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PERIMETERS AND AREAS OF PLANE FIGURES

You are already familair with a number of plane figures such as rectangle, square, parallelogram, triangle, circle, etc. You also know how to find perimeters and areas of these figures using different formulae. In this lesson, we shall consolidate this knowledge and learn something more about these, particularly the Heron's formula for finding the area of a triangle and formula for finding the area of a sector of a circle.

OBJECTIVES

After studying this lesson, you will be able to

- *find the perimeters and areas of some triangles and quadrilaterals, using formulae learnt earlier;*
- *use Heron's formula for finding the area of a triangle;*
- *find the areas of some rectilinear figures (including rectangular paths) by dividing them into known figures such as triangles, squares, trapeziums, rectangles, etc.;*
- *find the circumference and area of a circle;*
- *find the areas of circular paths;*
- *derive and understand the formulae for perimeter and area of a sector of a circle;*
- *find the perimeter and the area of a sector, using the above formulae;*
- *find the areas of some combinations of figures involving circles, sectors as well as triangles, squares and rectangles;*
- *solve daily life problems based on perimeters and areas of various plane figures.*

EXPECTED BACKGROUND KNOWLEDGE

- Simple closed figures like triangles, quadrilaterals, parallelograms, trapeziums, squares, rectangles, circles and their properties.
- Different units for perimeter and area such as m and m^2, cm and cm^2, mm and mm^2 and so on.
- Conversion of one unit into other units.
- Bigger units for areas such as **acres** and **hectares**.
- Following formulae for perimeters and areas of varioius figures:

  (i) Perimeter of a rectangle = 2 (length + breadth)

  (ii) Area of a rectangle = length × breadth

  (iii) Perimeter of a square = 4 × side

  (iv) Area of a square = $(\text{side})^2$

  (v) Area of a parallelogram = base × corresponding altitude

  (vi) Area of a triangle = $\frac{1}{2}$ base × corresponding altitude

  (vii) Area of a rhombus = $\frac{1}{2}$ product of its diagonals

  (viii) Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × distance between them

  (ix) circumference of a circle = $2\pi \times$ radius

  (x) Area of a circle = $\pi \times (\text{radius})^2$

20.1 PERIMETERS AND AREAS OF SOME SPECIFIC QUADRILATEALS AND TRIANGLES

You already know that the distance covered to walk along a plane closed figure (boundary) is called its **perimeter** and the measure of the region enclosed by the figure is called its **area.** You also know that perimeter or length is measured in linear units, while area is measured in square units. For example, units for perimeter (or length) are m or cm or mm and that for area are m^2 or cm^2 or mm^2 (also written as sq.m or sq.cm or sq.mm).

You are also familiar with the calculations of the perimeters and areas of some specific quadrilaterals (such as squares, rectangles, parallelograms, etc.) and triangles, using certain formulae. Lets us consolidate this knowledge through some examples.

Example 20.1: Find the area of square whose perimeter is 80 m.

Solution: Let the side of the square be a m.

So, perimeter of the square = $4 \times a$ m.

Therefore, $4a = 80$

or $a = \frac{80}{4} = 20$

That is, side of the square = 20 m

Therefore, area of the square = $(20\text{m})^2 = 400\ \text{m}^2$

Example 20.2: Length and breadth of a rectangular field are 23.7 m and 14.5 m respectively. Find:

(i) barbed wire required to fence the field

(ii) area of the field.

Solution: (i) Barbed wire for fencing the field = perimeter of the field

= 2 (length + breadth)

= 2(23.7 + 14.5) m = 76.4 m

(ii) Area of the field = length × breadth

$= 23.7 \times 14.5\ \text{m}^2$

$= 343.65\ \text{m}^2$

Example 20.3: Find the area of a parallelogram of base 12 cm and corresponding altitude 8 cm.

Solution: Area of the parallelogram = base × corresponding altitude

$= 12 \times 8\ \text{cm}^2$

$= 96\ \text{cm}^2$

Example 20.4: The base of a triangular field is three times its corresponding altitude. If the cost of ploughing the field at the rate of ₹ 15 per square metre is ₹ 20250, find the base and the corresponding altitutde of the field.

Solution: Let the corresponding altitude be x m.

Therefore, base = $3x$ m.

So, area of the field = $\frac{1}{2}$ base × corresponding altitude

$$= \frac{1}{2}\, 3x \times x\ \text{m}^2 = \frac{3x^2}{2}\ \text{m}^2 \qquad ....(1)$$

Also, cost of ploughing the field at ₹ 15 per m^2 = ₹ 20250

Therefore, area of the field = $\dfrac{20250}{15}$ m^2

$$= 1350 \text{ m}^2 \qquad \text{...(2)}$$

From (1) and (2), we have:

$$\frac{3x^2}{2} = 1350$$

or $$x^2 = \frac{1350 \times 2}{3} = 900 = (30)^2$$

or $$x = 30$$

Hence, corresponding altitutde is 30 m and the base is 3 × 30 m i.e., 90 m.

Example 20.5: Find the area of a rhombus whose diagonals are of lengths 16 cm and 12 cm.

Solution: Area of the rhombus $= \dfrac{1}{2}$ product of its diagonals $= \dfrac{1}{2} \times 16 \times 12 \text{ cm}^2$

$$= 96 \text{ cm}^2$$

Example 20.6: Length of the two parallel sides of a trapezium are 20 cm and 12 cm and the distance between them is 5 cm. Find the area of the trapezium.

Solution: Area of a trapezium = $\dfrac{1}{2}$ (sum of the two parallel sides)×distance between them

$$= \frac{1}{2}(20 + 12) \times 5 \text{ cm}^2 = 80 \text{ cm}^2$$

CHECK YOUR PROGRESS 20.1

1. Area of a square field is 225 m^2. Find the perimeter of the field.
2. Find the diagonal of a square whose perimeter is 60 cm.
3. Length and breadth of a rectangular field are 22.5 m and 12.5 m respectively. Find:

   (i) Area of the field

   (ii) Length of the barbed wire required to fence the field

4. The length and breadth of rectangle are in the ratio 3 : 2. If the area of the rectangle is 726 m^2, find its perimeter.
5. Find the area of a parallelogram whose base and corresponding altitude are respectively 20 cm and 12 cm.
6. Area of a triangle is 280 cm^2. If base of the triangle is 70 cm, find its corresponding altitude.
7. Find the area of a trapezium, the distance between whose parallel sides of lengths 26 cm and 12 cm is 10 cm.
8. Perimeter of a rhombus is 146 cm and the length of one of its diagonals is 48 cm. Find the length of its other diagonal.

20.2 HERON'S FORMULA

If the base and corresponding altitude of a triangle are known, you have already used the formula:

$$\text{Area of a triangle} = \frac{1}{2}\ \text{base} \times \text{corresponding altitude}$$

However, sometimes we are not given the altitude (height) corresponding to the given base of a triangle. Instead of that we are given the three sides of the triangle. In this case also, we can find the height (or altitude) corresponding to a side and calculate its area. Let us explain it through an example.

Example 20.7: Find the area of the triangle ABC, whose sides AB, BC and CA are respectively 5 cm, 6 cm and 7 cm.

Solution: Draw AD $\perp$ BC as shown in Fig. 20.1.

Let BD = x cm

So, CD = $(6 - x)$ cm

Now, from right triangle ABD, we have:

$AB^2 = BD^2 + AD^2$ (Pythagoras Theorem)

i.e. $25 = x^2 + AD^2$ ...(1)

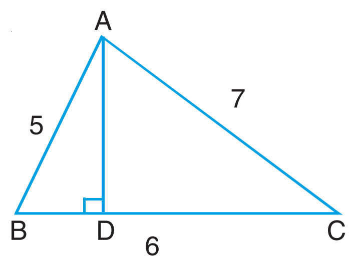


Fig. 20.1

Similarly, from right triangle ACD, we have:

$AC^2 = CD^2 + AD^2$

i.e. $49 = (6 - x)^2 + AD^2$ ...(2)

From (1) and (2), we have:

$49 - 25 = (6 - x)^2 - x^2$

i.e. $24 = 36 - 12x + x^2 - x^2$

or $12\,x = 12$, i.e., $x = 1$

Putting this value of x in (1), we have:

$$25 = 1 + AD^2$$

i.e. $AD^2 = 24$ or $AD = \sqrt{24} = 2\sqrt{6}$ cm

Thus, area of $\Delta ABC = \frac{1}{2}\ BC \times AD = \frac{1}{2} \times 6 \times\ 2\sqrt{6}\ \text{cm}^2 = 6\sqrt{6}\ \text{cm}^2$

You must have observed that the process involved in the solution of the above example is lengthy. To help us in this matter, a formula for finding the area of a triangle with three given sides was provided by a Greek mathematician Heron (75 B.C. to 10 B.C.). It is as follows:

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where, a, b and c are the three sides of the triangle and $s = \frac{a+b+c}{2}$. This formula can be proved on similar lines as in Example 20.7 by taking a, b and c for 6, 7 and 5 respectively.

Let us find the area of the triangle of Example 20.7 using this formula.

Here, $a = 6$ cm, $b = 7$ cm and $c = 5$ cm

So, $s = \frac{6+7+5}{2} = 9$ cm

Therefore, area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{9(9-6)(9-7)(9-5)}\ \text{cm}^2$$

$$= \sqrt{9 \times 3 \times 2 \times 3}\ \text{cm}^2$$

$= 6\sqrt{6}$ cm^2, which is the same as obtained earlier.

Let us take some more examples to illustrate the use of this formula.

Example 20.8: The sides of a triangular field are 165 m, 154 m and 143 m. Find the area of the field.

Solution: $s = \frac{a+b+c}{2} = \frac{(165+154+143)}{2}\ \text{m} = 231\ \text{m}$

So, area of the field $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{231 \times (231-165)(231-154)(231-143)} \text{ m}^2$$

$$= \sqrt{231 \times 66 \times 77 \times 88} \text{ m}^2$$

$$= \sqrt{11 \times 3 \times 7 \times 11 \times 2 \times 3 \times 11 \times 7 \times 11 \times 2 \times 2 \times 2} \text{ m}^2$$

$$= 11 \times 11 \times 3 \times 7 \times 2 \times 2 \text{ m}^2 = 10164 \text{ m}^2$$

Example 20.9: Find the area of a trapezium whose parallel sides are of lengths 11 cm amd 25 cm and whose non-parallel sides are of lengths 15 cm and 13 cm.

Solution: Let ABCD be the trapezium in which AB = 11 cm, CD = 25 cm, AD = 15 cm and BC =13 cm (See Fig. 20.2)

Through B, we draw a line parallel to AD to intersect DC at E. Draw BF $\perp$ DC.

Now, clearly BE = AD = 15 cm

BC = 13 cm (given)

and EC = (25 – 11) cm = 14 cm

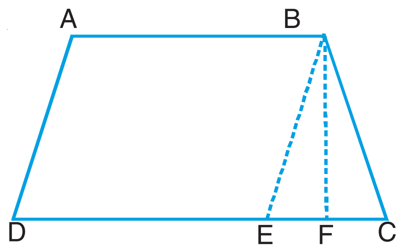


Fig. 20.2

So, for ΔBEC, $s = \dfrac{15+13+14}{2}$ cm = 21 cm

Therefore area of ΔBEC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21 \times (21-15)(21-13)(21-14)} \text{ cm}^2$$

$$= \sqrt{21 \times 6 \times 8 \times 7} \text{ cm}^2$$

$$= 7 \times 3 \times 4 \text{ cm}^2 = 84 \text{ cm}^2 \qquad ...(1)$$

Again, area of ΔBEC $= \dfrac{1}{2}$ EC $\times$ BF

$$= \frac{1}{2} \times 14 \times \text{BF} \qquad ...(2)$$

So, from (1) and (2), we have:

$$\frac{1}{2} \times 14 \times \text{BF} = 84$$

i.e., $\text{BF} = \dfrac{84}{7}$ cm = 12 cm

Therefore, area of trapezium ABCD $= \frac{1}{2}$ (AB + CD) × BF

$$= \frac{1}{2}(11 + 25) \times 12 \text{ cm}^2$$

$$= 18 \times 12 \text{ cm}^2 = 216 \text{ cm}^2$$

CHECK YOUR PROGRESS 20.2

1. Find the area of a triangle of sides 15 cm, 16 cm and 17 cm.
2. Using Heron's formula, find the area of an equilateral triangle whose side is 12 cm. Hence, find the altitude of the triangle.

20.3 AREAS OF RECTANGULAR PATHS AND SOME RECTILINEAR FIGURES

You might have seen different types of rectangular paths in the parks of your locality. You might have also seen that sometimes lands or fields are not in the shape of a single figure. In fact, they can be considered in the form of a shape made up of a number of polygons such as rectangles, squares, triangles, etc. We shall explain the calculation of areas of such figures through some examples.

Example 20.10: A rectangular park of length 30 m and breadth 24 m is surrounded by a 4 m wide path. Find the area of the path.

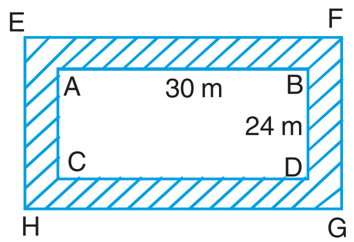


Fig. 20.3

Solution: Let ABCD be the park and shaded portion is the path surrounding it (See Fig. 20.3).

So, length of rectangle EFGH = (30 + 4 + 4) m = 38 m

and breadth of rectangle EFGH = (24 + 4 + 4) m = 32 m

Therefore, area of the path = area of rectangle EFGH – area of rectangle ABCD

$$= (38 \times 32 - 30 \times 24) \text{ m}^2$$

$$= (1216 - 720) \text{ m}^2$$

$$= 496 \text{ m}^2$$

Example 20.11: There are two rectangular paths in the middle of a park as shown in Fig. 20.4. Find the cost of paving the paths with concrete at the rate of ₹ 15 per m². It is given that AB = CD = 50 m, AD = BC = 40 m and EF = PQ = 2.5 m.

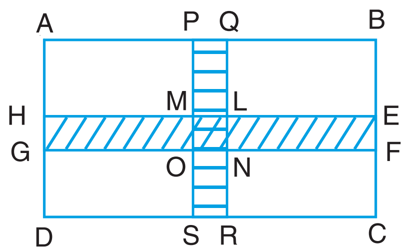


Fig. 20.4

Solution: Area of the paths = Area of PQRS + Area of EFGH – area of square MLNO

$$= (40 \times 2.5 + 50 \times 2.5 - 2.5 \times 2.5)\ \text{m}^2$$

$$= 218.75\ \text{m}^2$$

So, cost of paving the concrete at the rate of ₹ 15 per m^2 = ₹ 218.75 × 15

= ₹ 3281.25

Example 20.12: Find the area of the figure ABCDEFG (See Fig. 20.5) in which ABCG is a rectangle, AB = 3 cm, BC = 5 cm, GF = 2.5 cm = DE = CF., CD = 3.5 cm, EF = 4.5 cm, and CD || EF.

Solution: Required area = area of rectangle ABCG + area of isosceles triangle FGC

+ area of trapezium DCEF ...(1)

Now, area of rectangle ABCG = $l \times b$ = 5 × 3 cm^2 = 15 cm^2 ...(2)

For area of ΔFGC, draw FM ⊥ CG.

As FG = FC (given), therefore

M is the mid point of GC.

That is, $GM = \frac{3}{2} = 1.5$ cm

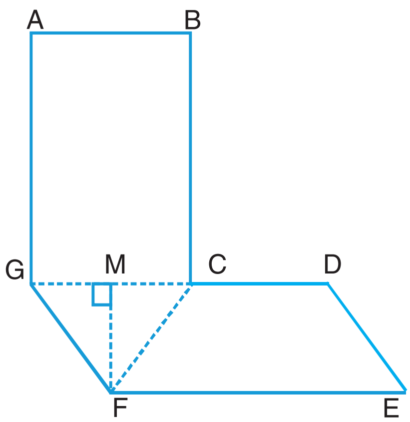


Fig. 20.5

Now, from ΔGMF,

$$GF^2 = FM^2 + GM^2$$

or $(2.5)^2 = FM^2 + (1.5)^2$

or $FM^2 = (2.5)^2 - (1.5)^2 = 4$

So, FM = 2, i.e., length of FM = 2 cm

So, area of $\Delta FGC = \frac{1}{2}\ GC \times FM$

$$= \frac{1}{2} \times 3 \times 2\ \text{cm}^2 = 3\ \text{cm}^2 \quad ...(3)$$

Also, area of trapezium CDEF = $\frac{1}{2}$ (sum of the parallel sides) × distance between them

$$= \frac{1}{2}(3.5 + 4.5) \times 2\ \text{cm}^2$$

$$= \frac{1}{2} \times 8 \times 2\ \text{cm}^2 = 8\ \text{cm}^2 \quad ...(4)$$

So, area of given figure

$$= (15 + 3 + 8) \text{ cm}^2 \qquad \text{[From (1), (2), (3) and (4)]}$$

$$= 26 \text{ cm}^2$$

CHECK YOUR PROGRESS 20.3

1. There is a 3 m wide path on the inside running around a rectangular park of length 48 m and width 36 m. Find the area of the path.

2. There are two paths of width 2 m each in the middle of a rectangular garden of length 80 m and breadth 60 m such that one path is parallel to the length and the other is parallel to the breadth. Find the area of the paths.

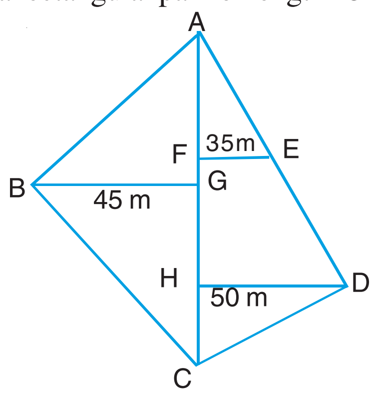


Fig. 20.6

3. Find the area of the rectangular figure ABCDE given in Fig. 20.6, where EF, BG and DH are perpendiculars to AC, AF = 40 m, AG = 50 m, GH = 40 m and CH = 50 m.

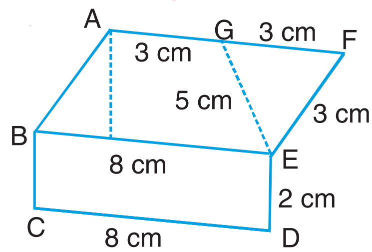


Fig. 20.7

4. Find the area of the figure ABCDEFG in Fig. 20.7, where ABEG is a trapezium, BCDE is a rectangle, and distance between AG and BE is 2 cm.

20.4 AREAS OF CIRCLES AND CIRCULAR PATHS

So far, we have discussed about the perimeters and areas of figures made up of line segments only. Now we take up a well known and very useful figure called circle, which is not made up of line segments. (See. Fig. 20.8). You already know that **perimeter (circumference) of a circle is $2\pi r$ and its area is πr^2,** where r is the radius of the circle and π is a constant equal to the ratio of circumference of a circle to its diameter. You also know that π is an irrational number.

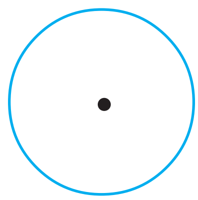


Fig. 20.8

A great Indian mathematician Aryabhata (476 - 550 AD) gave the value of π as $\frac{62832}{20000}$, which is equal to 3.1416 correct to four places of decimals. However, for practical purposes, the value of π is generally taken as $\frac{22}{7}$ or 3.14 approximately. Unless, stated otherwise, we shall take the value of π as $\frac{22}{7}$.

Example 20.13: The radii of two circles are 18 cm and 10 cm. Find the radius of the circle whose circumference is equal to the sum of the circumferences of these two circles.

Solution: Let the radius of the circle be r cm.

Its circumference = $2\pi r$ cm ....(1)

Also, sum of the circumferences of the two circles = $(2\pi \times 18 + 2\pi \times 10)$ cm

$= 2\pi \times 28$ cm ...(2)

Therefore, from (1) and (2), $2\pi r = 2\pi \times 28$

or $r = 28$

i.e., radius of the circle is 28 cm.

Example 20.14: There is a circular path of width 2 m along the boundary and inside a circular park of radius 16 m. Find the cost of paving the path with bricks at the rate of ₹ 24 per m². (Use $\pi = 3.14$)

Solution: Let OA be radius of the park and shaded portion be the path (See. Fig. 20.9)

So, OA = 16 m

and OB = 16 m – 2 m = 14 m.

Therefore, area of the path

$= (\pi \times 16^2 - \pi \times 14^2)\ m^2$

$= \pi(16 + 14)(16 - 14)\ m^2$

$= 3.14 \times 30 \times 2 = 188.4\ m^2$

So, cost of paving the bricks at ₹ 24 per m²

= ₹ 24×188.4

= ₹ 4521.60

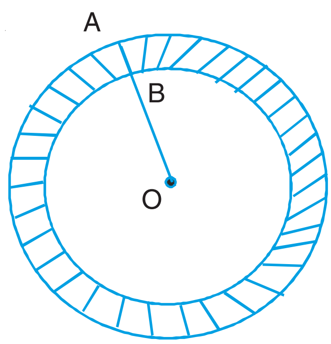


Fig. 20.9

CHECK YOUR PROGRESS 20.4

1. The radii of two circles are 9 cm and 12 cm respectively. Find the radius of the circle whose area is equal to the sum of the areas of these two circles.

2. The wheels of a car are of radius 40 cm each. If the car is travelling at a speed of 66 km per hour, find the number of revolutions made by each wheel in 20 minutes.

3. Around a circular park of radius 21 m, there is circular road of uniform width 7 m outside it. Find the area of the road.

20.5 PERIMETER AND AREA OF A SECTOR

You are already familar with the term **sector of a circle**. Recall that a part of a circular region enclosed between two radii of the corresponding circle is called a sector of the circle. Thus, in Fig. 20.10, the shaded region OAPB is a sector of the circle with centre O. $\angle AOB$ is called the **central angle** or simply the angle of the sector. Clearly, APB is the corresponding arc of this sector. You may note that the part OAQB (unshaded region) is also a sector of this circle. For obvious reasons, OAPB is called the **minor sector** and OAQB is called the **major sector** of the circle (with major arc AQB).

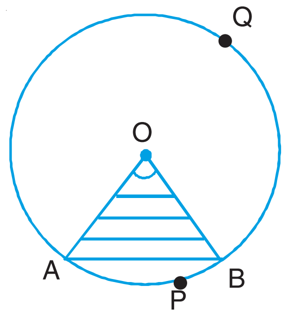


Fig. 20.10

Note: *unless stated otherwise, by sector, we shall mean a minor sector.*

(i) **Perimeter of the sector:** Clearly, perimeter of the sector OAPB is equal to OA + OB + length of arc APB.

Let radius OA (or OB) be r, length of the arc APB be l and $\angle AOB$ be θ.

We can find the length l of the arc APB as follows:

We know that circumference of the circle = $2\pi r$

Now, for total angle 360° at the centre, length = $2\pi r$

So, for angle θ, length $l = \dfrac{2\pi r}{360^\circ} \times \theta$

or $\quad l = \dfrac{\pi r \theta}{180^\circ} \quad$ ...(1)

Thus, perimeter of the sector OAPB = OA + OB + l

$$= r + r + \frac{\pi r \theta}{180^\circ} = 2r + \frac{\pi r \theta}{180^\circ}$$

(ii) **Area of the sector**

Area of the circle = πr^2

Now, for total angle 360°, area = πr^2

So, **for angle θ, area = $\dfrac{\pi r^2}{360^\circ} \times \theta$**

Thus, area of the sector OAPB = $\dfrac{\pi r^2 \theta}{360^\circ}$

Note: *By taking the angle as 360º – θ, we can find the perimeter and area of the major sector OAQB as follows*

$$\text{Perimeter} = 2r + \frac{\pi r(360^\circ - \theta)}{180^\circ}$$

and $$\text{area} = \frac{\pi r^2}{360^\circ} \times (360^\circ - \theta)$$

Example 20.15: Find the perimeter and area of the sector of a circle of radius 9 cm with central angle 35º.

Solution: Perimeter of the sector $= 2r + \dfrac{\pi r \theta}{180^\circ}$

$$= \left(2 \times 9 + \frac{22}{7} \times \frac{9 \times 35^\circ}{180^\circ}\right) \text{ cm}$$

$$= \left(18 + \frac{11 \times 1}{2}\right) \text{ cm} = \frac{47}{2} \text{ cm}$$

Area of the sector $= \dfrac{\pi r^2 \times \theta}{360^\circ}$

$$= \left(\frac{22}{7} \times \frac{81 \times 35^\circ}{360^\circ}\right) \text{cm}^2$$

$$= \left(\frac{11 \times 9}{4}\right) \text{cm}^2 = \frac{99}{4} \text{cm}^2$$

Example 20.16: Find the perimeter and area of the sector of a circle of radius 6 cm and length of the arc of the sector as 22 cm.

Solution: Perimeter of the sector $= 2r + \text{length of the arc}$

$$= (2 \times 6 + 22) \text{ cm} = 34 \text{ cm}$$

For area, let us first find the central angle θ.

So, $$\frac{\pi r \theta}{180^\circ} = 22$$

or $\frac{22}{7} \times 6 \times \frac{\theta}{180^\circ} = 22$

or $\theta = \frac{180^\circ \times 7}{6} = 210^\circ$

So, area of the sector $= \frac{\pi r^2 \theta}{360^\circ}$

$= \frac{22}{7} \times \frac{36 \times 210^\circ}{360^\circ}$

$= 66 \text{ cm}^2$

Alternate method for area:

Circumference of the circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 6 \text{ cm}$

and area of the circle $= \pi r^2 = \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

For length $2 \times \frac{22}{7} \times 6$ cm, area $= \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

So, for length 22 cm, area $= \frac{22}{7} \times \frac{6 \times 6 \times 7 \times 22}{2 \times 22 \times 6} \text{ cm}^2$

$= 66 \text{ cm}^2$

CHECK YOUR PROGRESS 20.5

1. Find the perimeter and area of the sector of a circle of radius 14 cm and central angle 30°.
2. Find the perimeter and area of the sector of a circle of radius 6 cm and length of the arc as 11 cm.

20.6 AREAS OF COMBINATIONS OF FIGURES INVOLVING CIRCLES

So far, we have been discussing areas of figures separately. We shall now try to calculate areas of combinations of some plane figures. We come across these type of figures in daily life in the form of various designs such as table covers, flower beds, window designs, etc. Let us explain the process of finding their areas through some examples.

Example 20.17: In a round table cover, a design is made leaving an equilateral triangle ABC in the middle as shown in Fig. 20.11. If the radius of the cover is 3.5 cm, find the cost of making the design at the rate of ₹ 0.50 per cm^2 (use $\pi = 3.14$ and $\sqrt{3} = 1.7$)

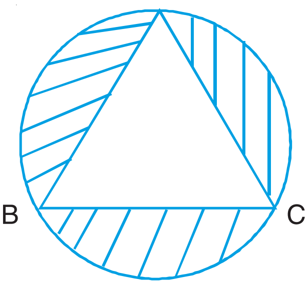


Fig. 20.11

Solution: Let the centre of the cover be O.

Draw $OP \perp BC$ and join OB, OC. (Fig. 20.12)

Now, $\angle BOC = 2\ \angle BAC = 2 \times 60^\circ = 120^\circ$

Also, $\angle BOP = \angle COP = \frac{1}{2}\ \angle BOC = \frac{1}{2} \times 120^\circ = 60^\circ$

Now, $\frac{BP}{OB} = \sin\angle BOP = \sin 60^\circ = \frac{\sqrt{3}}{2}$ [See Lessons 22-23]

i.e., $\frac{BP}{3.5} = \frac{\sqrt{3}}{2}$

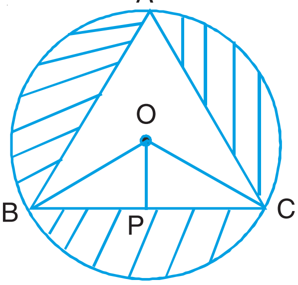


Fig. 20.12

So, $BC = 2 \times \frac{3.5\sqrt{3}}{2}\ \text{cm} = 3.5\sqrt{3}\ \text{cm}$

Therefore, area of $\Delta ABC = \frac{\sqrt{3}}{4} BC^2$

$$= \frac{\sqrt{3}}{4} \times 3.5 \times 3.5 \times 3\ \text{cm}^2$$

Now, area of the design = area of the circle – area of ΔABC

$$= (3.14 \times 3.5 \times 3.5 - \frac{\sqrt{3}}{4} \times 3.5 \times 3.5 \times 3)\ \text{cm}^2$$

$$= (3.14 \times 3.5 \times 3.5 - \frac{1.7 \times 3.5 \times 3.5 \times 3}{4})\ \text{cm}^2$$

$$= 3.5 \times 3.5 \left(\frac{12.56 - 5.10}{4} \right) \text{cm}^2$$

$$= 12.25 \left(\frac{7.46}{4} \right) \text{cm}^2 = 12.25 \times 1.865 \text{ cm}^2$$

Therefore, cost of making the design at ₹ 0.50 per cm^2

$$= ₹\ 12.25 \times 1.865 \times 0.50 = ₹\ 114.23 \text{ (approx)}$$

Example 20.18: On a square shaped handkerchief, nine circular designs, each of radius 7 cm, are made as shown in Fig. 20.13. Find the area of the remainig portion of the handkerchief.

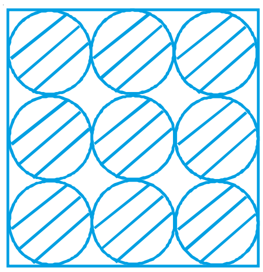


Fig. 20.13

Solution: As radius of each circular design is 7 cm, diameter of each will be 2×7 cm = 14 cm

So, side of the square handkerchief = $3 \times 14 = 42$ cm ...(1)

Therefore, area of the square = 42×42 cm^2

Also, area of a circle = $\pi r^2 = \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 154 \text{ cm}^2$

So, area of 9 circles = 9×154 cm^2 ...(2)

Therefore, from (1) and (2), area of the remaining portion

$$= (42 \times 42 - 9 \times 154) \text{ cm}^2$$

$$= (1764 - 1386) \text{ cm}^2 = 378 \text{ cm}^2$$

CHECK YOUR PROGRESS 20.6

1. A square ABCD of side 6 cm has been inscribed in a quadrant of a circle of radius 14 cm (See Fig. 20.14). Find the area of the shaded region in the figure.

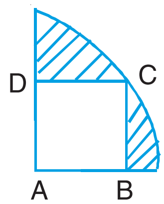


Fig. 20.14

2. A shaded design has been formed by drawing semicircles on the sides of a square of side length 10 cm each as shown in Fig. 20.15. Find the area of the shaded region in the design.

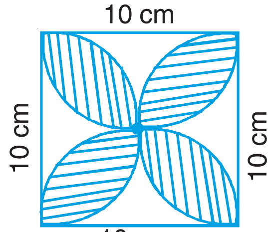


Fig. 20.14

LET US SUM UP

- Perimeter of a rectangle = 2 (length + breadth)
- Area of a rectangle = length × breadth
- Perimeter of a square = 4 × side
- Area of a square = $(\text{side})^2$
- Area of a parallelogram = base × corresponding altitude
- Area of a triangle = $\frac{1}{2}$ base × corresponding altitude

  and also $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b and c are the three sides of the triangle

  and $s = \frac{a+b+c}{2}$.
- Area of a rhombus = $\frac{1}{2}$ product of its diagonals
- Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × distance between them
- Area of rectangular path = area of the outer rectangle – area of inner rectangle
- Area of cross paths in the middle = Sum of the areas of the two paths – area of the common portion
- circumference of a circle of radius r = 2 πr
- Area of a circle of radius r = πr^2
- Area of a circular path = Area of the outer circle – area of the inner circle
- Length l of the arc of a sector of a circle of radius r with central angle θ is $l = \frac{\pi r\theta}{180^\circ}$
- Perimeter of the sector a circle with radius r and central angle $\theta = 2r + \frac{\pi r\theta}{180^\circ}$
- Area of the sector of a circle with radius r and central and $\theta = \frac{\pi r^2\theta}{360^\circ}$

- Areas of many rectilinear figures can be found by dividing them into known figures such as squares, rectangles, triangles and so on.
- Areas of various combinations of figures and designs involving circles can also be found by using different known formulas.

TERMINAL EXERCISE

1. The side of a square park is 37.5 m. Find its area.
2. The perimeter of a square is 480 cm. Find its area.
3. Find the time taken by a person in walking along the boundary of a square field of area 40 000 m^2 at a speed of 4 km/h.
4. Length of a room is three times its breadth. If its breadth is 4.5 m, find the area of the floor.
5. The length and breadth of a rectangle are in the ratio of 5 : 2 and its perimeter is 980 cm. Find the area of the rectangle.
6. Find the area of each of the following parallelograms:

   (i) one side is 25 cm and corresponding altitude is 12 cm

   (ii) Two adjacent sides are 13 cm and 14 cm and one diagonal is 15 cm.
7. The area of a rectangular field is 27000 m^2 and its length and breadth are in the ratio 6:5. Find the cost of fencing the field by four rounds of barbed wire at the rate of ₹ 7 per 10 metre.
8. Find the area of each of the following trapeziums:

S. No.	Lengths of parellel sides	Distance between the parallel sides
(i)	30 cm and 20 cm	15 cm
(ii)	15.5 cm and 10.5 cm	7.5 cm
(iii)	15 cm and 45 cm	14.6 cm
(iv)	40 cm and 22 cm	12 cm

9. Find the area of a plot which is in the shape of a quadrilateral, one of whose diagonals is 20 m and lengths of the perpendiculars from the opposite corners on it are of lengths 12 m and 18 m respectively.
10. Find the area of a field in the shape of a trapezium whose parallel sides are of lengths 48 m and 160 m and non-parallel sides of lengths 50 m and 78 m.

11. Find the area and perimeter of a quadrilateral ABCCD in which AB = 8.5 cm, BC = 14.3 cm, CD = 16.5 cm, AD = 8.5 cm and BD = 15.4 cm.

12. Find the areas of the following triangles whose sides are

    (i) 2.5 cm, 6 cm and 6.5 cm

    (ii) 6 cm, 11.1 cm and 15.3 cm

13. The sides of a triangle are 51 cm, 52 cm and 53 cm. Find:

    (i) Area of the triangle

    (ii) Length of the perpendicular to the side of length 52 cm from its opposite vertex.

    (iii) Areas of the two triangles into which the given triangle is divided by the perpendicular of (ii) above.

14. Find the area of a rhombus whose side is of length 5 m and one of its diagonals is of length 8 m.

15. The difference between two parallel sides of a trapezium of area 312 cm^2 is 8 cm. If the distance between the parallel sides is 24 cm, find the length of the two parallel sides.

16. Two perpendicular paths of width 10 m each run in the middle of a rectangular park of dimensions 200 m $\times$ 150 m, one parallel to length and the other parallel ot the breadth. Find the cost of constructing these paths at the rate of ₹ 5 per m^2

17. A rectangular lawn of dimensions 65 m $\times$ 40 m has a path of uniform width 8 m all around inside it. Find the cost of paving the red stone on this path at the rate of ₹ 5.25 per m^2.

18. A rectangular park is of length 30 m and breadth 20 m. It has two paths, each of width 2 m, around it (one inside and the other outside it). Find the total area of these paths.

19. The difference between the circumference and diameter of a circle is 30 cm. Find its radius.

20. A path of uniform width 3 m runs outside around a circular park of radius 9 m. Find the area of the path.

21. A circular park of radius 15 m has a road 2 m wide all around inside it. Find the area of the road.

22. From a circular piece of cardboard of radius 1.47 m, a sector of angle 60^o has been removed. Find the area of the remaining cardboard.

23. Find the area of a square field, in hectares, whose side is of length 360 m.

24. Area of a triangular field is 2.5 hectares. If one of its sides is 250 m, find its corresponding altitude.

25. A field is in the shape of a trapezium of parallel sides 11 m and 25 m and of non-parallel sides 15 m and 13 m. Find the cost of watering the field at the rate of 5 paise per 500 cm^2.

26. From a circular disc of diameter 8 cm, a square of side 1.5 cm is removed. Find the area of the remaining poriton of the disc. (Use $\pi = 3.14$)

27. Find the area of the adjoining figure with the measurement, as shown. (Use $\pi = 3.14$)

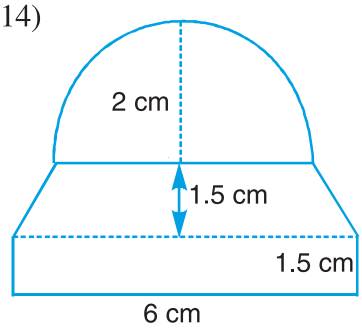


Fig. 20.16

28. A farmer buys a circular field at the rate of ₹ 700 per m^2 for ₹ 316800. Find the perimeter of the field.

29. A horse is tied to a pole at a corner of a square field of side 12 m by a rope of length 3.5 m. Find the area of the part of the field in which the horse can graze.

30. Find the area of the quadrant of a circle whose circumference is 44 cm.

31. In Fig. 20.17, OAQB is a quadrant of a circle of radius 7 cm and APB is a semicircle. Find the area of the shaded region.

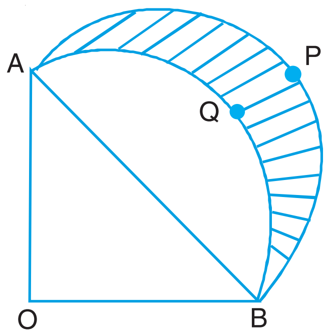


Fig. 20.17

32. In Fig 20.18, radii of the two concentric circles are 7 cm and 14 cm and $\angle AOB = 45^\circ$, Find the area of the shaded region ABCD.

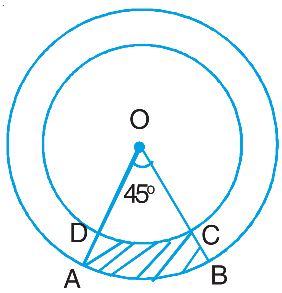


Fig. 20.18

33. In Fig. 20.19, four congruent circles of radius 7 cm touch one another and A, B, C, and D are their centres. Find the area of the shaded region.

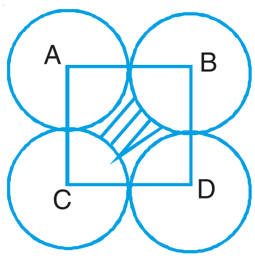


Fig. 20.19

34. Find the area of the flower bed with semicircular ends of Fig. 20.20, if the diameters of the ends are 14cm, 28 cm, 14 cm and 28 cm respectively.

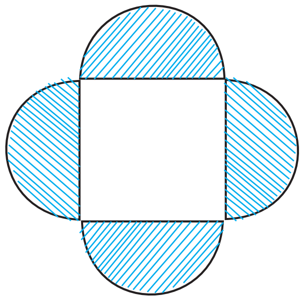


Fig. 20.20

35. In Fig 20.21, two semicircles have been drawn inside the square ABCD of side 14 cm. Find the area of the shaded region as well as the unshaded region.

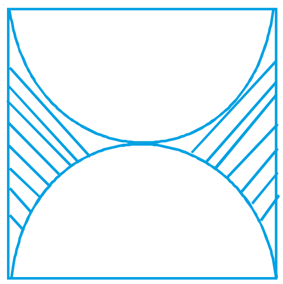


Fig. 20.21

In each of the questions 36 to 42, write the correct answer from the four given options:

36. The perimeter of a square of side a is

(A) a^2 (B) $4a$ (C) $2a$ (D) $\sqrt{2}\,a$

37. The sides of a triangle are 15 cm, 20 cm, and 25 cm. Its area is

(A) 30 cm^2 (B) 150 cm^2 (C) 187.5 cm^2 (D) 300 cm^2

38. The base of an isosceles triangle is 8 cm and one of its equal sides is 5 cm. The corresponding height of the triangle is

(A) 5 cm (B) 4 cm (C) 3 cm (D) 2 cm

39. If a is the side of an equilateral triangle, then its altitude is

(A) $\frac{\sqrt{3}}{2}a^2$ (B) $\frac{\sqrt{3}}{2a^2}$ (C) $\frac{\sqrt{3}}{2}a$ (D) $\frac{\sqrt{3}}{2a}$

40. One side of a parallelogram is 15 cm and its corresponding altitude is 5 cm. Area of the parallelogram is

(A) 75 cm^2 (B) 37.5 cm^2 (C) 20 cm^2 (D) 3 cm^2

41. Area of a rhombus is 156 cm^2 and one of its diagonals is 13 cm. Its other diagonal is

(A) 12 cm (B) 24 cm (C) 36 cm (D) 48 cm

42. Area of a trapezium is 180 cm^2 and its two parallel sides are 28 cm and 12 cm. Distance between these two parallel sides is

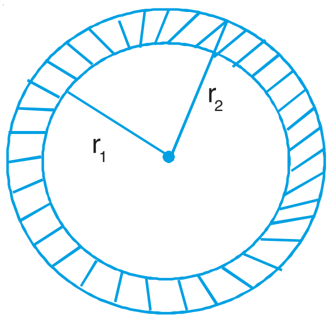
(A) 9 cm (B) 12 cm (C) 15 cm (D) 18 cm

43. Which of the following statements are true and which are false?

(i) Perimeter of a rectangle is equal to length + breadth.

(ii) Area of a circle of radus r is πr^2.

(iii) Area of the circular shaded path of the adjoining figure is $\pi r_1^2 - \pi r_2^2$.



(iv) Area of a triangle of sides a, b and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where s is the perimeter of the triangle.

(v) Area of a sector of circle of radius r and central angle 60° is $\frac{\pi r^2}{6}$.

(vi) Perimeter of a sector of circle of radius 5 cm and central angle 120° is 5 cm + $\frac{10\pi}{3}$ cm

44. Fill in the blanks:

(i) Area of a rhombus = $\frac{1}{2}$ product of its ____________________

(ii) Area of a trapezium = $\frac{1}{2}$ (sum of its ________) × distance between ______

(iii) The ratio of the areas of two sectors of two circles of radii 4 cm and 8 cm and central angles 100° and 50° respectively is __________

(iv) The ratio of the lengths of the arcs of two sectors of two circles of radii 10 cm and 5 cm and central angles 75° and 150° is ______________.

(v) Perimeter of a rhombus of diagonals 16 cm and 12 cm is __________

ANSWERS TO CHECK YOUR PROGRESS

20.1

1. 60 m
2. $15\sqrt{2}$ cm
3. (i) 281.25 m^2 (ii) 70 m
4. 110 m [Hint $3x \times 2x = 726 \Rightarrow x = 11$ m]
5. 240 cm^2
6. 80 cm
7. 190 cm^2
8. 55 cm, 1320 cm^2

20.2

1. $24\sqrt{21}$ cm^2
2. $36\sqrt{3}$ cm^2; $6\sqrt{3}$ cm

20.3

1. 648 m^2
2. 276 m^2
3. 7225 m^2
4. $\left(27+\frac{5}{4}\sqrt{11}\right)$ cm^2

20.4

1. 15 cm
2. 8750
3. 10.78 m^2

20.5

1. Perimeter = $35\frac{1}{2}$ cm; Area = $\frac{154}{3}$ cm^2
2. Perimeter = 23 cm, Area = 33 cm^2

20.6

1. 118 cm^2

2. $4\times\frac{1}{2}\pi\times5^2 - 10\times10$ cm^2

   $= (50\pi - 100)$ cm^2

ANSWERS TO TERMINAL EXERCISE

1. 1406.25 m^2 2. 14400 cm^2 3. 12 minutes
4. 60.75 m^2 5. 49000 cm^2 6. (i) 300 cm^2 (ii) 168 cm^2
7. ₹ 1848
8. (i) 375 cm^2 (ii) 97.5 cm^2 (iii) 438 m^2 (iv) 372 cm^2
9. 300 m^2 10. 3120 m^2 11. 129.36 cm^2
12. (i) 7.5 cm^2 (ii) 27.54 cm^2
13. (i) 1170 cm^2 (ii) 45 cm (iii) 540 cm^2, 630 cm^2
14. 24 m^2 15. 17 cm and 9 cm 16. ₹ 17000
17. ₹ 7476 18. 400 m^2 19. 7 cm
20. 198 m^2 21. 176 m^2 22. 1.1319 m^2
23. 12.96 ha 24. 200 m 25. ₹ 216
26. 47.99 cm^2 27. 22.78 cm^2 28. $75\frac{3}{7}$ m
29. $\frac{77}{8}$ m^2 30. $\frac{77}{2}$ cm^2 31. $\frac{49}{2}$ cm^2
32. $\frac{231}{4}$ cm^2 33. 42 cm^2 34. 1162 cm^2
35. 42 cm^2, 154 cm^2 36. (B) 37. (B)
38. (C) 39. (C) 40. (A)
41. (B) 42. (A)
43. (i) False (ii) True (iii) False
    (iv) False (v) True (vi) False
44. (i) diagonals (ii) parallel sides, them (iii) 1 : 2
    (iv) 1 : 1 (v) 40 cm.