.00 PM is 2.076×10^7.

CHECK YOUR PROGRESS 8.7

1. The population of a town is 281250. What will be its population after 3 years, if the rate of growth of population is 4% per year?
2. The cost of a car was ₹ 4,36,000 in January 2005. Its value depreciates at the rate of 15% in the first year and then at the rate of 10% in the subsequent years. Find the value of the car in January 2008.

3. The cost of machinery is ₹ 360000 today. In the first year the value depreciates by 12% and subsequently, the value depreciates by 8% each year. By how much, the value of machinery has depreciated at the end of 3 years?
4. The application of manure increases the output of a crop by 10% in the first year, 5% in the second year and 4% in the third year. If the production of crop in the year 2005 was 3.5 tons per hectare, find the production of crop per hectare in 2008.
5. The virus of a culture decreases at the rate of 4% per hour due to a medicine. If the virus count in the culture at 9.00 AM was 3.5×10^8, find the virus count at 11.00 AM on the same day.
6. Three years back, the population of a village was 50000. After that, in the first year, the rate of growth of population was 5%. In the second year, due to some epidemic, the population decreased by 10% and in the third year, the population growth rate was noticed as 4%. Find the population of the town now.

LET US SUM UP

- Percent means 'per hundred'.
- Percents can be written as fractions as well as decimals and vice-versa.
- To write a percent as a fraction, we drop the % sign and divide the number by 100.
- To write a fraction as a percent, we multiply the fraction by 100, simplify it and suffix the % sign.
- To determine the specific percent of a number or quantity, we change the percent to a fraction or a decimal and then multiply.
- When the selling price is more than the cost price of the goods, there is a profit (or gain).
- When the selling price is less than the cost price of the goods, there is a loss.

$$\text{Profit (Gain)} = \text{S.P.} - \text{C.P.} \quad ; \quad \text{Loss} = \text{C.P.} - \text{S.P.}$$

$$\text{Gain\%} = \frac{\text{Gain}}{\text{C.P.}} \times 100 \quad ; \quad \text{Loss\%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \frac{100 + \text{Gain\%}}{100} \times \text{C.P.} \quad ; \quad \text{S.P.} = \frac{100 - \text{Loss\%}}{100} \times \text{C.P.}$$

- The simple interest (I.) on a principal (P) at the rate of R% for a time T years, is calculated, using the formula

$$\text{I.} = \text{P} \times \text{R} \times \text{T}$$

- Discount is a reduction in the list price of goods.
- Discount is always calculated on the marked price of the goods
- (Marked price – discount), gives the price, which a customer has to pay while buying an article.
- Two or more successive discounts are said to form a discount series.
- A discount series can be reduced to a single discount.
- Sales tax is charged on the sale price of goods.
- An instalment plan enables a person to buy costlier goods.
- In the case of compound interest

  Amount (A) = P (1 + R)^n, where P is the Principal, R = rate% and n = time.
- Compount interest is greater than simple interest, except for the first conversion period.
- If V_o is the value of an article in the beginning and V_n is its value after 'n' conversion periods and 'r' be the rate of growth/depreciation per period, then

$$V_n = V_o\left(1+\frac{r}{100}\right)^n \text{ in case of growth, and}$$

$$V_n = V_o\left(1-\frac{r}{100}\right)^n \text{ in case of depreciation.}$$

- If the rate of growth/depreciation varies for each conversion period, then

$$V_n = V_o\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\cdots \text{ for growth, and}$$

$$V_n = V_o\left(1-\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1-\frac{r_3}{100}\right)\cdots \text{ for depreciation.}$$

TERMINAL EXERCISE

1. Write each of the following as a percent

   (a) $\frac{9}{20}$ (b) $\frac{7}{10}$ (c) 0.34 (d) 0.06

2. Write each of the following as a decimal:

   (a) 36% (b) 410% (c) 2% (d) 0.35%

3. Write each of the following as fraction:

   (a) 0.12% (b) 2.5% (c) 25.5% (d) 255%

4. Find each of the following:

   (a) 23% of 500 (b) 2.5% of 800 (c) 0.4% of 1000 (d) 115% of 400

5. What percent of 700 is 294?

6. By what percent is 60 more than 45?

7. What number increased by 10% of itself is 352?

8. Find the number whose 15% is 270.

9. What number decreased by 7% of itself is 16.74?

10. If three fourth of the students of a class wear glasses, what percent of the students of the class do not wear glasses?

11. There are 20 eggs in a fridge and 6 of them are brown. What percent of eggs are not brown?

12. 44% of the students of a class are girls. If the number of girls is 6 less than the number of boys, how many students are there in the class?

13. During an election, 70% of the population voted. If 70,000 people cast their votes, what is the population of the town?

14. A man donated 5% of his monthly income to a charity and deposited 12% of the rest in a Bank. If he has Rs. 11704 left with him, what is his monthly income?

15. Ratan stores has a sale of ₹ 12000 on Saturday, while Seema stores had a sale of ₹ 15000 on that day. Next day, they had respective sales of ₹ 15000 and ₹ 17500. Which store showed more improvement in Sales?

16. A candidate has to secure 45% marks in aggregate of three papers of 100 marks each to get through. He got 35% marks in the first paper and 50% marks in the second paper. At least how many marks should he get in third paper to pass the examination?

17. The price of sugar rises by 25%. By how much percent should a householder reduce his consumption of sugar, so as not to increase his expenditure on sugar?

18. By selling 90 ball pens for ₹ 160, a person loses 20%. How many ball pens should he sell for Rs. 96, so as to have a gain of 20%?

19. A vendor bought bananas at 6 for 5 rupees and sold them at 4 for 3 rupees. Find his gain or loss percent.

20. A man bought two consignments of eggs, first at ₹ 18 per dozen and an equal number at ₹ 20 per dozen. He sold the mixed egges at ₹ 23.75 per dozen. Find his gain percent.

21. A man sells an article at a gain of 10%. If he had bought it for 10% less and sold it for ₹ 10 more, he would have gained 25%. Find the cost price of the article.

22. A pair of socks is marked at ₹ 80 and is being offered at ₹ 64. Find the discount percent being offered.

23. A dealer buys a table listed at ₹ 1800 and gets a discount of 25%. He spends ₹ 150 on transportation and sells it at a profit of 10%. Find the selling price of the table.

24. A T.V. set was purchased by paying ₹ 18750. If the discount offered by the dealer was 25%, what was the marked price of the TV set?

25. A certain sum of money was deposited for 5 years. Simple interest at the rate of 12% was paid. Calculate the sum deposited if the simple interest received by the depositor is ₹ 1200.

26. Simple intrest on a sum of money is $\frac{1}{3}$rd of the sum itself and the number of years is thrice the rate percent. Find the rate of interest.

27. In what time will ₹ 2700 yield the same interest at 4% per annum as ₹ 2250 in 4 years at 3% per annum?

28. The difference between simple interest on a sum of money for 3 years and for 2 years at 10% per annum is ₹ 300. Find the sum.

29. Find the sum which when invested at 4% per annum for 3 years will becomes ₹ 70304, when the interest is compounded annually.

30. the difference between compound interest and simple interest at 10% per annum in 2 years (compounded annually) is ₹ 50. Find the sum.

31. A sum of money becomes ₹ 18522 in three years and ₹ 19448.10 in 4 years at the same rate of interest, compounded annually. Find the sum and the rate of interest per annum.

32. Find the sum of money which will amount to ₹ 26460 in six months at 20% per annum, when the interest is compounded quarterly.

33. At what rate percent per annum will a sum of ₹ 12000 amount to ₹ 15972 in three years, when the interest is compounded annually?

34. The price of a scooter depreciates at the rate of 20% in the first year, 15% in the second year and 10% afterwards, what will be the value of a scooter now costing ₹ 25000, after 3 years.

35. The population of a village was 20,000, two years ago. It increased by 10% during first year but decreased by 10% in the second year. Find the population at the end of 2 years.

ANSWERS TO CHECK YOUR PROGRESS

8.1

1. (a) 48% (b) 45% (c) $41\frac{2}{3}\%$ (d) 40% (e) 20%
   (f) 30% (g) 36% (h) 126% (i) 288% (j) 98.48%
2. (a) $\frac{53}{100}$ (b) $\frac{17}{20}$ (c) $\frac{27}{160}$ (d) $\frac{137}{4000}$ (e) $\frac{1}{16}$
   (f) $\frac{7}{10}$ (g) $\frac{63}{400}$ (h) $\frac{1}{40000}$ (i) $\frac{947}{2000}$ (j) $\frac{21}{4000}$
3. (a) 97% (b) 73.5% (c) 3% (d) 207% (e) 80%
   (f) 175% (g) 2.5% (h) 325.75% (i) 15.2% (j) 300.15%
4. (a) 0.72 (b) 0.41 (c) 0.04 (d) 1.25 (e) 0.09
   (f) 4.1 (g) 3.5 (h) 1.025 (i) 0.00025 (j) 0.1025
5. 50% 6. 90% 7. 6.25% 8. 47.5% 9. 30%
10. 5%

8.2

1. (a) 200 (b) 564
2. Rs. 2625 3. 175, 100, 125, 100 4. 25%
5. 56.25% 6. 500 7. 36 minutes
8. 6000 9. Rs. 40, Rs. 32 10. 4% decrease
11. B 12. 10.8%
13. Rs. 13200, Rs. 12000, Rs. 10000

8.3

1. $33\frac{1}{3}\%$ profit 2. 10% 3. 25% 4. 120
   5. Rs. 2576 6. 21 7. 4% loss 8. Rs. 1108.80
   9. 12% gain 10. 15% gain

8.4

1. Rs. 318.75 2. 20% 3. Rs. 15390 4. 25%
5. Rs. 724.50 6. 10% loss 7. Rs. 2185 8. 10.4%

8.5

1. Rs. 16240 2. Rs. 3744 3. Rs. 5600 4. 11%

5. 4 years 6. 10% 7. Rs. 1100, 5% 8. $9\frac{5}{7}\%$

9. 24 years 10. b

8.6

1. Rs. 1951 2. Rs. 1951 3. Rs. 2522 4. Rs. 24000

5. Rs. 630 6. Rs. 46875 7. Rs. 80000 8. $1\frac{1}{2}$ years

9. 20% 10. Rs. 1600, 5%

8.7

1. 316368 2. Rs. 300186 3. Rs. 291456

4. 4.2042 tons/hectare 5. 3.2256×10^{8} 6. 49140

ANSWERS TO TERMINAL EXERCISE

1. (a) 45% (b) 70% (c) 34% (d) 6%
2. (a) 0.36 (b) 4.10 (c) 0.02 (d) 0.0035
3. (a) $\frac{3}{2500}$ (b) $\frac{1}{40}$ (c) $\frac{51}{200}$ (d) $\frac{51}{20}$
4. (a) 115 (b) 20 (c) 4 (d) 460
5. 42% 6. 25% 7. 320 8. 1800
9. 18 10. 25% 11. 70% 12. 50
13. 1 Lakh 14. Rs. 14000 15. Ratan Stores 16. 50
17. 20% 18. 36 19. 60% gain 20. 25%
21. Rs. 400 22. 20% 23. Rs. 1650 24. Rs. 25000
25. Rs. 2000 26. $3\frac{1}{3}\%$ 27. $2\frac{1}{2}$ years 28. Rs. 3000
29. Rs. 62500 30. Rs. 5000 31. Rs. 16000, 5% 32. Rs. 24000
33. 10% 34. Rs. 13500 35. 19800