(C 103)

Open Basic Education (Adult)

MATHEMATICS

Level - C (Equivalent to Class 8)

National Institute of Open Schooling

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Message from Chairman

Dear learner,

We can not hypothesize of the progress of any class/society and nation without education. Those countries are developed and progressive whose literacy rate is higher. Due to illiteracy not only we remain away from the activities happening around the world but also we are not capable to understand, to change and to intervene in the social development process due to illiteracy. The change in the society is only possible when the whole society is educated. Literacy does not mean only to read and write but to join people in the larger part of the society and it's activities and thereby bringing them in the main stream of the nation.

After independence many plans/schemes were launched to bring reforms in the field of education. As a result of these plans/schemes upward trend was observed in the level of education but at the same time numbers of illiterates have risen. To meet this challenge, "National Literacy mission" was launched on 5th may, 1988 throughout the country. A "complete literacy Abhiyan" was started but the number of illiterates could not be reduced.

In the next phase, on 8th September, 2009, under the National Literacy Mission, "Saakshar Bharat" programme was launched. Under this scheme along with formed education-vocational education, skill development/practical knowledge and the education related to moral value was also included.

The main objective of this mission is to continue the education of neo-literates. To achieve this objective, National Institute of Open Schooling has joined hand with "Saakshar Bharat". Under this scheme, NIOS will develop curriculum, Self Learning Material for the equivalency programme and after evaluation certificates to the neo-literates will also be issued. The passed out of Basic Literacy Assessment or those who could not continue their education due to any reason are covered under this programme. The provision has been made to upgrade the level of learning of the passed out of the Basic Literacy Assessment upto the Class 3.

Mathematics has been regarded as a dull subject but if this is made functional to real life then it could be made interesting. Keeping this in view, the content in this book has been developed in such a way that it is related to life situations such as knowledge of numbers understanding their pattern, writing them in numbers and understanding the process of addition, subtraction, multiplication and division are dealt in a practical way. Knowledge of fraction, metric system all has been dealt in a way relating to life situations. Knowledge of counting for money, dealing with kilogram, litre, meter etc are taught by relating to practical life situations

While developing this book- it has been kept in mind that age level experiences of adult learners is more than the learners from the formal system. Many things they learn through social and family activities, for example keeping the records of income-expenditure of labour, selling and purchasing the things, exchange of commodity, addition, subtraction of kilometers, litre, kilogram etc but they to not know formal writing and reading.

In the development of this book, it has been kept in mind the level of competencies and learners abilities, so that whatever knowledge transferred/transacted it is used by the learner immediately.

In this book, some questions under section "Let's see what you have learnt" have been given. At the end of the lesson an exercise is given. After every two lesson an assessment sheet is given. In the end a model question paper is given. The neo-literates can make their self assessment by doing these questions

A special thank to every intellectual who has helped in making this book interesting and useful. I fully believe that learners will like this book and will learn a lot from it. I wish for their bright future ahead. Any suggestions for improvement in the book are welcome.

Chairman

National Institute of Open Schooling

Contents

Module - I

Arithmetic

In the ancient time, mankind used to count cattle and personal belongings by using tallies(/), stone pieces or knotting the threads. Later on, invented numbers, called 'Counting Numbers' or 'Natural Numbers' were used. Different civilisations developed different groups of symbols (called 'Numerals') to represent these counting numbers.

Following symbols were used for representing numbers:

Roman:	I	II	III	IV	V	VI	VII	VIII	IX	X	C
Egypt:	I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII	∩	𓍢
Devnagri:											
Hindu-Arabic: (International)	1	2	3	4	5	6	7	8	9	10	00

Hindu- Arabic system was developed in India and then it was adopted in Arab and Europe.

Indians invented Zero and the Place - Value concept credit goes to India for inventing zero and the place-value concept. In this way Hindu- Arabic Decimal System of counting in which ten symbols (images) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are used, was established. Using these symbols any number, how large it may be, can be written. In this Module of Arithmetic, we shall discuss about the Natural Numbers, Integers and Whole Numbers. With reference to these numbers we shall learn the four fundamental operations of Addition, Subtraction, Multiplication and Division and will discuss about their properties.

Factors and multiples of numbers, Highest Common Factor and Lowest Common Multiple, Prime and Composite Numbers, Co- Prime Numbers will also be done in this Module. Rule of testing divisibility of any natural number by 2, 3, 4, 5, 6, 8, 9 or 11 is also discussed.

You will learn to represent Natural numbers Integers and Whole Numbers on a Number Line in this Module.

1

NATURAL AND WHOLE NUMBERS

On some auspicious occasion some guests are expected at your home and for their stay at night you are to arrange some beds. If you do not know counting, you will be in a big trouble. Before arrival of the guests, you will not come to know that whether your arrangement is sufficient or not. In this way to get rid of such troubles, mankind invented counting numbers, which are called 'Natural Numbers'. If in the group of Counting Numbers, we add the number 'zero' then we get the set of 'Whole Numbers'. Later on mankind learned to add, subtract, multiply and divide Natural and Whole Numbers and their properties.

From this lesson you will learn:

- Operations on Natural Numbers and whole numbers and their properties
- Highest Common Factor (HCF) and Lowest common Multiple (LCM) of numbers
- Divisibility of numbers

1.1 Natural Numbers and Whole Numbers

We know that counting numbers 1, 2, 3, 4 ..., are called Natural Numbers. If zero (0) be also taken along with these numbers then the group thus formed is called the set of whole numbers.

We know the method of adding, subtracting, multiplying and dividing two natural numbers. Let us learn about the properties of these operations.

1.1.1 Addition or Sum

We know that $7 + 8 = 15$

$12 + 6 = 18$

$55 + 43 = 98$

Note

We observe that 15, 18, 98 are also natural numbers. So, concluded that if two natural numbers be added then sum will also be a natural number. In general form, if a and b are two natural numbers, then (a+b) will also be a natural number.

Suppose, Naresh has two pencils and Arun has three pencils. Naresh gives his two pencils to Arun, then Arun will have 5 pencils (i.e. $2 + 3 = 5$), but if Arun gives his three pencils to Naresh, then Naresh will have 5 pencils because $(3 + 2 = 5)$

Again $4 + 8 = 12$ and $8 + 4 = 12$, so $4 + 8 = 8 + 4$

$47 + 33 = 80$ and $33 + 47 = 80$, so, $47 + 33 = 33 + 47$

So, it is concluded that by adding two numbers in any order, sum will remain same.

In general, if a and b be two natural numbers then $a + b = b + a$

If we are to add three numbers, then we add the third number to the sum of two numbers. For example, if 3, 5 and 7 are to be added then sum is obtained as follows

$3 + 5 + 7 = (3 + 5) +7 = 8 + 7 = 15$

Here, bracket is indicating that the sum of the numbers written in the bracket be added first then third number be added to the sum.

If 5, 7 be added first and 3 be added to the sum then we write in the following manner

$$3 + 5 + 7 = 3 + (5 +7)= 3 + 12 = 15$$

Sum is equal to the sum obtained earlier. Let us verify it by taking some other natural numbers.

$$(1 + 3) + 8 = 4 + 8 = 12$$

$$1 + (3 + 8) = 1 + 11 = 12$$

Thus, $(1 + 3) + 8 = 1 + (3 + 8)$

In general form, $(a + b) + c = a +(b+ c)$, where a, b, c are natural numbers.

Sometimes we can add three numbers easily by using this property.

For example, find the sum of 235, 233 and 367.

$$235 + 233 + 367 = 235 + (233 + 367)$$

$$= 235 + 600$$

$$= 835$$

If in it, 235 +233 be obtained first, then it will become bit difficult to obtain the sum.

Example 1.1: Find the sum: 325 + 467 + 175

Sol. 325 + 467 + 175 = (325 + 175) + 467

= 500 + 467

= 967

Example 1.2: Find the sum 517 + 473 + 527

Sol. 517 + 473 + 527 = 517 + (473 + 527)

= 517 + 1000

= 1517

Intext Questions 1.1

1. Find the sum of each of the following and verify the sum obtained by reversing the order of the numbers:

   (i) 573 617 (ii) 2145 1355

   (iii) 243 357 (iv) 12345 34521

2 Fill in the blanks to make each of the following statements true:

   (i) 105 + 513 = + 105

   (ii) 345 + (118 + 202) = (345 +) + 202

   (iii) (108 + 413) + 517 = (517 +) + 413

   (iv) 2344 + (1432 + 4224) = (1432 + 2344) +

3. Find the sum of 15, 27 and 58 by grouping all possible ways.

4. Add the numbers in easier way

   (i) 537 368 463 (ii) 2493 3676 1324

1.2 Subtraction (Difference)

Subtraction (Difference) is the reverse process of addition. In it, we subtract the number of objects of a group from the number of objects in another given group . Thus it is clear that a number of the objects of a smaller group can be subtracted from the number of objects of a larger group. For example 78 - 43 = 35

In fact, 43 subtracted from 78 means to find such a number which when added to 43 to get 78.

So, 78 - 43 = 35 can be written as 43 + 35 = 78.

Let us learn the properties of Subtraction (Difference).

24 - 8 = 16, 47 - 17 = 30, 258 - 143 = 115

16, 30, 115 are all natural numbers. Can we say that difference of any two natural numbers will always be a natural number? Clearly no, such natural number exist which when added to 27 may give 15. So, 15 - 27 is not a natural number.

Thus, a natural number will be obtained on subtraction, if number to be subtracted is smaller than the other number.

Example 1.3: Perform the subtraction operation in the following. Verify your answer by performing the corresponding addition.

(i) 3251 - 539

(ii) 987654 - 78937

Solution: (i) 3251 - 539 = 2712

Verification 539 + 2712 = 3251

(ii) 987654 - 78937 = 908717

Verification 78937 + 908717 = 987654

Example 1.4: Find the difference between smallest number of 7 - digits and largest number of 5 - digits.

Solution: Smallest number of 7 - digits = 1000000

Largest number of 5 - digits = 99999

Therefore difference = 1000000 - 99999 = 900001

Example 1.5: Out of a sum of ₹5000, I paid ₹1,200 as electricity bill, ₹500 as School fee of my children and ₹1800 to the milk man. How much money is left with me?

Solution: Total amount spent = ₹ (1200 + 500 + 1800) = ₹ 3500

Amount of money left = ₹(5000 - 3500) = ₹1500

Intext Questions 1.2

1. Perform subtraction operation on the following and verify your answer by performing corresponding addition:

(i) 97 54 (ii) 576 247

(iii) 4276 1352 (iv) 59432 27654

2. Find the difference between smallest 6 - digit number and largest 4 - digit number.

3. In a group of buffalos, cows and sheep there are 536 animals. If number of buffalos is 218 and number of cows be 79, then find the number of sheep.

1.3 Multiplication

Let us take 4 boxes having 5 balls each.

Total number of balls 5 +5 +5 +5 = 20

Above stated addition fact can be written as '4 times 5' or '$4 \times 5 = 20$'

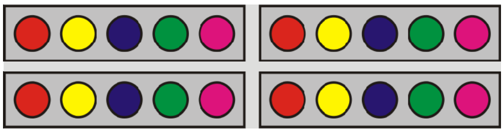


Figure 1.1

Similarly, if you have 6 bunches of 3 bananas each, then total number of bananas = 3 +3 + 3 + 3 + 3 + 3 = 18

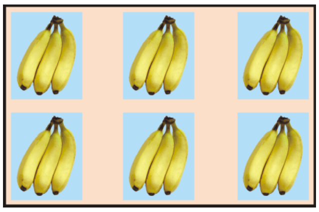


Figure 1.2

This can be presented by $6 \times 3 = 18$ also.

So we can say that multiplication is repeated addition.

Now we shall learn the properties of multiplication on Natural Numbers.

We know that

$7 \times 6 = 42$

$15 \times 10 = 150$

$27 \times 12 = 324$

Since 42, 150 and 324 are natural numbers.

So, if a and b are two natural numbers then $a \times b$ is also a natural number.

Again look at the following:

$3 \times 4 = 12$	and	$4 \times 3 = 12$	$\therefore$	$3 \times 4 = 4 \times 3$
$11 \times 8 = 88$	and	$8 \times 11 = 88$	$\therefore$	$11 \times 8 = 8 \times 11$
$23 \times 12 = 276$	and	$12 \times 23 = 276$	$\therefore$	$23 \times 12 = 12 \times 23$

We observe that order does not matter in multiplication i.e. $a \times b = b \times a$

Now let us multiply 3 natural numbers. This can be done in two ways.

Again observe the following:

(i) $5 \times 7 \times 6 = (5\times7) \times6 = 35\times6 = 210$

(ii) $5 \times 7 \times 6 = 5 \times (7\times6) = 5\times42 = 210$

So, $(5\times7) \times6 = 5 \times (7\times6)$

Hence, we observe that

$(a \times b) \times c = a \times (b \times c)$ when a, b, c are natural numbers.

Again we know that

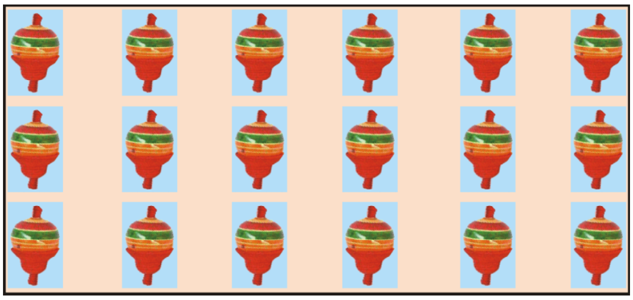
$9 \times 1 = 9$, $15 \times 1 = 15$, $27 \times 1 = 27$, $93 \times 1 = 93$

This means if any natural number is multiplied by 1 then same number is obtained.

In general form, if 'a' is a natural number, then

$a \times 1 = 1 \times a = a$

Now let us consider the following examples, where $3 \times 6 = 18$ has been depicted.



3x 6 = 18

Figure1.3

Now, if we fold the paper in such a manner that the folding line may change the figure as shown below:

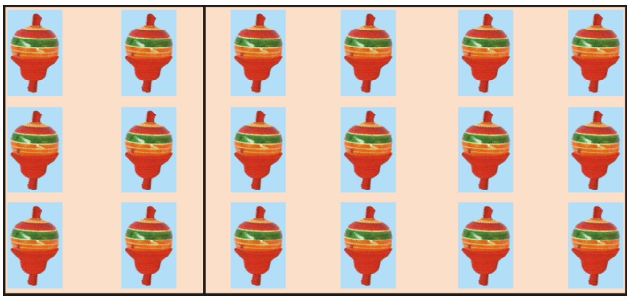


Figure 1.4

$3 \times 2 = 6$ and $3 \times 4 = 12$

From it we observe that

$3 \times 6 = 3 \times (2+4) = 3 \times 2 + 3 \times 4$

Note

For any three natural numbers a, b and c, $a \times (b + c) = a \times b + a \times c$

For verifying this result, let us consider an example.

$4 \times (5 + 6) = 4 \times 11 = 44$

And $4 \times 5 + 4 \times 6 = 20 + 24 = 44$

$\therefore\ 4 \times (5 + 6) = 4 \times 5 + 4 \times 6$

Similarly, we can see that

$5 \times (8 - 3) = 5 \times 8 - 5 \times 3,\ 12 \times (15 - 3) = 12 \times 15 - 12 \times 3$

We use this property for multiplication as follows:

$$\begin{aligned} 86 \times 43 &= 86 \times (40 + 3) \\ &= 86 \times 40 + 86 \times 3 \\ &= 3440 + 258 \\ &= 3698 \end{aligned}$$

This process can also be shown as:

$$\begin{array}{r l} 8\ 6 & \\ \times\ 4\ 3 & \\ \hline \boxed{2\ 5\ 8} & \text{On multiplying by 3} \\ 3\ 4\ 4\ 0 & \text{On multiplying by 40} \\ \hline 3\ 6\ 9\ 8 & \\ \hline \end{array}$$

On adding the sums we got 3698.

So, $86 \times 43 = 3698$

Example 1.6: Find the value of : $798 \times 65 + 798 \times 35$

Solution: $798 \times 65 + 798 \times 35 = 798 \times (65 + 35)$

$= 798 \times 100$

$= 79800$

Example 1.7: Find the value of: $2357 \times 143 - 43 \times 2357$

Solution: $2357 \times 143 \quad 43 \times 2357 = 2357 \times 143 \quad 2357 \times 43$

$= 2357 \times (143 \quad 43)$

$= 2357 \times 100$

$= 235700$

Example 1.8: Find the product: 725×94

Solution: $725 \times 94 = 725 \times (100 \quad 6)$

$= 725 \times 100 \quad 725 \times 6$

$= 72500 \quad 4350$

$= 68150$

Intext Questions 1.3

1. Fill in the blanks:

   (i) $247 \times \quad = 33 \times 247$

   (ii) $12 \times 45 \times 97 = 97 \times \quad \times 45$

   (iii) $578 \times 1 = 1 \times$

   (iv) $57 \times 36 = 57 \times 30 + 57 \times$

   (v) $213 \times 37 = 213 \times 40 \quad 213 \times$

2. Using the properties find the value:

   (i) $344 \times 6 + 344 \times 4$

   (ii) $247 \times 17 \quad 247 \times 7$

   (iii) $1025 \times 1275 \quad 275 \times 1025$

   (iv) $239 \times 6 + 239 \times 3 + 239$

3. Find the product by grouping:

   (i) $4 \times 1527 \times 25$

   (ii) $125 \times 278 \times 8$

   (iii) $250 \times 37 \times 4$

4. Using the properties find the product:

   (i) 273×51

   (ii) 3045×99

1.4 Division

We know the process of dividing a number by a smaller(or same) number . Now, we shall learn the properties of Division operation.

1. If 20 are to be divided equally among 5 children, then we will give 4 toys to each child, since $20 \div 5 = 4$

   But if we have 21 toys then can we divide them among 5 children equally? We cannot do so.

   $\therefore$ 21 is not completely divisible by 5.

   Thus, in natural numbers division may or may not be feasible.

2. Note: $45 \div 15 = 3$ (a natural number)

   But $15 \div 45$ is not a natural number.

   $\therefore\ 45 \div 15 \neq 15 \div 45$

In general form, for two natural numbers 'a' and 'b'

$a \div b \neq b \div a$

3. We know that $24 \div 6 = 4$, $4 \times 6 = 24$

   In general form, if a, b and c are natural numbers, $a \div b = c$ then $b \times c = a$

   Again if 25 is to be divided by 6, then $25 = 6 \times 4 + 1$.

Divisor) Dividend (Quotient
....
Remainder

Dividend = Divisor × Quotient + Remainder

Example 1.9: Divide 3475 by 234 and verify the answer by multiplication.

$$\begin{array}{r|l|l} 234 & 3475 & 14 \\ & -234 & \\ \hline & 1135 & \\ & -936 & \\ \hline & 199 & \end{array}$$

$\therefore$ On dividing 3475 by 234, quotient is 14 and remainder is 199

Verification 234 x 14 + 199 = 3276 + 199

= 3475 = Divisor

Intext Questions 1.4

1. Divide and verify the answer:

   (i) $3345 \div 15$ (ii) $9457 \div 43$

2. Find the value of the following:

   (i) $241 + (790 \div 79)$

   (ii) $(73 \div 73) + 45$

   (iii) $347 - (249 \div 249)$

   (iv) $(3125 \div 25) \div 25$

3. Cost price of 13 watches is ₹14400. Find the cost price of 1 watch.

1.5 Properties of operations on Whole Numbers

We know that on subtracting a natural number from itself we get zero, which is not a natural number. Natural numbers along with zero are called whole numbers.

1. Properties of operations which are true for Natural numbers are also true for whole numbers also.

2. Zero is a special number and needs attention.

   For an whole number 'a' ,

   (i) $a + 0 = 0 + a = a$

   (ii) $a - 0 \neq 0 - a$

   (iii) $a \times 0 = 0 \times a = 0$

   (iv) $a \div 0$ is not defined, since there is no such whole number which when multiplied by zero may give a.

Intext Questions 1.5

1. Write smallest whole number. Can you write largest whole number?

2. Fill in the blanks:

   (a) $473 + 0 =$ -----

   (b) $473 - 0 =$ -----

   (c) $473 \times 0 =$ -----

   (d) $473 \div 0 =$ -----

   (e) $(425 \times 1575) \times 0 =$ -----

1.6 Factors and Multiples

We know that

$18 = 1 \times 18$

$= 2 \times 9$

$= 3 \times 6$

1, 2, 3, 6, 9, 18 are such numbers that if 18 be divided by any of these numbers then remainder will always be zero.

So numbers 1, 2, 3, 6, 9, 18 are called factors of 18.

And number 18 is called the multiple of numbers 1, 2, 3, 6, 9, 18.

Similarly, numbers 1, 3, 5, 15 are factors of 15 and 15 itself is a multiple of 1, 3, 5, 15.

Thus, a factor of a natural number is the number, which may divide it exactly and natural number is called the multiple of each of its factors.

Hence, we conclude that

(i) 1 is a factor of every number or every number is a multiple of 1.

(ii) Every number is a factor of itself and is its own multiple also.

All multiples of 2 are called Even Numbers.

2, 4, 6, 8, 10 ... are even numbers; numbers which are not the multiples of 2 are called Odd Numbers.1, 3, 5, 7,9, 11, 13, 15 ... are all odd numbers.

Example 1.10: Write odd and even numbers separately:

1, 12, 14, 19, 44, 159, 240, 3451, 4437, 135792

Solution: Even Numbers: 12, 14, 44, 240, 135792

Odd Numbers: 1, 19, 159, 3451. 4437

Note: If at unit's place there be 2, 4,6,8, or 0, then the number is even. Look at the following numbers and their factors:

Numbers	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5

6	1, 2, 3, 6
7	1, 7
8	1 2 4 8
9	1 3 9
10	1 2 5 10

We note that the numbers 2, 3, 5, 7 have only two factors. Such numbers are called Prime Numbers.

Numbers 4, 6, 8, 9, 10 ... have more than two factors. Such numbers are called Composite Numbers. Number 1 has only one factor. Therefore it is neither prime nor composite number.

Note: Number 2 is the smallest prime number and it is the only even prime number.

Example 1.11: Write all prime numbers between 1 and 50.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

We know that 1 is not a prime number. Now 2 is a prime number. Encircle it. Now cross out all multiples of 2. Encircle 3. Cross out all multiples of 3. Similarly repeat the process with 5, 7 ... . We have prime numbers between 1 and 50:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

1.7 Twin Primes

When difference between two prime numbers is 2 then it is called pair of primes or Twin Primes.

Some Twin Primes are

(i) 3, 5 (ii) 5, 7 (iii) 11, 13

(iv) 17, 19 (v) 29, 31 (vi) 41, 43 -----

A popular Mathematician Goldbech gave a property that all even numbers greater than 4 can be expressed as sum of two odd prime numbers.

For example, 6 = 3 + 3

8 = 3 + 5

10 = 3 + 7 or 5 + 5

Note

12 = 5 + 7

14 = 3 + 11 or 7 + 7

16 = 3 + 13 or 5 + 11

1.7.1 Co-Prime Numbers

Two natural numbers are called Co-Prime Numbers, if they do not have any common factor other than 1.

For example,

(i) 2, 3 (ii) 2, 5 (iii) 3, 4

(iii) 3, 5 (v) 3, 7 (vi) 2, 7 (vii) 3, 8

Example 1.12: Write the factors of each of the following:

(i) 24 (ii) 30 (iii) 65 (iv) 95

Solution: (i) $24 = 1 \times 24$

$= 2 \times 12$

$= 3 \times 8$

$= 4 \times 6$

Factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

(ii) $30 = 1 \times 30$

$= 2 \times 15$

$= 3 \times 10$

$= 5 \times 6$

Factors of 30 are 1, 2, 3 5,6, 10, 15, 30.

(iii) $65 = 1 \times 65$

$= 5 \times 13$

Factors of 65 are 1, 5, 13, and 65.

(iv) $95 = 1 \times 95$

$= 5 \times 19$

Factors of 95 are 1, 5, 19 and 95.

Intext Questions 1.6

1. Write all factors of each of the following;

(i) 50 (ii) 64 (iii) 144 (iv) 243

2. Write four multiples of each of the following:

(i) 11 (ii) 18 (iii) 23 (iv) 49

3. Verify that if the following numbers are divisible by 27?

(i) 72900 (ii) 2430 (iii) 54793 (iv) 13527

1.7.2 Prime Factors

Let us write factors of 36

$$36 = 2 \times 18$$
$$= 2 \times 2 \times 9$$
$$= 2 \times 2 \times 3 \times 3$$

So we have prime factorisation of 36.This method of finding factors is known as prime factorisation method.

Example 1.13: Find the prime factorisation of the following numbers:

(i) 48 (ii) 120 (iii) 210 (iv) 440

Solution: (i)

2	48
2	24
2	12
2	6
3	3
	1

$\therefore \quad 48 = 2 \times 2 \times 2 \times 2 \times 3$

(ii)

2	120
2	60
2	30
3	15
5	5
	1

$\therefore \quad 120 = 2 \times 2 \times 2 \times 3 \times 5$

(iii)

2	210
3	105
5	35
7	7
	1

$\therefore \quad 210 = 2 \times 3 \times 5 \times 7$

(iv)

2	440
2	220
2	110
5	55
11	11
	1

$\therefore \quad 440 = 2 \times 2 \times 2 \times 5 \times 11$

From the examples given above we observe that every composite number can be uniquely expressed as a product of prime numbers.

Intext Questions 1.7

1. Write the prime factors for each of the following:

   (i) 12 (ii) 34 (iii) 56 (iv) 98

   (v) 136 (vi) 945 (vii) 540 (viii) 7325

2. Write the greatest 5-digit number and express it as the product of prime factors.

3. Write the smallest 4-digit number and express it as the product of prime factors.

1.8 Highest Common factor (HCF) and Least Common Multiple (LCM)

Factors of 16 are: 1, 2, 4, 8, 16

and factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36

Common factors of 16 and 36 are: 1, 2 and 4

$\therefore$ HCF = 4

HCF of 2 or more numbers is that number, which is the greatest amongst the common factors of the numbers.

Another method of finding HCF of two numbers is by writing the two numbers as the product of prime factors, and then finding the product of all the common factors. Every common factor is to be taken in least number.

For example,

$16 = 2 \times 2 \times 2 \times 2$

$36 = 2 \times 2 \times 3 \times 3$

We observe that prime factor 2 comes atleast 2 times in both the products.

$\therefore$ HCF $= 2 \times 2 = 4$

Finding HCF by Division method

There are two methods of finding HCF by Division method.

First method:

For example we are to find the HCF of 16 and 36 then

- Divide 16 and 36 by smallest prime number 2
- Then divide 8 and 18 by 2
- Now it is not possible to divide 4 and 9 both by any prime number

2	16, 36
2	8, 18
	4, 9

Therefore, HCF is the product of only those prime numbers which divide all the numbers simultaneously.

So, HCF of 16 and 36 $= 2 \times 2 = 4$

Let us understand by taking another example

To find HCF of 60, 90 and 210

- Divide 60, 90 and 210 by 2
- Divide 30, 45 and 105 by 3
- Divide 10, 15 and 35 by 5

2	60, 90, 210
3	30, 45, 105
5	10, 15, 35
	2, 3, 7

As now it is not possible to divide 2, 3 and 7 by any prime number simultaneously, so HCF of 60, 90 and 210 $= 2 \times 3 \times 5 = 30$

1.8.1 Least Common Multiple (LCM)

We know that LCM of two numbers is that smallest number, which is a multiple of every number i.e. it is divisible by all the numbers. For example if we are to find LCM of 12 and 16 then we will write their multiples as under:

Multiples of 12 are : 12, 24, 26, 48 , 60, 72, 84, 96 , -----

Multiples of 16 are : 16, 32, 48 , 64, 80, 96 , 112, -----

Common multiples of 12 and 16 are 48, 96 …

∴ Smallest common multiple of 12 and 16 = 48

∴ LCM = 48

LCM of two numbers can be found by using prime factorisation method also as under:

12 = 2 x 2 x 3

16 = 2 x 2 x 2 x 2

∴ LCM = 2 x 2 x 3 x 2 x 2 = 48

i.e. as a first step take all the common factors of the given numbers and multiply with remaining factors of both the numbers.

Finding LCM by Division method

To find LCM by division method

- As a first step divide all the numbers by the smallest prime factor common to atleast 2 numbers and write the quotients exactly below those numbers.
- For a number which is not divisible, copy it as it is below it.
- Continue the process till we have co-prime numbers below every number.

  We multiply all the divisors and co-prime numbers written in the last line to get desired LCM.

  For example to find LCM of 12, 16 and 24

2	12, 16, 24	All the numbers are divisible by 2.
2	6, 8, 12	All the numbers are divisible by 2.
2	3, 4, 6	Two numbers are divisible by 2.
2	3, 2, 3	Only one number is divisible by 2.
3	3, 1, 3	Two numbers are divisible by 3.
	1, 1, 1	

LCM is the product of all the divisors,

therefore, LCM of 12, 16 and 24 = 2 × 2 × 2 × 2 × 3 = 48

Let us understand by taking another example.

If we want to find LCM of 16, 32 and 64.

2	16, 32, 64
2	8, 16, 32
2	4, 8, 16
2	2, 4, 8
2	1, 2, 4
2	1, 1, 2
	1, 1, 1

$\therefore$ LCM of 16, 32 and 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Let us discuss one more example.

Six bells ring together. If they ring again after an interval of 2, 4, 6, 8, 10 and 12 seconds respectively, then after how much time will they ring together again?

To find the time after which they will ring again together, we need to find the LCM.

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
3	1, 1, 3, 2, 5, 3
2	1, 1, 1, 2, 5, 1
5	1, 1, 1, 1, 5, 1
	1, 1, 1, 1, 1, 1

Therefore LCM of 2, 4, 6, 8, 10 and 12 = $2 \times 2 \times 2 \times 3 \times 5 = 120$

All the bells will ring together again after 120 seconds or 2 minutes.

Now let us establish relation between LCM and HCF of 60 and 84.

$$\therefore\ 60 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 5$$

$$84 = \boxed{2} \times \boxed{2} \times \boxed{3} \times 7$$

$\therefore$ HCF = 2 x 2 x 3 = 12

Note

LCM = 2 × 2 × 3 × 5 × 7 = 420

Now 60 × 84 = 5040

and LCM xHCF = 420 ×12 = 5040

∴ Product of two numbers = Product of their HCF and LCM

∴ first numbers × second number = HCF and LCM

Therefore for two natural numbers

(i) $\text{HCF} = \dfrac{\text{Product of the two numbers}}{\text{LCM}} = \dfrac{\text{first number} \times \text{second number}}{\text{LCM}}$

(ii) $\text{LCM} = \dfrac{\text{Product of the two numbers}}{\text{HCF}} = \dfrac{\text{first numbers} \times \text{second number}}{\text{HCF}}$

Example 1.14: Find the LCM and HCF of 234 and 592.

Solution: We know that 234 = 2 × 3 × 3 × 13

and 592 = 2 × 2 × 2 × 2 × 37

HCF = 2

$$\text{LCM} = \frac{234 \times 592}{2}$$

= 117 × 592

= 69264

Example 1.15: HCF of two numbers is 128 and their LCM is 14976. If one of the numbers is 1664 then find the other number.

Solution: We know that

first number × second number = HCF × LCM

$$\text{Second Number} = \frac{\text{HCF} \times \text{LCM}}{\text{First Number}}$$

$$= \frac{128 \times 14976}{1664}$$

Second Number = 1152

Intext Questions 1.8

1. Find the HCF of the following numbers:

   (i) 60, 75 (ii) 36, 40 (iii) 36, 60, 72

   (iv) 144, 180, 384 (v) 276, 1242 (vi) 625, 3125, 15625

2. Find the HCF and LCM of the following numbers and verify that product of the two numbers = HCF x LCM

   (i) 145, 232 (ii) 117, 221 (iii) 27, 90

   (iv) 420, 660 (v) 135, 162

1.9 Divisibility Rules

To find that whether a number is divisible by another number or not, we do not need to actually perform the operation of division. We have some rules to examine this.

1.9.1 Divisibility by 2

If in any number digit at the unit's place is any one of the digits 0, 2, 4, 6 or 8 then number is divisible by 2. For example, each of the numbers 31240, 43572, 98764, 83246, 97698 is divisible by 2.

1.9.2 Divisibility by 3

A number is divisible by 3 for which sum of the digits is divisible by 3. For example, sum of the digits of number 12639 is 1 + 2 + 6 + 3 +9 is 21 which is divisible by 3, therefore 12639 is divisible by 3.

1.9.3 Divisibility by 4

A number is divisible by 4 for which number formed by its tens and units digits is divisible by 4. For example, number 54764 is divisible by 4 because 64 is divisible by 4. Number 876952 is divisible by 4 because 52 is divisible by 4. Number 1357642 is not divisible by 4 because 42 is not divisible by 4.

1.9.4 Divisibility by 5

A number is divisible by 5 if digit at the unit's place is either 0 or 5. For example, numbers 6215, 3570, 2495, 36840 are divisible by 5.

1.9.5 Divisibility by 6

A number is divisible by 6 if it is divisible by 2 as well as 3. For example, number 4320 is divisible by 2 (because its units digit is 0) and it is divisible by 3 also (4 + 3 +2 +0 = 9, is divisible by 3). Therefore number 4320 is divisible by 6.

1.9.6 Divisibility by 8

A number is divisible by 8 for which number formed by its hundreds, tens and units digits is divisible by 8. For example, number 5690248 is divisible by 8 because 248 is divisible by 8.

1.9.7 Divisibility by 9

A number is divisible by 9 for which sum of the digits is divisible by 9. For example, number 87642 is divisible by 9 because 8 + 7 + 6 +4 +2 is 27 which is divisible by 9.

1.9.8 Divisibility by 11

A number is divisible by 11 if difference in the sum of the digits at odd places and sum of digits at even places is either 0 or divisible by 11. For example, to examine the divisibility by 11 for 2080217062

sum of the digits at even places = 6 + 7 + 2 + 8 + 2 = 25

sum of the digits at odd places = 2 + 0 + 1 + 0 + 0 = 3

Difference = 25 - 3 = 22 which is divisible by 11.

∴ Number 2080217062 is divisible by 11.

Intext Questions 1.9

1. Using divisibility rules examine that following numbers are divisible by 2, 3, 5 or 9, or not.

   (i) 612 (ii) 276 (iii) 2650 (iv) 79124

   (v) 872645 (vi) 524781

2. For following numbers examine their divisibility by 4 and 8.

   (i) 63712 (ii) 763452 (iii) 51342 (iv) 35056

   (v) 234976 (vi) 2971

3. For following numbers examine their divisibility by 6.

(i) 297144 (ii) 46523 (iii) 9087248

(iv) 2070 (v) 35274 (vi) 93162

4. For following numbers examine their divisibility by 11.

(i) 83721 (ii) 438750 (iii) 723405

(iv) 3178965 (v) 70169803 (vi) 10000001

5. Which of the following statements are true?

(i) Even number is always divisible by 4.

(ii) A number divisible by 9 is always divisible by 3.

(iii) A number divisible by 6 is always divisible by 3.

(iv) A number divisible by 3 is always divisible by 9.

(v) A number divisible by 2 is always divisible by 6.

(vi) A number divisible by both 3 and 5 is always divisible by 15.

(vii) A number divisible by 3 and 6 is always divisible by 18.

(viii) A number divisible by 8 is always divisible by 4.

Let us Revise

1. If a, b and c are Whole numbers then
   - a + b and a x b will be Whole nnumbers.
   - a - b may or may not be a Whole nnumber.

2. Factor of a number always divides the number.
   - Multiple of a number is always divisible by the number.
   - Number 2 is the only even prime number.
   - HCFxLCM = Product of numbers.
   - HCF of 2 or more numbers is always a factor of their LCM.

Note

3. Divisibility

A number is

- divisible by 2 if its unit's digit is any one of the numbers 0, 2, 4, 6 or 8.
- divisible by 3 if sum of the digits is divisible by 3.
- divisible by 4 if number formed by its tens and units digits is divisible by 4.
- divisible by 5 if units digit is either 0 or 5.
- divisible by 6 if it is divisible by 2 as well as 3.
- divisible by 8 if number formed by its hundreds, tens and units digits is divisible by 8.
- divisible by 9 if sum of the digits is divisible by 9.
- divisible by 11 if difference in the sum of the digits at odd places and sum of digits at even places is either 0 or divisible by 11.

Exercise

1. Find the sum by taking some convenient order:

   (a) $3376 + 1808 + 2348 + 92 + 2652 + 1024$

   (b) $6254 + 1297 + 446 + 103$

2. Fill in the blanks:

   (i) $(400 + 7)\ (500 \quad 1) = 499 \times$

   (ii) $770 + 990 + 660 = 110 \times$

   (iii) $93 \times (100 \quad 9) = 91 \times (100 \qquad)$

   (iv) $(25 + 5)\ (25 \quad 5) = 625$

3. Which of the following statements are true?

   (i) Every Natural number is a Whole number.

   (ii) Every Whole number is a Natural number.

(iii) Zero is such a number that which when multiplied to any number gives the same number.

(iv) One is such a number that which when multiplied to any number gives the same number.

(v) It is always possible to divide a Whole number by another Whole number.

4. Find the value:

(i) $3457 \times 648 + 3457 \times 230 + 122 \times 3457$

(ii) $5641 \times 157 \quad 5641 \times 7 \quad 5641 \times 50$

5. Using properties find the values:

(i) $347 \times 7 + 347 \times 3$

(ii) $2136 \times 159 - 2136 \times 59$

(iii) $746 \times 10 \times 541 - 441 \times 7460$

6. Write the prime numbers between 50 and 100.

7. Write a twin prime between 50 and 100.

8. Examine the divisibility by 11 for the following numbers:

(i) 9020814 (ii) 70169803 (iii) 618618

(iv) 25926857 (v) 723715806

Note

Answers

Intext Questions 1.1

1	(i) 1190	(ii) 3500	(iii) 600	(iv) 46866
2	(i) 513	(ii) 118	(iii) 108	(iv) 4224
3	100			
4	(i) 1368	(ii) 7493		

Intext Questions 1.2

1	(i) 43	(ii) 329	(iii) 2924	(iv) 31778
2	90001			
3	239			

Intext Questions 1.3

1	(i) 33	(ii) 12	(iii) 578	(iv) 6
	(v) 3			
2	(i) 3440	(ii) 2470	(iii) 1025000	(iv) 2390
3	(i) 152700	(ii) 278000	(iii) 37000	
4	(i) 13923	(ii) 301455		

Intext Questions 1.4

1	(i) 223 0	(ii) 219 40		
2	(i) 251	(ii) 46	(iii) 346	(iv) 5
3	900			

Intext Questions 1.5

1.	0, No		
2.	(a) 473	(b) 473	(c) 0
	(d) Not possible	(e) 0	

Intext Questions 1.6

1 (i) 1 2 5 10 25 50

(ii) 1 2 4 8 16 32 64

(iii) 1 2 3 4 6 9 12 18 24 36 48 72 144

(iv) 1 3 9 27 81 243

2 (i) 11 22 33 44

(ii) 18 36 54 72

(iii) 23 46 69 92

(iv) 49 98 147 196

3. Numbers 72900, 2430, 13527 are divisible by number 27.

Intext Questions 1.7

1 (i) $2 \times 2 \times 3$ (ii) 2×17 (iii) $2 \times 2 \times 2 \times 7$

(iv) $2 \times 7 \times 7$ (v) $2 \times 2 \times 2 \times 17$ (vi) $5 \times 3 \times 3 \times 3 \times 7$

(vii) $2 \times 2 \times 3 \times 3 \times 3 \times 5$ (vi) $5 \times 5 \times 293$

2 $99999 = 3 \times 3 \times 11111$

3 $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

Intext Questions 1.8

1 (i) 15 (ii) 4 (iii) 12 (iv) 12

(v) 138 (vi) 625

2 (i) 29, 1160 (ii) 13, 1989 (iii) 9, 270 (iv) 60, 4620

(v) 27, 810

Intext Questions 1.9

1. (i) 612: divisible by 2, 3 and 9, not divisible by 5.

(ii) 276: divisible by 2 and 3, not divisible by 5and 9.

(iii) 2650: divisible by 2 and 5, not divisible by 3 and 9.

(iv) 79124: divisible by 2, not divisible by 3, 5 and 9.

(v) 872645: divisible by 5, not divisible by 2, 3 and 9.

(vi) 524781: divisible by 3 and 9, not divisible by 2 and 5.

Note

2. (i) divisible by 4 and 8.

   (ii) divisible by 4, not by 8.

   (iii) not divisible by 4 and 8.

   (iv) divisible by 4 and 8.

   (v) divisible by 4 and 8.

   (vi) not divisible by 4 and 8.

3. (i) divisible (ii) not divisible (iii) not divisible

   (iv) divisible (v) divisible (vi) divisible

4. (i) divisible (ii) not divisible (iii) not divisible

   (iv) not divisible (v) divisible (vi) divisible

5. (i) False (ii) True (iii) True (iv) False

   (v) False (vi) True (vii) False (viii) True

Exercise

1 (a) 11300 (b) 8100

2 (i) 407 (ii) 22 (iii) 7

  (iv) 25 (v) 111

3. (i) True (ii) False (iii) True

   (iv) False (v) False

4 (i) 3457000 (ii) 564100

5 (i) 3470 (ii) 213600 (iii) 746000

6 53 59 61 67 71 73 79 83 89 97

7 59 61 71 73

8. (i) is divisible (ii) is divisible (iii) is divisible

   (iv) is divisible (v) is divisible

2

INTEGERS

While studying operations on Whole numbers we observed that it is not always possible to subtract a number from another number. For example 15-17, 12-15, 8-10 can not be expressed by a whole number.

To express such operations we need to extend the Number system.

From this lesson, you will learn

- Representing Integers on a Number line
- Putting Integers in order
- Finding the absolute valueof Integers
- Operations on Integers andtheir properties

2.1 Creating a new number for every natural number

For 1 we create -1 (called negative 1) in such a manner that 1 + (-1) = 0, therefore 1 and (-1) are called inverse of each other. On the similar lines by creating -2 for 2, -3 for 3 and -4 for 4 etc. we get the following collection of numbers.

0, 1, -1, 2, -2, 3, -3 … These numbers are called Integers. Numbers1, 2, 3, 4… are called positive integers and -1, -2, -3, -4… are called negative integers. Number Zero (0) is the only integer which is neither positive nor negative.

2.2 Representation of Integers on Number line

We know that negative integers are the opposite of positive integers; therefore these can be represented on a number line in the opposite directions. It means that positive integers are represented on the right hand side of zero and negative integers are represented on the left hand side of zero. You can observe that that 2 and -2 are on the right hand side and left hand side respectively of zero and are at equal distance from zero.

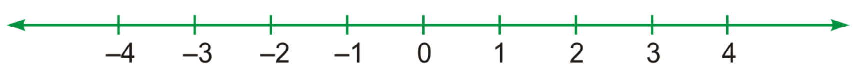


Figure 2.1

2.2 Ordering Integers on Number line

You know that +9 is greater than +7, because on number line distance of +9 from zero is more than that of +7. On the right side of the number line any integer which is at more distance from zero will be greater. Conversly on the left side of the number line any integer which is at more distance from zero will be smaller.

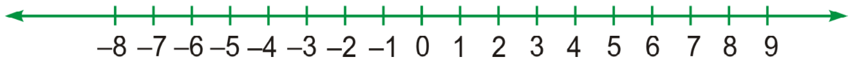


Figure 2.2

+5 > +3 because, +5 is at more distance in comparison to +3 on the right side of zero.

-5 < -3 because, -5 is at more distance in comparison to -3 on the left side of zero.

From this we can conclude that -

- Every positive integer is greater than every negative integer.
- On the right side of zero on the number line whichever integer is at more distance from zero will be greater.
- On the left side of zero on the number line whichever integer is at more distance from zero will be smaller.
- Zero is smaller than every positive integer.
- Zero is greater than every negative integer.

Let us learn from some other examples-

- On comparing +7 and -3 we get +7 > -3, because every positive integer is greater than every negative integer.
- On comparing -10 and -13 we get -10 > -13, because on number line -13 is at more distance from zero than -10. Therefore -13 is smaller.
- On comparing 0 and -8 we get 0 > -8, because zero is greater than every negative integer.

  We know that on number line every integer represented on the right is greater than integer on its left side, e.g. 4 > 2 because 4 is on the right side of 2 on the number line. Similarly we have 0 > -1 and -2 > -3.

2.4 Addition and Subtraction of integers using Number line

We know that to represent 5 on number line we are to move five steps from zero on its right side and for -5, five steps on its left side.

To represent '3 + 5' on the number line, firstly we will reach at 3 after moving 3 steps from zero on its right and then moving 5 steps on the right we reach at 8.

Therefore 3 + 5 = 8

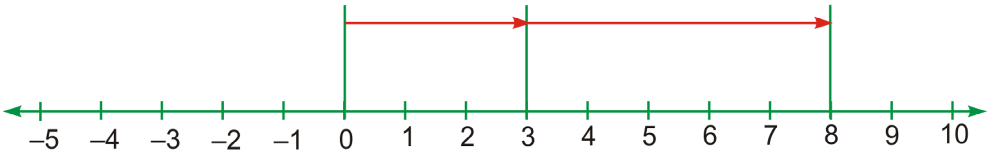


Figure 2.3

Now for representing '-3 + 5' on the number line, first we will reach at -3 after moving 3 steps from zero on its left and then moving 5 steps on the right of '-3' we reach at 2.

Therefore -3 + 5 = 2

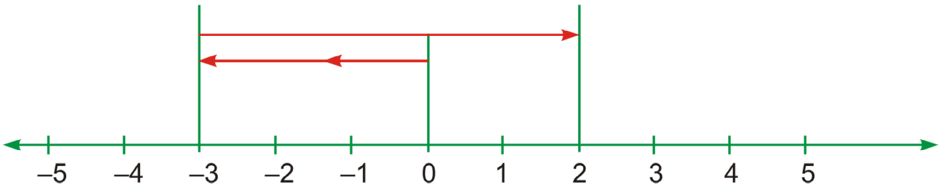


Figure 2.4

Now let us represent (-3) + (-5) on the number line.

To represent '-3' we will move 3 steps from zero on its left and then after moving 5 steps further on the right of '-3' we reach at '-8'.

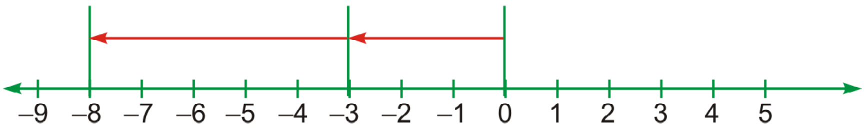


Figure 2.5

Therefore (-3) + (-5) = -8

Subtraction of two integers

If we wish to subtract 5 from 3 with the help of number line, then we are to find a number which when added to '3' gives 5. On the number line when we move 2 steps from '3'on its right, we get '5'.

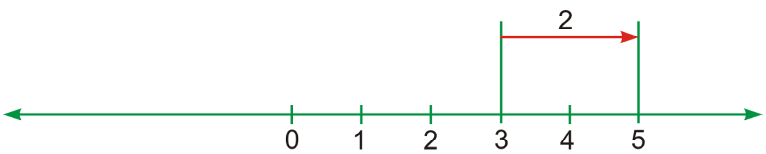


Figure 2.6

Therefore (5)-(3) = 2

Now for subtracting (-3) from '5' we are to find a number which when added to '-3' gives 5. In other words, we need to know the number of steps to be moved on the right side of '-3' to reach at 5. We will have to move 8 steps to the right of '-3'.

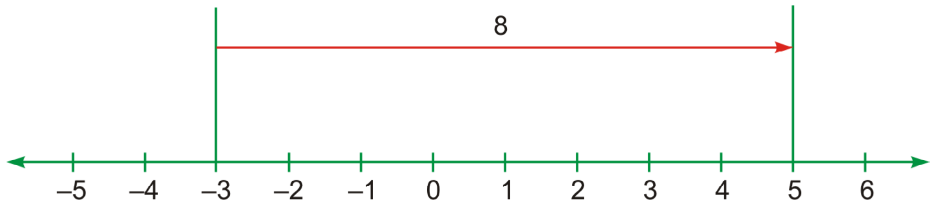


Figure 2.7

Therefore (5)-(-3) = 8

Similarly to subtract '-5' from '-3' we are to move on the right side of '-5' to reach '-3', and for this we need to move 2 steps to the right.

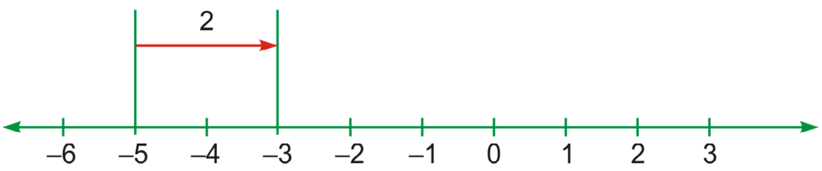


Figure 2.8

Therefore (-3)-(-5) = 2

If '-3' is to be subtracted from '-5' then by moving two steps from '-3' on its left side we reach at '-5'.

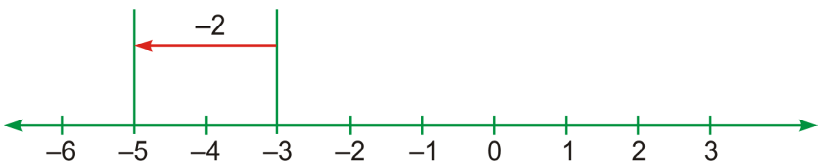


Figure 2.9

Therefore (-5)- (-3) = -2

Intext Questions 2.1

1. From the following pairs of numbers which number is smaller?

   (i) 5, -5 (ii) -12, -8

   (iii) 0, -3 (iv) 405, -517

2. Write the integers between:

   (i) -3 and 3 (ii) 0 and 5

   (iii) -4 and 0 (iv) -7 and -1

Note

3. In each of the following write > or < in place of '*', so that statement is true:

   (i) -3 * -7 (ii) 0 * 4

   (iii) -3 * 2 (iv) -8 * 8

4. Find the value:

   (i) -4 + 7 (ii) 6 + (-8)

   (iii) -2 + (-7) (iv) 7 - (-2)

   (v) -8 - (-3) (vi) 0 - (-5)

2.5 Absolute value of an Integer

Absolute value of an integer is its that numerical value in which we ignore its sign '+' or '-'. On number line an absolute value of an integer means the distance of that integer from 'zero', in which we do not care for the sign.

Therefore, absolute value of + 3 is 3

absolute value of - 3 is 3

absolute value of 0 is 0

To represent absolute value of a number, we place the number between two vertical line segments.

Therefore, absolute value of -5 is written as $|-5|$.

Therefore, $|7| = 7; |-7| = 7$

2.6 Operations on Integers

2.6.1 Addition of Integers

We have got the following results of addition of two integers using number line.

5 + 4 = 9,

7 + (-3) = 4

(-6) + (-3) = -9,

(-5) + (2) = -3

In this way we can conclude that

(i) To add two positive integers or two negative integers we add their absolute values and put the sign of the addends with the sum.

(ii) To add a positive integer and a negative integer, first we find the difference between their absolute values and then put the sign of the number with greater absolute value with the difference.

Note

Example 2.1: Add -537 and -231

Solution: $(-537) + (-231) = -(|-537| + |-231|)$

$= -(537 + 231)$

$= -768$

Example 2.2: Add 405 and -227

Solution: $(405) + (-227) = +(|405| - |-227|)$

$= +(405 - 227)$

$= 178$

Example 2.3: Add -349 and 127

Solution: $(-349) + (127) = -(|-349| + |127|)$

$= -(349 - 127)$

$= -222$

Example 2.4: Find the addition of -15, -47 and 84

Solution: $(-15) + (-47) + 84 = [-15 + (-47)] + 84$

$= -[|-15| + |-47|] + 84$

$= -[15 + 47] + 84$

$= -62 + 84 = 22$

2.6.2 Properties of Addition of Integers

1. Please concentrate on addition of the following integers:

   (i) $5 + (-8) = -3$

   (ii) $15 + (-11) = 4$

   (iii) $-6 + (-7) = -13$

   The sums -3, -4 and -13 are also integers.

Therefore if a and b are two integers then a + b also is an integer.

2. (i) $4 + (-5) = -1$ and $(-5) + 4 = -1$

   (ii) $(-4) + (-5) = -9$ and $(-5) + (-4) = -9$

Therefore a + b = b + a, where 'a' and 'b' are integers.

3. (i) -3 + (-5) + 4 = (-3) + (-5) + 4 = (-8) + 4 = -4

   Or -3 + (-5) + 4 = (-3) + (-5) + 4 = (-3) + (-1) = -4

Therefore (a + b) + c= a + (b + c), where a, b and c are integers.

4. (-3) + 0 = -3 and 0 + (-3) = -3

Therefore a + 0= 0 + a = a, a is an integer.

2.6.3 Subtraction of Integers

Using number line we have seen that 5 - (-3)= 8, 3 - (-2) = 5

5 - (-3)= 8, or 5 + (inverse of -3) = 5 + (+3) = 8

Therefore, when we have two same signs (either both positive or both negative) in multiplication we always get a positive number. If we have opposite signs we always get a negative number.

So a - (-b) = a + b; a + (-b) = a - b

a + (+b) = a + b; a - (+b) = a - b

Example 2.5: Subtract -15 from 18

Solution: 18 - (-15) = 18 + 15 = 33

Example 2.6: Subtract -27 from -45

Solution: (-45) - (-27) = -45 + 27 = -18

Note: To find the value of multinomial expressions (with positive and negative numbers) we add all the positive integers and add all the negative integers separately, and then find the sum of both.

Example 2.7: Find the value of -17 + 25 - (-37) + (- 28) + (-15)

Solution: We can re-write the given expression as

-17 + 25 + 37 + (- 28) + (-15) = -17 + (- 28) + (-15) + 25 + 37

= -60 + 62 = 2

2.6.4 Properties of Subtraction of Integers

1. We have seen that difference of two integers is again an integer.

   Therefore **if a, b be two integers then a - b is also an integer.**

2. a - 0 = a, where a is any integer.

2.6.3 Multiplication of Integers

Look at the following products of integers

$3 \times (-4) = (-4) + (-4) + (-4)$

$= -12 = -(3 \times 4)$

Similarly $(-3) \times 5 = (-3) + (-3) + (-3) + (-3) + (-3)$

$= -15 = -(3 \times 5)$

Therefore to find the product of a positive and a negative integer we find the product of their absolute values and put a negative sign with it.

Example 2.8: Find the value of $(-15) \times 8$

Solution: $(-15) \times 8 = -(15 \times 8) = -120$

Look at the following table of multiplications of integers

$(-5) \times 3 = -15$

$(-5) \times 2 = -10$

$(-5) \times 1 = -5$

$(-5) \times 0 = 0$

$(-5) \times (-1) = ?$

$(-5) \times (-2) = ?$

We see that when second integer decreases by 1 then product increases by 5. Therefore $(-5) \times (-1)$ must be 5 more than 0 (i.e. 5) and

$(-5) \times (-2)$ must be 5 more than 5 (i.e. 10).

In this way $(-5) \times (-1) = 5$ or (5×1)

$(-5) \times (-2) = 10$ or (5×2)

Therefore if both the integers are positive (or negative) then their product is a positive integer which is the product of their absolute values.

Example 2.9: Find the value of $(-25) \times (-40)$.

Solution: $(-25) \times (-40) = +(25 \times 40)$

$= 1000$

2.6.6 Properties of Multiplication of Integers

1. $3 \times (-4) = -12, \; -5 \times (-6) = 30$

Note

Products -12 and 30 are also integers.

Therefore if a, b are integers then a × b is also an integer.

2. $4\times(-5) = -20$, $(-5)\times4 = -20$

   In this way $4\times(-5) = (-5)\times4$

   Therefore, $a \times b = b \times a$, where a, b are integers.

3. $(3 \times 4)\times(-5) = 12\times(-5) = -60$

   and $3 \text{ x } [4 \times (-5)] = 3\times(-20) = -60$

   In this way $(3 \times 4)\times(-5) = 3 \times [4 \times(-5)]$

   Therefore, $(a \text{ x } b) \times c = a \times (b \times c)$, where a, b, c are integers.

4. $a\times0 = 0 \times a = 0$, a is any integer.

5. $a\times1 = 1\times a = a$, a is any integer.

6. $-2 \times [(-6) + 5] = -2 \times [-1] = 2$

   And$(-2)\times(-6) + (-2)\times5 = +12 + (-10) = 2$

So $-2\times[(-6) + 5] = (-2)\times(-6) + (-2)\times5$

Therefore, $a\times(b + c) = (a \times b) + (a \times c)$, where a, b, c are integers.

2.6.7 Division of Integers

We know that division is reverse of multiplication. Therefore, dividing 45 by (-5) means that by which (-5) be multiplied to get the product 45.

$(-57) \div 19 = -3$, because $-3\times19 = -57$

and $(-40) \div (-8) = 5$, because $5\times(-8) = -40$

Therefore we conclude that

(i) If two integers are positive or both are negative then their quotient is a positive integer which is the quotient of absolute values of the two integers.

(ii) Quotient of a positive integer and a negative integer is a negative integer having absolute value as the quotient of absolute values of the two integers.

Example 2.10: Divide 80 by -16.

Solution: $80\div(-16) = -(|-80| \div |-16|)$

$= -(80 \div 16) = -5$

2.6.8 Properties of Division of Integers

(-15) ÷5 = -3, which is an integer

But (-17) ÷ 5 is not an integer.

1. So if a and b are two integers then a ÷ b is not always an integer.
2. 0 ÷ a = 0, where a ÷ 0 is not defined.

Intext Questions 2.2

1. Add the integers:

   (i) -312 and 217 (ii) -425 and -308

   (iii) -231 and 231 (iv) 125 and -45

2. Find the sum:

   (i) 200 + (135) + (65)

   (ii) 15 + 135 + (250)

3. Subtract:

   (i) 17 from -13 (ii) -25 from 18

   (iii) -115 from -25 (iv) -315 from 0

   (v) 0 from -412

4. Evaluate:

   (i) 35 (28)

   (ii) 17 18 (45)

5. Find the following products:

   (i) 3 × (13) (ii) (115) × 4

   (iii) (27) × (30) (iv) 5 × (8) × 4

   (v) (317) × (225) × 0

6. Verify each of the following:

   (i) 15 × (5) × 20 = 15 × (5) × 20

   (ii) 28 × (11 + (9) = 28 × 11 + 28 × (9)

7. Divide:

   (i) (-85) by 17 (ii) 72 by (-12)

8. Fill in the blanks:

   (i) 88 ÷ = 8 (ii) 108 ÷ = 9

   (iii) 144 ÷ = 16 (iv) ÷ 12 = 8

2.7 Use of grouping symbols

To simplify expressions with two or more than two operations we perform the operations in the following order: First divide, then multiply, then add and in the end subtract.

For example: 24 - 6 ÷ 3 x 4 = 24 - 2 x 4

= 24 - 8

= 16

To determine the operation to be performed at the first place, we use brackets (grouping symbols).

For example: 49÷ (3 + 4) = 49÷ 7 = 7.

When we need more than one bracket, then we use the following brackets:

Symbol	Name
( )	Small bracket
{ }	Medium bracket
[ ]	Big bracket

Left side of every symbol is its beginning and right side denotes its end. Sequence of their use is as [{( )}].

When [{( )}] has been used then at the first place we remove the inner most brackets by performing the operations mentioned in them. After that the brackets next to these are removed.

Example 2.11: Simplify {15 + (5-8)} ÷ 6

Solution: {15 + (5-8)} ÷ 6 or {15 - 3}÷ 6 = 12 ÷ 6 = 2

If there is no operation symbol between any number and the brackets then it is taken as 'multiplication'.

For example, 5 (43 - 13) = 5 x (43 - 13)

= 5 x 30 = 150

Example 2.12: Evaluate5-[12 + {9 - (17 - 3)}]

Solution: 15 -[12 + {9 - (17 - 3)}] = 15 -[12 + {9 - 14}]

Note

$= 15 - [12 - 5]$

$= 15 - 7$

$= 8$

Intext Questions 2.3

1. Find the value of:

   (i) $42 + 45 \div 9$ (ii) $320 - 120 \div 8$

   (iii) $13 - (15 - 18 \div 3)$ (iv) $(-10) + (-6) \div (-2) \times 3$

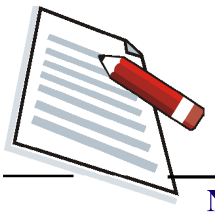
2. Simplify:

   (i) $30 + \{20 - 15 - (8 - 3)\}$

   (ii) $29 - [14 + \{16 - (12 - 4)\}]$

Let us Revise

- Zero is greater than every negative integer and smaller than every positive integer.
- Every positive integer is greater than every negative integer.
- Absolute value of an integer is its only numerical value in which we ignore its sign '+' or '-'.
- Sum of two negative integers is a negative integer, whose absolute value is same as the sum of their absolute values.
- To add a positive integer and a negative integer, we find the difference between their absolute values and then put the sign of the number with greater absolute value.
- After subtracting the integer b from the integer a we gets a - b.
- To find the product (or quotient) of a positive and a negative integer we find the product (or quotient) of their absolute values and put a negative sign with it.
- Product (or quotient) of two positive integers or two negative integersis a positive integer which is the product (or quotient) of the absolute values of the two integers.
- To simplify expressions with two or more than two operations we perform the operations in the order: first division, then multiplication, then addition and then subtraction.
- To remove brackets at the first place we remove small brackets, then medium brackets and in the end large brackets.



Exercise

1. Represent the following integers on the number line:

(i) -7   (ii) -3

(iii) 0   (iv) 5

(v) 7

2. Using number line write down which integer is

(i) 5 more than 2   (ii) 4 less than -3

(iii) 7 more than -8   (iv) 5 less than 3

3. Find the sum:

(i) 253 + (-133)   (ii) (-625) + (-3512) + 625

4. Subtract:

(i) 65 from (-34)   (ii) -30 from (-45)

(iii) sum of (-450) and 210 from 240   (iv) 395 from 0

5. Simplify:

(i) (-7) x 8 + (-7) x 12   (ii) 14 x (-12) + 16 x(-12)

6. Find the quotient:

(i) 21 ÷ (-3)   (ii) (-21) ÷ 3

(iii) (-64) ÷ (-16)   (iv) 0÷ 3215

7. Simplify:

(i) 16 + 8 ÷ 4 - 2 x 3   (ii) (-16) ÷ (-8) + (-4)

Answers

Intext Questions 2.1

1 (i) 5   (ii) 12   (iii) 3   (iv) 517

2 (i) 2 1 0 1 2   (ii) 1 2 3 4

(iii) 3 2 1   (iv) 6 5 4 3 2

3 (i) 3 > 7   (ii) 0 < 4   (iii) 3 < 2   (iv) 8 < 8

4 (i) 3   (ii) 2   (iii) 9   (iv) 9

(v) 5   (vi) 5

Note

Intext Questions 2.2

1 (i) 95 (ii) 733 (iii) 0 (iv) 80

2 (i) 0 (ii) 100

3 (i) 30 (ii) 43 (iii) 90 (iv) 315

(v) 412

4 (i) 7 (ii) 10

5 (i) 39 (ii) 460 (iii) 810 (iv) 160

(v) 0

7 (i) 5 (ii) 6 (iii) 7 (iv) 1

8 (i) 11 (ii) 12 (iii) 9 (iv) 96

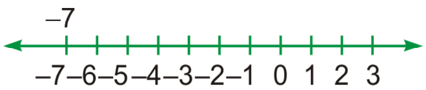
Intext Questions 2.3

1 (i) 47 (ii) 305 (iii) 4 (iv) 1

2 (i) 30 (ii) 7

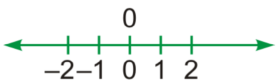
Exercise

1 (i)



(ii)

(iii)



(iv) (v)

2 (i) 7 (ii) 7 (iii) 1 (iv) 2

3 (i) 120 (ii) 3512

4 (i) 99 (ii) 15 (iii) 480 (iv) 395

5 (i) 140 (ii) 360

6 (i) 7 (ii) 7 (iii) 4 (iv) 0

7 (i) 12 (ii) 2

3

SQUARE, SQUAREROOT AND CUBE, CUBE ROOT

You have already studied about Natural numbers, Whole Numbers and Integers. You have also studied about operations like addition, subtraction, multiplication and division in these numbers.

From this lesson, you will learn

- Finding out square and square root of Natural numbers.
- Square of an even number is even number and square of an odd number is an odd number.
- Finding a square root of a number using factorisation method.
- Finding a square root of a number using division method.
- Solving some problems through square root.
- Meaning of Cube and Cube root.
- Finding cube root of a perfect cube number by prime factorisation method.

3.1 Squares of numbers

You already know that

$1 \times 1 = 1$

$2 \times 2 = 4$

$3 \times 3 = 9$

$7 \times 7 = 49$

$10 \times 10 = 100$

In the above examples numbers have been multiplied by themselves. Results found are their products.

Note

This result is known as its square. It means that square of 1 is 1, square of 2 is 4, square of 3 is 9, etc.

Therefore $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ …

In the following table squares of numbers from 1 to 20 have been given.

Natural number	Square	Natural number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

We know that product of two negative numbers is a positive number. Therefore

$(\ 1) \times (\ 1) = 1$

$(\ 2) \times (\ 2) = 4$

$(\ 3) \times (\ 3) = 9$

In this way we observe that square of a negative number is a positive number.

Square of a negative number is a positive number.

Example 3.1: Find the squares of the following numbers:

(i) 27 (ii) 13 (iii) 36

Solution:

(i) Square of 27 = $27^2 = 27 \times 27 = 729$

(ii) $(-13)^2 = (-13) \times (-13) = 169$

(iii) $(-36)^2 = (-36) \times (-36) = 1296$

Intext Questions 3.1

1. Find the squares of the following numbers:

   (i) 9 (ii) 25 (iii) 8 (iv) 19

3.2 Squares of Natural numbers

Let us consider squares of Natural numbers:

$1^2 = 1$ $2^2 = 4$

$3^2 = 9$ $4^2 = 16$

$5^2 = 25$ $6^2 = 36$

$9^2 = 81$ $10^2 = 100$

Here we observe that

1. Square of an odd number is always an odd number.
2. Square of an even number is always an even number.

Example 3.2: Observe the following pattern carefully:

$2^2 = 4 = 3 \times 1 + 1$

$3^2 = 9 = 3 \times 3$

$4^2 = 16 = 3 \times 5 + 1$

$5^2 = 25 = 3 \times 8 + 1$

$6^2 = 36 = 3 \times 12$

$7^2 = 49 = 3 \times 16 + 1$

What do you conclude from this pattern? Justify your conclusion by giving an example.

Solution: By observing this pattern we come to know that square of every number greater than 1 can be written as either a multiple of 3 or (a multiple of 3) + 1.

For example,

$8^2 = 64 = 3 \text{ x } 21 + 1$

and $9^2 = 81 = 3 \text{ x } 27$

Intext Questions 3.2

1. Find the square of each of the following:

   (i) 67 (ii) 83 (iii) 101 (iv) 71

2. Which of the following numbers are square of a Natural number?

   64, 40, 36, 35

3. Which of the following numbers have even numbers as their square?

   31, 48, 115, 526, 1250

4. Which of the following numbers have odd numbers as their square?

   309, 5002, 4709, 484, 1111

5. Observe the following pattern carefully:

   $2^2 = 2 \times 2$

   $3^2 = 2 \times 4 + 1$

   $4^2 = 2 \times 8$

   $5^2 = 2 \times 12 + 1$

   $6^2 = 2 \times 18$

   $7^2 = 2 \times 24 + 1$

   Extend this pattern and write next two items.

3.3 Square root

In the earlier section we studied about squares of numbers. 4 is a perfect square number, because it is square of 2. In other words we can say that square root of 4 is 2. Square of 4 is 16. Therefore square root of 16 is 4.

Because square of 5 is 25, therefore square root of 25 is 5.

Therefore, square root of a number 'a' is that number, which when multiplied by itself gives number 'a' as the product. We use the surd $\sqrt{\ }$ for positive square root.

$\therefore \sqrt{16} = 4, \quad \sqrt{36} = 6, \quad \sqrt{100} = 10$ etc.

Also we know that

$(\ 2) \times (\ 2) = 4 \qquad (\ 3) \times (\ 3) = 9$

$(\ 4) \times (\ 4) = 16$

It means that square root of 4 is (-2) also.

It means that square root of 9 is (-3) also.

It means that a square root of 16 is (-4) also.

From this we come to know that every number has two square roots. One of which is positive and other is negative.

But in this lesson we will discuss only positive square roots.

Can you think of a number which when multiplied by itself gives a product a negative number?

Your answer will be 'No'.

So we can say that

Square root of any negative number cannot be found.

3.4 Finding a square root of a perfect square by factorisation method

We know that 3 x 3 = 9

So $\sqrt{9} = \sqrt{3 \times 3} = 3$

Similarly $5 \times 5 \times 5 \times 5 = 625$

and $\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5} = 5 \times 5 = 25$

and $2 \times 2 \times 3 \times 3 = 36$, therefore $\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3} = 2 \times 3 = 6$

From these examples, we observe that if in the prime factorisation of a number any factor comes twice, then it comes once in its square root. Therefore, for finding the square root of a number we find the product bytaking a prime factor from the pairs of its prime factors.

In this technique of finding square root, we follow the following steps:

(i) First of all do prime factorisation of the given number.

(ii) Then make pairs of like factors.

(iii) Then find the product by taking one number from each pair. The resulting product is the desired square root.

Example 3.3: Find the square root of 324.

Solution:

Note

2	324
2	162
3	81
3	27
3	9
	3

$\therefore\ 324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$\therefore\ \sqrt{324} = 2 \times 3 \times 3$

$= 18$

Example 3.4: Find the square root of 2601.

Solution:

3	2601
3	867
17	289
	17

$\therefore\ 2601 = 3 \times 3 \times 17 \times 17$

$\therefore\ \sqrt{2601} = 3 \times 17 = 51$

Intext Questions 3.3

Find the square root of each of the following using factorisation method:

1 1296   2 4225   3 50176   4 5184   5 160000

3.5 Finding a square root of a perfect square by division method

In the previous section, we found the square root of numbers by Factorisation method but when numbers are very large or their factors are not easy to find, in that situation we apply the division method. Let us find the square root of 1296 by division method.

We observe that

$30^2 = 900$

and $40^2 = 1600$

So, square root of 1296 is a number which is between 30 and 40. It means ten's digit of the square root will be 3, unit's digit in 96 is 6 and also 4 x 4 = 16 and 6 x 6 = 36.

Therefore unit's digit is either 4 or 6.

It can be verified that

$34 \times 34 = 1156$ and $36 \times 36 = 1296$

Therefore $\sqrt{1236} = 36$

We observe that number of digits in the square root is two, whereas number of digits in the given number is 4. By division method we can solve this example as follows:

	36
3	12,96
	–9
66	396
	–396
	0

Steps of the technique:

1. Beginning from the unit's digit pair the digits of the number. Here there are two such pairs.
2. Find the greatest such number whose square is either 12 or less than 12. Such number is 3. Write 3 as the divisor.

   Write its square 9 below 12 and write 3 in the quotient also.
3. Subtract 9 from 12 to find the first remainder 3 and write the next pair of numbers on the right hand side of this remainder. Now dividend is 396.
4. Write double of 3 i.e. 6 in the ten's place of the probable divisor.
5. Now think of a number which after writing in the unit's place with 6 and then multiplied by this number yields the product 396.
6. In this way next divisor is 66, which when multiplied by 6 gives 396.
7. Write 6 in the quotient on the right side of 3 and subtract 396 from divisor 396. Now balance is 0.

   $\therefore \sqrt{1296} = 36$

   Now, let us apply this method to find a square root of 6-digit number.

   Assume that number is 290521.

Three pairs of the number 290521 are 29, 05, and 21.

So its square root will be a 3-digit number.

	5 3 9
5	29, 05, 21
	–25
103	405
	–309
1069	9621
	–9621
	0

Steps of the technique:

1. Beginning from the unit's digit divide the number in pairs of digits. Here there are three such pairs.

2. Find the greatest such number that its square is either 29 or less than 29. Such number is 5. Write 5 as the divisor. Write its square 25 below 29 and write 5 in the quotient also at hundred's place.

3. Write the remainder 4 after subtract 25 from 29 and write the next pair of numbers on the right hand side of this remainder. Now dividend is 405.

4. Write double of 5 i.e. 10 in the ten's place of the next divisor.

5. Now observe that 40 ÷ 10 = 4, which means use 104 as the next divisor.

   But 104 x 4= 416 is more than 405.

   ∴ 104 cannot be the divisor. Therefore, take 103 as the next divisor.

6. Multiply 103 by 3. Subtract the product 309 from 405. Write 3 in the quotient at the ten's place.

7. Write the next pair on the right side of the remainder 96 found in step 6. Now next dividend becomes 9621.

8. Write 106, double of 53 in the divisor leaving space for unit's digit.

9. Now 96 ÷ 10 = 9 + …

   ∴ now consider 1069 as the next divisor.

Note

10. Because 1069 x 9 = 9621, so next divisor is 1069. Subtract 9621 from dividend 9621 and we get 0 as the last balance.

11. Write 9 at the unit's place in the quotient also.

$\therefore \sqrt{290521} = 539$.

Remark: Number of digits in the square root is always same as number of pairs of digits in the perfect square number.

Example 3.5: Find the square root of 49284.

Solution:

	2 2 2
2	4, 9 2, 8 4
	– 4
4 2	0 9 2
	–8 4
4 4 2	8 8 4
	–8 8 4
	0

$\therefore \sqrt{49284} = 222$

Note: In this example there are two pairs and one digit 4 is left out. Therefore here we consider of a square root of 4.

Example 3.6: Find the square root of 256036.

Solution:

	50 6
5	25, 60, 36
	–25
1006	06036
	–06036
	0

$\therefore \sqrt{256036} = 506$

Intext Questions 3.4

Find the square root of each of the following:

1 4489

2 61504

3 207936

4 314721

5 152497801

5. 152497801

3.6 Some problems based on Square root

In this section we will use square root to solve some day-to-day life problems.

Example 3.7: There are 529 students in a school. They are to stand for prayer in such a way that there are as many studentss in a line as number of lines. Find the number of lines or number of students in each line.

Solution: Suppose number of lines is x.

Then number of students in every line will also be x.

$\therefore$ Total number of students $= x \times x = x^2$

$\therefore x^2 = 529$

$x = \sqrt{529}$

$= \sqrt{23 \times 23}$

$= 23$

$\therefore$ Number of lines = 23

and Number of students in each line = 23

Example 3.8: 2304 apples are to be packed in boxes in such a way that number of apples in each box is same as the number of boxes. Find the number of boxes and number of apples in each box.

Solution: Suppose number of boxes= x

Then number of apples in each box= x

$\therefore$ Total number of apples $= x \times x = x^2$

$\therefore x^2 = 2304$

or $x = \sqrt{2304}$

$= \sqrt{4 \times 4 \times 4 \times 4 \times 3 \times 3}$

Note

$= 4 \times 4 \times 3$

$= 48$

$\therefore$ Number of boxes = 48

and Number of apples in each box = 48

Example 3.9: Area of a playground of square shape is 12100 square meter. Find the length of each side of the playground.

Solution: Suppose length of each side of square playground = x meter

$\therefore$ Area of the square = $x \times x$ square meter = x^2 square meter

$\therefore$ $x^2 = 12100$

$x = \sqrt{12100}$

$= \sqrt{12100} = \sqrt{11 \times 11 \times 10 \times 10} = 11 \times 10$

$= 11 \times 10 = 110$

$\therefore$ Length of each side = 110 meter

Intext Questions 3.5

1. In a game 16 artists are made to stand in such a way that there are as many artists in a rows as there are number of rows. Find the number of artists in each row.

2. 4096 plants are to be sowed in a park in such a way that number of plants in every row is same as the number of rows. Find the number of rows in which plants may be sowed.

3. There are 2601students in a school. They stand for prayer in such a way that number of students in a row is same as the number of rows. How many students stand in each row?

4. Area of a square playground is 36100 square meter.Find the length of each side of the playground.

3.7 Cube and Cube root

Product of a number and the number itself is called square of that number. Now if we multiply this product again by the original number then new product is cube of the original number.

e.g. $2 \times 2 \times 2 = 8$ or $2^3 = 8$

and $7 \times 7 \times 7 = 343$ or $7^3 = 343$

Here 8 and 343 are the cubes of 2 and 7respectively.

Understand it in this way

Number	Three times multiplication	Exponential Form	Cubic Number
1	$1 \times 1 \times 1$	1^3	1
2	$2 \times 2 \times 2$	2^3	8
3	$3 \times 3 \times 3$	3^3	27
4	$4 \times 4 \times 4$	4^3	64
⋮	⋮	⋮	⋮

In the above table 1, 8, 27, 64, … are the cubes of integers 1, 2, 3, 4 etc. respectively.

Looking at the table we come to know that cubes of even numbers are even and cubes of odd numbers are odd numbers.

3.8 Perfect Cube numbers

Consider number 8

$$8 = 2 \times 2 \times 2$$

Similarly $64 = 4 \times 4 \times 4$

8 is a cube of 2 and 64 is a cube of 4.

64 can be written like this also:

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

If a number can be written as product of triplet and no factor or pair of factors is left out then such numbers are called Cubic numbers.

For example $125 = 5 \times 5 \times 5$ has been written as product of a triplet, therefore it is a perfect cube.

But $81 = 3 \times 3 \times 3 \times 3$, when we make triplet, a number 3 is left. So 81 is not a perfect cube.

Note

Let us take another example:

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2$

$= 2^3 \times 3^3 \times 2$

After making triplets of 2 and 3, factor '2' is left out, so 432 is not a perfect cube.

3.9 Making any number a Perfect Cubic Number

After breaking 432 in to prime factors and making triplets of 2 and 3, factor '2' remained left out. Now if we multiply this number by 2x 2 then we will have one triplet of 2 and Number 432 x 2x 2 = 1728 will become a perfect cubic number.

Even by dividing 432 by 2 we could have got a perfect cubic number.

Then $432 \div 2 = \dfrac{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2}{2}$

= 2 x 2 x 2 x 3 x 3 x 3 = 216 will be a perfect cubic number.

For example,

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Find a smallest possible number by which we multiply 256 to get a perfect cubic number.

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^3 \times 2^3 \times 2 \times 2$

Here, we observe that after taking two triplets of 2 prime factors, 2×2 is left out. Now if we multiply 256 by 2 then there will be one more triplet of 2 and number $256 \times 2 = 512$ will become a perfect cubic number. So required smallest number is 2.

Similarly, let us take another example:

2	10584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	7
	1

Find a smallest possible number by which we divide 10584 so that quotient becomes a perfect cube.

$105684 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7$

$= 2^3 \times 3^3 \times 7 \times 7$

Therefore, by dividing 105684 by 7x 7 or 49 quotient 216 will be a perfect cubic number.

So required smallest number = 49.

Intext Questions 3.6

1. Find the cube:

   (i) +19 (ii) +11 (iii) +12 (iv) +10

2. Which of the following numbers is a perfect cube?

   (i) 2197 (ii) 36125 (iii) 43200 (iv) 13824

3. Find smallest possible number by which we multiply 500000 so that the product is a perfect cube.

4. Find the smallest possible number by which we divide 165375so that the quotient becomes a perfect cubic number.

3.10 Cube root

We know that $5^2 = 25$, therefore we said that square root of 25 is 5. We have seen that cube of 4 is 64, in other words cube root of 64 is 4. Similarly we can say that cube root of 8 is 2, because cube of 2 is 8.

To denote cube root of a number we use the symbol $\sqrt[3]{}$. So $\sqrt[3]{27}$ means 'cube root of 27' and $\sqrt[3]{125}$ means 'cube root of 125'.

Therefore $\sqrt[3]{8} = 2$

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{125} = 5$$

3.11 Finding a cube root of a perfect cube by Prime Factorisation method

To find the cube root of a number we express the number as the product of prime factors and then make the triplets of like factors. For finding the cube root we take one number from each triplet and find their product.

Let us take an example

$$\sqrt[3]{216} = \sqrt[3]{2\times2\times2\times3\times3\times3}$$

$$= \sqrt[3]{\underline{2\times2\times2}\times\underline{3\times3\times3}}$$

$$= 2 \times 3$$

$$= 6$$

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

Similarly, to find the cube root of 17576

$$\sqrt[3]{17576} = \sqrt[3]{2\times2\times2\times13\times13\times13}$$

$$= \sqrt[3]{\underline{2\times2\times2}\times\underline{13\times13\times13}}$$

$$= 2 \times 13$$

$$= 26$$

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

Intext Questions 3.7

1. Find the cube root:

   (i) 13824 (ii) 35937 (iii) 46656

   (iv) 343 × 216 (v) 125 × 1331

2. Find the smallest possible number by which we divide 5400so that the quotient becomes a perfect cubic number. Also find the cube root of the quotient.

Let us Revise

- If a number is multiplied by itself then the product is known as square of the number.
- Square of number x is denoted by x^2.
- Square of positive as well as negative numbers are always positive.
- A natural number is known as a perfect square if it is square of a natural number.
- Square root of a number x is the number which when multiplied by itself gives the product x.
- Square root of x is denoted by $\sqrt{x}$.
- Surd symbol $\sqrt{\ }$ is used for positive square root of a number.
- Every positive number has two square roots, one of which is positive and the other is negative.
- We cannot find square root of a negative number.
- There are two methods for finding square root - Factor method and Division method.
- If 'a' is a whole number then a^3 is called its cube.
- If 'a' is a whole number and $a = x^3$ then x is called cube root of number a.

- Cubes of positive numbers are positive and cubes of negative numbers are negative.
- Cubes of even numbers are even and cubes of odd numbers are odd.

Exercise

1. Find the squares of the following numbers:

   (i) 83 (ii) 139 (iii) 311

2. Which of the following numbers are perfect squares and which are not perfect squares?

   (i) 441 (ii) 960 (iii) 1250 (iv) 2116

3. Find the square root of each of the following numbers using Factor method.

   (i) 3364 (ii) 3025 (iii) 774400 (iv) 69696

4. Find the square root of each of the following numbers using Division method.

   (i) 546121 (ii) 480249 (iii) 346921

5. 1024 Oranges have been arranged in a wooden box in such a way that number of Oranges in its every row is same as the number of rows in the box. Find the number of Oranges in each row.

6. 7056 apples have been packed in some boxes in such a way that number of apples in every box is same as the number of boxes. Find the number of boxes and number of apples in each box.

7. In a school ₹63001was collected for contribution to Gujrat relief fund. If every student collected as many rupees as the numer of students in the school then find the number of students in the school.

8. Area of a square is 65536 sq cm. Find the length of its each side.

9. In an army soldiers are made to stand in such a way that number of rows is same as the number of soldiers in every row. After doing this 16 soldiers are left out. How many soldiers are there in each row if total number of soldiers is 5200?

10. Find the cubes.

    (i) 21 (ii) 25 (iii) 27 (iv) 40

11. Find the cube roots.

    (i) 5832 (ii) 13824

12. Volume of a cuboidal box is 1331 cubic meters; find the measure of the side of the box.

Answers

Intext Questions 3.1

1. (i) 81 (ii) 625 (iii) 64 (iv) 361

Intext Questions 3.2

1. (i) 4489 (ii) 6889 (iii) 10201 (iv) 5041
2. 64 and 36
3. 48, 526 and 1250
4. 309, 4709 and 1111
5. $8^2 = 2 \times 32$ and $9^2 = 2 \times 40 + 1$

Intext Questions 3.3

1. 36 2. 65 3. 224

4. 72 5. 400

Intext Questions 3.4

1. 67 2. 248 3. 456

4. 561 5. 12349

Intext Questions 3.5

1. 4 2. 64 3. 51 4. 190 meter

Intext Questions 3.6

1. (i) 6589 (ii) 1331 (iii) 1728 (iv) 1000
2. (i) is a perfect cubic number.

   (ii) is not a perfect cubic number.

   (iii) is not a perfect cubic number.

   (iv) is a perfect cubic number.
3. by 250
4. by 49

Intext Questions 3.7

1. (i) 24 (ii) 33 (iii) 36

   (iv) 42 (v) 55

2. Dividing by 200, cube root = 3

Exercise

1. (i) 6889 (ii) 16641 (iii) 96721

2. Perfect squares: (i) 441 and (iv) 2116

   Not perfect squares: (ii) 960 and (iii) 1250

3 (i) 58 (ii) 55 (iii) 880 (iv) 264

4 (i) 739 (ii) 693 (iii) 589

5 32

6. Number of boxes = 84 and number of apples in a box = 84

7 251

8 256cm

9 72

10 (i) 9261 (ii) 15625 (iii) 19683 (iv) 64000

11 (i) 18 (ii) 24

12 11 meter

Note

Module - II

Algebra

From very famous book of Al-khawarizmi 'HisabAl-jabrwa'lmuqabalah' we get the European form of Al-Jabr as Algebra.

Translation of the title is The Science of Calculation by Completion and Balancing. These words are used with reference to the systematic solutions of Linear and Quadratic Equations. Name of this branch of mathematics was evolved from his Algebra book.In addition to it, great Indian Mathematician Aryabhatta (AD 476), Brahamgupt (AD 598) Mahaveer (AD 850) also made significant contributions towards the development of Algebra.

Mathematician, Astronomer, Cerographist Muhammad ibnMûsâ al-Khwârizmî was born in nearly 770 AD in a small village at the south of Akas river in Pharas (situated in Uzbekistan, Rawewa)

In this module, you will learn to represent numbers with alphabets. These alphabets are called 'Variables' and they have different numerical values.

You will learn to multiply variables with numbers, to differentiate between like and unlike terms, adding and subtracting like terms and multiplying two or more variables.

You will be introduced to the concept of Algebraic expressions and will learn to identify Monomial, Binomial and Trinomial. For given values of variable (variables) you will be able to find the value of any expression. You will be able to perform fundamental operations addition, subtraction and multiplication in Algebraic expressions (not containing more than three terms).

You will be able to differentiate Identities and Equations and will learn to solve linear equations in one variable. You will be in a position to solve simple daily life problems with the help of Linear Equations.

In the end you will learn four special products and simplify special products using these products and you will learn to find their values also.

4

INTRODUCTION TO ALGEBRA

You have already gone through the Arithmetic Module. So, you are familiar with fundamental operations of Mathematics. These operations are Addition, Subtraction, Multiplication and Division on numbers. If it is so, then you are having a good knowledge of Arithmetic. In Algebra, we use symbols and numbers to write statements. In other words we can say that Algebra is the general form, in which variables are used involving numbers.

In Algebra, along with numbers, symbols like - x, y, z etc. are used for variables. These symbols are called variables which represent numbers. Use of symbols helps us in writing results in short and general form. In real life, we use the Algebraic techniques in solving problems using given information, while dealing with one or two unknown numbers called variables.

From this lesson, you will learn:

- To represent numbers with variables
- To multiply or divide an variables with number
- Addition and subtraction of like terms
- Multiplication of two variables

4.1 Constant and Variable

In daily life situations you have seen that number of minutes in one hour is 60, number of days in a week is 7, and number of months in a yearis 12. In these information values are fixed.

Let us now search some new ideas. You also know that number of days in all months of a year is not same. Some months have 30 days and some have 31 and February month has 29 or 28 days depending on the fact that it is leap year or not. Do you think this example is different from the first three examples- in what way? In first three examples values were fixed and everyone knows them. In case of months, except February number of days in all other months is fixed, which are free from the type of the year. All these can be represented by numbers. All these are the examples of fixed numbers. These fixed numbers are called 'Constants'.

An alphabet always has a constant numerical value.

But number of days of February is not known, till its year is not known. Can you think of some other example, in which value is not fixed? Have you verified the cost of same item from different shops? It is possible thatit is not same on all shops. Similarly temperature of different places at different time may not be same. It is different at day and night times.

So, number of days of February, cost price of item at different shops, temperature at different times and places cannot be determined by a fixed number. These are called 'Variables'.

A variable may have different values.

Variables are represented by alphabets x, y, z, … etc.

Remark: Alphabet representing a variable is called basic alphabet.

Let us observe the following situations, where you will be requiring an alphabet.

Suppose you are having some toffees. Actual number of toffees is unknown. If we add five more toffees in it, then how many toffeeswillyou have?

These will be (Number of toffees + 5) toffees.

If out of these you eat 3 toffees, then you will be left with:

(Number of toffees + 5 - 3) toffees

i.e. (Number of toffees + 2) toffees

If in the beginning you had 7 toffees, then at the end you would have been left with (7+2) toffees.

Similarly, if you had 10 toffees in the beginning, then at the end you would have left with (10+2) toffees. Can you tell, if in the beginning you had 8 toffees then you would have been left with how many toffees? Answer is very simple (98+2) toffees.

This number depends on the 'number of toffees', which you had in the beginning. Since the number of toffees you had in the beginningis not known,so , instead of writing 'number of toffees' repeatedly we represent it by 'n' , where 'n' is any number. Hence number of toffees left at the end will be n+2, when n is representing number of toffees in the beginning. n is called a variable term. So, an alphabet (or variable) is used for that number whose actual value is unknown. In other words in Algebra, alphabets represent numbers.

Note

Similarly, number of days in February, cost price of an item at different shops, temperature at different times and at different places can be represented by d, p and t. It is not necessary that we should take first alphabet of the word. You can represent it by any alphabet.

Let us represent the following situations with alphabets.

4.2 Double of a number

In Arithmetic, if you are asked to find two times or double of 5, then what will be your answer?

Without hesitation, you can say 'this is 10', which can be represented by 2 x 5.

Similarly two times 4 is 2 x 4 or 8 and two times 10 is 2 x 10 or 20.

Let us write it in tabular form:

How many times	Given number	Value
2 times	1	2×1
	2	2×2
	3	2×3
	....	.....
	10	2×10
		.....

Are you observing the pattern in this table? You will find that 2 is being multiplied with corresponding respective number 1, 2, 3, 4, ------, 10 (which is the given number)

i.e. 2 x 1

2×2

2×3

2×10

Thus we can say that 'Double of unknown number'

= 2 x unknown number

Let us represent the unknown number by 'n'.

Then two times unknown number = 2 x n = 2n

Product of a number and an alphabet or that of two alphabets is written without putting multiplication sign between them.

Normally we do not put multiplication symbol between number and alphabet (variable) or two alphabets (variables).

Now instead of finding double of a number, we want to find three times, then what will be its form? As before 'unknown' number n will be multiplied by 3. We will get 3xn. This can be written as 3n also.

Now you think 'to get 6 times n', what will be its form?

Let us now take another example. Suppose you have 12 meter long rod.

If you cut it in two equal parts, then what will be the length of each part?

Each part will be 6 meter long.

6 meter can be written as 12÷2 meters or it can be represented by $\frac{1}{2}\times 12$ meters also, which is the length of the rod.

If we assume length of the rod as n meter, then $\frac{1}{2}$ part = $\frac{1}{2}$ n.

Similarly $\frac{1}{3}$ of n or one-third part = $\frac{1}{3}$n and $\frac{1}{4}$ part = $\frac{1}{4}$ n.

4.3 Perimeter of a square

Let us think over the following example. Venktesh daily goes to a square shaped park near his house for morning walk. Daily he goes round the square park once. If each side of the parkis 1 km then can you compute that every morning how much distance he walks? For knowing this distance you find the sum of four sides of the square. In this case sum of the four sides of the park

= (1 + 1 + 1 + 1) km

= 4 km

Therefore, Venktesh daily walks 4km.

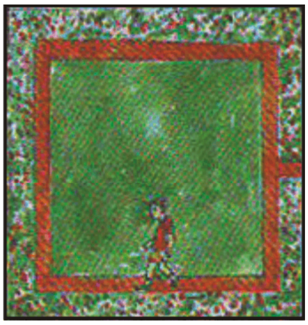


Figure 4.1

Sum of the length of four sides of this square is called its 'Perimeter'.

From this example total distance covered by Venktesh

= Perimeter of the square park

= 4km

If each side of the park would have been 2 km, then its perimeter = (2 + 2 + 2 + 2) km = 8 km, which is 4 times 2 km.

Again if length of each side of the park be 5 km, then its perimeter is (5 + 5 + 5 + 5) km = 20 km, which is 4 times 5 km

Now let us compute the perimeter of squares with different lengths of sides

Length of side	1k.m.	2k.m.	5k.m.	10k.m.
Perimeter	4k.m.	8k.m.	20k.m.	40k.m.

If you look at this pattern, then you will find that in each case, it is 4 times the length of the side.

4km can be written as 4 x 1km

8km can be written as 4 x 2km

20km can be written as 4 x 5km

40km can be written as 4 x 10km

Thus, if we represent length of the side of the square by 'L', then perimeter will be 4 x (length of side) = 4L.

To understand thoroughly, we solve few more examples.

Example 4.1: Express the following situations by using alphabets:

(a) Raman's age is double the age of his younger sister

(b) One- fourth the length of a rod

(c) Fare of a child is part $\frac{1}{2}$ of the fare of the distance between two stations

Solution: (a) suppose age of Raman's younger sister is x years.

Raman's present age = 2 × (present age of younger sister)

= 2 × (x years)

= 2x years

Note

(b) Suppose length of rod = L

One-fourth of the length of the rod = $\frac{1}{4}$ x (length of rod)

$$= \frac{1}{4} \text{ x L}$$

$$= \frac{1}{4} \text{ L}$$

(c) Suppose fare between two stations = ₹R

Fare for the child = part of fare = ₹ $\frac{1}{2}$ x R

$$= ₹\frac{1}{2}\text{R}$$

Intext Questions 4.1

1. Express the following situations using alphabets
   (a) Diameter of a circle is double of its radius.
   (b) One-third of the age of any person.
   (c) If you know the cost of one kg of rice, then find the cost of 5 kg of rice.

4.4 Fundamental operations on numbers and variables

You already know the fundamental operations- addition, subtraction, multiplication and division in Arithmetic. You can easily say that sum of 2 and 3 is 5. Difference between 5 and 3 is 5 - 3 =2 and product of 5 and 2 is 5 x 2 = 10. We can perform these operations in Algebra also, but its expression is different. Let us see how it can be done.

4.4.1 Addition and Subtraction

Suppose in the market, you purchased 3 balloons and 2 toys for your sister. If cost of 3 balloons be ₹3

And that of 2 toys be ₹10, then can you compute the amount spent on purchases?

For knowing the answer, you are to add cost of 3 balloons to the cost of 2 toys.

Figure 4.2

i.e. ₹3 + ₹10 = ₹13

Let us see, if cost of 3 balloons keep on changing but cost of 2 toys remainsconstant, then what will happen?

Cost of three balloons	Cost of two toys	Total expenditure
(i) ₹ 6	₹ 10	₹ 16
(ii) ₹ 10	₹ 10	₹ 20
(iii) ₹ 12	₹ 10	₹ 22

If you look at the pattern, then you will find that because of the varying cost of 3 balloons (because cost of 2 toys is same) total expenditure is not constant. Since cost of 3 balloons is varying, so we assume it to be an alphabet ₹x, where x is a variable.

Total expenditure = ₹x + ₹10 = ₹(x + 10)

In Algebra sum of variable and number 10 is written as x + 10 or 10 + x (Recall the addition property of changing order in Arithmetic). Similarly, number 7 more than t = 7 + t or t +7

Now we take our example again and change the cost of toys also.

Cost of 3balloons	Cost of 2 toys	Total expenditure
(i) ₹x	₹12	₹x + ₹12 = ₹(x + 12)
(ii) ₹x	₹16	₹x + ₹16 = ₹(x + 16)
(iii) ₹x	₹20	₹x + ₹20 = ₹(x + 20)

In this case due to varying cost of toys, total expenditure is not constant, since cost of two toys is changing, we represent it with second variable y.

Total expenditure is written as ₹x + ₹y = ₹(x + y).

Recall the subtraction operation on numbers. Here, 15 - 3, is showing 3 less than 15 or subtraction of 3 from 15. In the same manner in Algebra, difference between y and 3 or 3 less than y is written as (y - 3). In other words, constant number 3 subtracted from y gives (y - 3). In same manner subtraction of variable y from the other variable x, is represented by x - y.

4.4.2 Multiplication and Division

We know that three times x is shown by 3x and 5 times t by 5t. These are the examples of multiplying a variable with a number. In Algebra, we do not put multiplication sign between a constant and a variable or between two variables.

In section 4.2, we have learnt that double of n is written as 2n.

Let us see what will happen on multiplying n by 1, 3, 5 … 10

Give Number	How Many Times	Value
n	1	1 × n
	3	3 × n
	4	4 × n
	5	5 × n
	....	....
	....	....
	10	10 × n

Are you looking some pattern in the above table? You will find that numbers 1, 3, 4, 5, …10 are being always multiplied with n. So, on multiplying by the changing number 'How many times' value of the number is not remaining constant. So it is represented by another variable m.

Product of two variables m and n is mn.

In Arithmetic you have learned the Division Process on two numbers. On dividing two numbers, such as 25÷2, dividing 25 by 2 is written as $\frac{25}{2}$ (25 upon 2) .

Similarly, in Algebra, for dividing some variable with some number or some variable with some variable '÷' is used.

x÷6 is read as x divided by 6 and is written as $\frac{x}{6}$. Similarly 10÷y is written as $\frac{10}{y}$ and read as 10 divided by y.

When some x is divided by y, we write it as $\frac{x}{y}$ and read it as x divided by y.

4.5 Terms and Co-efficient

Combination of the product or division of a number and a variable or numbers and variable is called a Term.

Examples of terms are: 5, x, -3x, and $\frac{5}{x}$

Multiples of the variable is called Co-efficient of the variable.

For Example: In -3x, -3 is the co-efficient of x. Similarly in $\frac{x}{3}$, co-efficient of x is $\frac{1}{3}$ but in $\frac{5}{x}$, co-efficient of $\frac{1}{x}$ is 5, because $\frac{5}{x}$ can be written as $5 \times \left(\frac{1}{x}\right)$.

4.6 Like and Unlike Terms

Combining 2 apples and 3 apples you can say 5 apples, whereas you cannot combine 2 bananas and 1 toy. In same way x and 2x are alike and we can combine them, but 2x and 3y are unlike and their sum is 2x +3y.

Look at terms x, $2x$, $\frac{x}{3}$, in which in every term alphabet number (variable) is x. These are called Like terms.

Thus two or more terms are called like terms if their variable is same, whatsoever their co-efficients may be.

Two or more terms are called like terms, if at the most they are different with numerical coefficients.

Look at the terms 3t and 7z. Their variables are different; they are called Unlike Terms.

Terms with different variables are called Unlike Terms.

Example 4.2: Write the following by using numbers and variables;

(i) Add two variables p and q.

(ii) Subtract 2 from z.

(iii) Add 3 to the product of7 and z.

(iv) multiplying x by 3, subtract 2 from the product thus obtained.

(v) Divide the difference of p and q by 3.

Solution: (i) required sum is p+q

(ii) Required difference is z - 2

(iii) Product of 7 and z is 7z and on adding 3 it will become 7z +3

(iv) Product of x and 3 is 3x and on subtracting 2 from it, it becomes 3x -2

(v) Difference of p and q is p - q.

Note

On dividing p - q with 3 it will be $\frac{p-q}{3}$.

Required answer is $\frac{p-q}{3}$

Example 4.3: Write the coefficient of each of the following terms:

$$3z, -5t, \frac{3}{5}q, 7.5m$$

Solution: In the term 3z, coefficient of z is 3, since in it z is the only variable and 3 is a number.

In term -5t, coefficient of t is -5.

In terms $\frac{3}{5}q$, and 7.5m,coefficients of q and m are $\frac{3}{5}$ and 7.5 respectively.

You have seen that coefficients are taken with sign.

Example 4.4: From the following terms, identify like and unlike terms:

(i) $7d$ and $\frac{1}{7}d$, $3x$ and $-\frac{3}{5}y$, $\frac{7}{10}q$, and $-\frac{1}{5}q$

(ii) b and $-\frac{1}{3}a, \frac{1}{4}m$, and m, $\frac{2}{3}y$ and $\frac{1}{2}z$

Solution: (i) Like terms are: 7d and $\frac{1}{7}d$; $\frac{7}{10}q$ and $-\frac{1}{5}q$

Where as unlike terms are: 3x and $-\frac{3}{5}y$

(ii) $\frac{m}{4}$ and m are Like Terms and, b and $-\frac{1}{3}a$, $\frac{2}{3}y$ and $\frac{z}{2}$ are unlike terms,

4.6.1 Addition of Like Terms

While adding like terms we are to add the coefficients of each term. You must keep in mind the following rules:

For example: (+5) + (+3) = 5 + 3 = 8

(+5) + (-3) = 5 - 3 = 2

(-5) + (+3) = -5 + 3 = -2

(-5) + (-3) = - 5 - 3 = - 8

Example 4.5: Find the sum of the terms in each:

(i) x, 2x

(ii) 5x, -2x

Solution:

(i) Co-efficient of x and 2x are 1 and 2 respectively.

Sum of the co-efficient = 1 + 2 = 3

$\therefore$ x + 2x = (1 + 2) x = 3x

$\therefore$ Required sum = 3x.

(ii) Co-efficient of 5x and -2x are 5 and -2 respectively.

Sum of the co-efficient = 5+(- 2) =5 - 2 = 3

5x +(- 2x) = (5- 2) x = 3x

4.6.2 Subtraction of like Terms

Subtraction of like terms is done in the same way as addition of like terms is done. In Subtraction following rules are followed:

For example: (+5) - (+3) = 5 - 3 = 2

(+5) - (-3) = 5 + 3 = 8

(-5) - (+3) = -5 - 3 = -8

(-5) - (-3) = - 5 +3 = - 2

Example 4.6: In each of the following subtract second term from the first term:

(i) x, 2x

(ii) 5x, -2x

Solution:

(i) Co-efficient of x and 2x are 1 and 2 respectively.

Difference of the co-efficient = 1 - 2 = - 1

$\therefore$ x - 2x = (1 - 2) x = -x

$\therefore$ Required difference = -x.

(ii) Co-efficient of 5x and -2x are 5 and -2 respectively.

Difference of the co-efficient = 5-(- 2) =5 + 2 = 7

5x - (- 2x) = (5 + 2) x = 7x

Note

4.6.3 Multiplication of Variables

You know that multiplication is another form of repeated addition. So rules of addition are true in multiplication. You have also learnt that when number is multiplied by itself, then it can be written in exponential form also.

Thus, 3 x 3 can be written as 3^2.

This is read as 3 raised to the power 2.

4 x 4x 4 is written as 4^3. It is read as 4 raised to the power 3

Similarly in Algebra,

$x \times x = x^2$ it is read as x raised to the power 2.

$y \times y \times y \times y = y^4$ it is read as y raised to the power 4.

In x^2, x is called the base and 2 is called exponent or say exponent of x. y is called the base and 4 is called the exponent or say exponent of y. In the above examples, there is only one variable. Let us learn to multiply two or more than two such terms which have two or more variables.

For example, $3x \times 5x \times y \times z$

You will have to follow the following technique:

(i) Multiply all numbers with proper sign 3 x 5 = 15

(ii) Identify repeated variables, in $15x \times x \times y \times z$, x is the repeated variable.

(iii) Add the exponents of the same variable, $15x^{1+1}$ yz $(x \times x = x^{1+1} = x^2)$

We obtained the required product as 15 x^2 yz.

Similarly, product of $-7x \times y^2 \times 5z$

$$= (-7 \times 5)x \times y^2z$$

$$= -35\,x\,y^2z$$

You will note that multiplication of numbers satisfies the following rules:

$(+) \times (+) = (+)$	For Example	$(+2) \times (+3) = (+6)$
$(+) \times (-) = (-)$		$(+2) \times (-3) = (-6)$
$(-) \times (+) = (-)$		$(-2) \times (+3) = (-6)$
$(-) \times (-) = (+)$		$(-2) \times (-3) = (+6)$ etc.

Example 4.1 : Find the product of terms in each

(a) 4x and 3y

(b) $25p$ and $-\frac{1}{5}p$

(c) $-\frac{2}{3}r^2$ and $-3\,r^3$

(d) $-\frac{2}{3}s$ and $-\frac{3}{2}t$

Solution:

(i) Required Product $= 4x \times 3y$

$= (3 \times 4)\ x \times y = 12\ xy$

(ii) Required Product $= 25p \times \frac{-1}{5}p$

$= 25p \times \left(\frac{-1}{5}p\right)$

$= \left\{25 \times \left(\frac{-1}{5}\right)\right\} p^{1+1}$

$= -5p^2 \left[\therefore\ 25 \times \left(\frac{-1}{5}\right) = -5\right]$

(iii) Required Product $= \frac{-2}{3}r^2 \times (-3r^3)$

$= \left\{\left(\frac{-2}{3}\right)(-3)\right\} r^{2+3}$

$= 2\,r^5$

(iv) Required Product $= -\frac{2}{3}s \times \left(-\frac{3}{2}t\right)$

$= \left\{\frac{-2}{3} \times \left(-\frac{3}{2}\right)\right\} st$

$= st$

Intext Questions 4.2

1. Write product in expanded form

(a) $3x^3$ (b) $8a^2b$ (c) $-7a^2bc^3$

Note

2. In the following multiply the terms:

   (a) $2q^2$, $11t^3$

   (b) $2m^3$, $-3t$

   (c) $-2y^2$, $\frac{1}{2}z^2$

   (d) $-\frac{1}{3}b^2$, $-12a^2$

Let us Revise

- A constant number has a fixed value.
- Variable has different values.
- For writing variables, alphabets x, y, z … are used.
- Normally, we do not put multiplication sign between variable and constant. Similarly multiplication sign is not put between two variables.
- In a term, leaving the variable, number with sign is called the co-efficient of the variable.
- Two terms are like terms, if they are different in at the most in numerical coefficient. Two terms are unlike if their variables are different.
- For addition or subtraction of two like terms, their numerical coefficients are added or subtracted.
- In a term $3x^2$, 3 is called the co-efficient of x^2., in x^2, x is called the base and 2 is called the exponent of x.
- For multiplying two or more terms, following steps are to be followed:

  (i) Multiply all coefficients with their signs

  (ii) Add the exponents of same variable.

  (iii) Keep other variables unchanged.

Exercise

1. Write the following in statements:

   (a) $7x$ (b) $x + 5$ (c) $\frac{x}{3}$ (d) $a + 2b$

   (e) $7x - 11$

2. Fill in the blanks:

   (a) Age of a girl is x years. After 3 years, her age will be -------- years.

   (b) Vinit's age is x years. After y years, his age will be ----- years.

   (c) Age of a boy is y years. 5 years ago, his age was -------- years.

   (d) Vinay distributed x toffees to y children on his birthday. Each child got -------- toffees.

   (e) Prity's age is 2 years more than three times the age of Anju, then Anju's age is ------- years.

3. Express the following statements by using fundamental operations:

   (a) Subtract 5 from the sum of x and t.

   (b) Add 2 times p to three times q.

   (c) Add three times the product of a and b to half of d.

   (d) Divide the difference of l and m by the difference of p and q.

4. Identify like and unlike terms from the following:

   (a) $x, -2x$ (b) $x, -6z$ (c) $\frac{1}{2}x, -3y$

   (d) $\frac{1}{3}n, -\frac{1}{5}n$ (e) $2x^2, 3x$ (f) $5y^2, -7y^2$

5. In the following add the terms:

   (a) $\frac{1}{2}q, \frac{1}{2}q$ (b) $x, -2y$

   (c) $3a, -b, -2b$ (d) $7, 3x, 2$

6. From the following multiply the terms:

   (a) p, r (b) $y, -x$

   (c) $-a^2, a$ (d) $-\frac{2}{5}x^2, -\frac{5x}{2}$

Note

Answers

Intext Questions 4.1

1. (a) Diameter of a circle = 2xr =2r, where r is the radius.

   (b) Suppose age of a man is y years.

   $\therefore \frac{1}{3}$ Part of man's age = $\frac{1}{3}$ x y years = $\frac{1}{3}$ y years

   (c) Suppose cost price of 1 kg of rice = ₹R

   ∴ Cost price of 5 kg of rice = ₹R × 5

   = ₹5R

Intext Questions 4.2

1 (a) $3 \times x \times x \times x$

  (b) $8 \times a \times a \times b$

  (c) $-7 \times a \times a \times b \times c \times c \times c$

2 (a) $22q^2t^3$ (b) $-6m^3t$ (c) $-y^2z^2$ (d) $4a^2b^2$

Exercise

1 (a) Product of x and 7 (b) Sum of x and 5

  (c) x divided by 3. (d) Double of b plus a

  (e) 7 times x minus 11

2 (a) $x + 2$ (b) $x + y$ (c) $y - 5$ (d) $\frac{x}{y}$ (e) $3t + 2$

3. (a) $x + t - 5$ (b) $3q + 2p$ (c) $\frac{1}{2}d + 3ab$ (d) $\frac{(l - m)}{p - q}$

4. Like pairs are: (a), (d) and (f)

   Unlike pairs are: (b), (c) and (e)

5 (a) q (b) $x - 2y$ (c) $3a - 3b$ (d) $3x + 9$

6 (a) pr (b) $-xy$ (c) $-a^3$ (d) x^3

5

ALGEBRAIC EXPRESSIONS AND OPERATIONS

Whenever you wish to say something, you use your expression to express your views. 'I study Mathematics' is an expression in English. '4 + 10', '6 - 3' are the examples of Mathematical expressions. These are called Mathematical Expressions also.

Exactly in the same manner 4x, xy, ab - 8, $a^2x^2 + yz$ are the examples of Algebraic Expressions.

From this lesson, you will learn:

- Terms of Algebraic Expression
- Coefficients of the terms containing two or more variables
- Different types of Algebraic Expressions
- Finding the value of an expression for a given value of the variable
- Methods of adding or subtracting Algebraic Expressions
  - (i) Grouping the like terms
  - (ii) Column method
- Multiplying two Algebraic Expressions

5.1 Concept of Algebraic Expressions

Let us consider the following example. In your daily life you go to the market number of times. Suppose you purchased 2kg rice, 5 kg floor and 1 kg grams pulse. Further suppose that rate of rice is ₹x per kg, rate of floor is ₹y per kg and cost of gram pulse is ₹z per kg. Can you tell what amount of money you spent? For purchasing all these items, you spent ₹(2x + 5y + z).

Here, 2x + 5y + z is an algebraic expression. Thus Algebraic Expressions are formed by the combination of four basic operations on constants and variables.

$2x,\ 3t - 4s,\ p + 3q - 3n,\ x^2y + y^2z,\ x + \frac{1}{x},\ x^2y^2 + y^2z^2 - z^2x^2$, are the examples of Algebraic Expressions.

Note

Think of the expression $x^2y^2 + y^2z^2 - z^2x^2$. In it x^2y^2 has been separated by the symbol '+', y^2z^2 has been separated by the symbols '+' and z^2x^2 has been separated by the symbol '–'.

Parts of the Algebraic Expressions, separated by the symbols '+' or '–' are called its terms.

Remark: Expression with no symbol are treated with '+' sign. For example, 2x means +2x etc. Thus x^2y^2, y^2z^2 and $-z^2x^2$ are the terms of the expression $x^2y^2 + y^2z^2 - z^2x^2$ and these are 3 in number. Similarly in 3t - 4s number of terms is 2 and these terms are 3t and -4s.

Following examples will help you in understanding Algebraic Expressions and their terms:

Algebraic Expression	Number of terms	Terms
$-7x$	1	$-7x$
$\frac{2}{f} + q$	2	$\frac{2}{f}$, q
$3x^2y - yz + 6$	3	$3x^2y, -yz, 6$
$4t + \frac{1}{2}ft^2$	2	$4t,\ \frac{1}{2}ft^2$
$abc + 2fgh - af^2 - bg^2 - ch^2$	5	$abc, 2fgh, -af^2, -bg^2, -ch^2$

Intext Questions 5.1

1. Write the terms and number of terms of each of the following Algebraic Expressions:

(i) 3t (ii) $x^2 + 3xy$ (iii) $t^2 + 3t + \frac{1}{t^2}$

(iv) $a^3 - b^3 + 3$ (v) $ab + bc - ca$ (vi) $x^2 + y^2 + z^2 + 2hxy$

5.2 Co-efficient of the terms with two or more variables

Recall the concept of 'the co-efficient of the terms with one variable'. The same concept can be expanded further, when term has more than one variable. This can be seen in the following manner.

Look at the term 15*xy*. This can be written as $15 \times x \times y$ also. Thus 15, *x* and y are its factors. 15 is its numerical factor and *x*, y are its variable factors. Out of these any one can be considered as co-efficient of product of remaining factors (with sign).

In this way, in 15xy, co-efficient of x is 15y, in 15xy co-efficient of y is 15x and in 15xy, coefficient of 5x is 3y. Exactly in same way in $\frac{3}{7}$st co-efficient of t is $\frac{3}{7}$s and in $5x^2y$ co-efficient of x^2 is 5y. If you look at the Algebraic expression -3xyz + 5, then its first term -3xyz can be written as -3 × x×y × z, since factors of -3xyz are -3, x, y and z. Now second term of this expression is 5 and it does not have any variablefactor. This is called constant term.

Term of an Algebraic Expression which has no variable factor is called Constant Term.

Example 5.1: Find the co-efficient

(a) of t in t   (b) of m in $\frac{2}{3}m^2n^2$

(c) of x^2 in $-25\,x^3yz$ (d) of yz in $-25\,x^3yz$

(e) of 5xyz in $-25\,x^3yz$

Solution: (a) Factors of term t are 1 and t and therefore it can be written as 1 x t. So coefficient of 't' of the term 't' is 1.

(b) $\frac{2}{3}m^2n^2$ can be written as $\frac{2}{3} \times m \times m \times n \times n$.

So, in $\frac{2}{3}m^2n^2$ coefficient of m is $\frac{2}{3}mn^2$.

(c) $-25\,x^3yz$ can be written as $-25 \times x \times x \times x \times y \times z$

So, in $-25\,x^3yz$ coefficient of x^2 is $-25xyz$.

(d) It is clear from the above example (c) that in $-25\,x^3yz$ coefficient of yz is $-25\,x^3$.

(e) $-25\,x^3yz$ can be written as $-5 \times x \times x \times x \times y \times z \times 5$ also

So, in $-25\,x^3yz$ coefficient of 5xyz is $-5x^2$.

5.3 Like and Unlike terms of Algebraic Expressions

In previous chapter, we learnt that terms with same variables in same form are called Like Terms. For example in 3a, 5a, -7a there is a single variable 'a' in same form. These are called like terms.

Concept of like terms can be expanded to two or more variables too.

Look at the algebraic expression $3x^2y + 2yz^2 - 5x^2y + 7zx^2$.

Note

Term $3x^2y$ can be written as $3 \times x \times x \times y$ and $-5x^2y$ can be written as $-5 \times x \times x \times y$.

If you look at the variable factors of $3x^2y$ and $-5x^2y$ then you will find that except constant factors, all variable factors are same. (3 and -5 are respectively constant factors of $3x^2y$ and $-5x^2y$)

Terms of the expression, in which variable factors are alike, are called 'Like Terms'.

Now consider the terms $2yz^2$ and $7zx^2$. $2yz^2$ can be written as $2 \times y \times z \times z$ and $7zx^2$ can be written as $7 \times z \times x \times x$. You can easily observe that terms $2yz^2$ and $7zx^2$ do not have the same variable factors. These are called unlike terms.

Terms of the expression, in which variable factors are unlike, are called 'Unlike Terms'.

In this way we can show that

$x^3y, -x^3y, \frac{1}{3}x^3y$ are like terms and

$abc, \frac{1}{7}abc, -35\,abc$ are also like terms.

But $3x^2y, 4xy, -x^2y^2$ are unlike terms, since in them variable factors are different.

Example 5.2: In the following expressions identify like and unlike terms:

(i) $x + \frac{1}{x} + \frac{1}{7}x - \frac{1}{xy}$ (ii) $a^2y^2 - 2a^2y^2 + y - \frac{7}{y}$

(iii) $5mn - 3m^3n^3 + 5m^2n^2 + \frac{1}{2}mn$

Solution: x and $\frac{1}{7}x$ are like terms, but $\frac{1}{x}, -\frac{1}{xy}$ are unlike terms., since in $\frac{-1}{xy} = \frac{-1}{x} \times \frac{1}{y}$ and in it $\frac{1}{y}$ is such a factor which is not in the other term $\frac{1}{x}$

Similarly x and $\frac{1}{x}$; x and $-\frac{1}{xy}$ are also unlike terms.

(ii) $a^2y^2, -2a^2y^2$ are like terms but $y, -\frac{7}{y}$ are unlike terms. Exactly in same way a^2y^2 and y; a^2y^2 and $\frac{-7}{y}$ are also unlike terms.

(iii) $5mn, \frac{1}{2}mn$ are like terms and $5m^2n^2, -3m^3n^3$ are also like terms but $5mn$ and $-3m^3n^3$; $5mn$ and $5m^3n^3$; $-3m^3n^3$ and $\frac{1}{2}mn$; $5m^3n^3$ and $\frac{1}{2}mn$ are unlike terms.

Note

Intext Questions 5.2

1. Find the co-efficient of

   (i) a^2 in $a^2 y^2 z$ (ii) st in $\frac{3}{7}s^3t^3$

   (iii) 5t in $-15qr^2 t^2$ (iv) $x^3 y^2$ in $7 x^5 y^3 z^2$

2. In the following algebraic expressions, identify the numerical factors and variable factors:

   (i) $3\frac{y^2}{x^2}+5$ (ii) $\frac{5}{x}-3$ (iii) $2a^2b-\frac{1}{7}$ (iv) $-\frac{3}{7}st^3-\frac{5}{7}$

3. In the following terms which are like and which are unlike;

   (i) 1, t (ii) x, y

   (iii) $\frac{1}{3}x^2y, -y^2x, 5xy$ (iv) $\frac{x}{y}, -\frac{7x}{y}, \frac{x}{7y}$

   (v) $a^2b^2c^2, -b^2c^2a^2$

4. In the following expressions identify like and unlike terms:

   (i) $x^2 - y^2 + 3x^2 - 4xy$ (ii) $5x-3y+\frac{3}{5}x+5$

   (iii) $xyz - yxz + zxy + x^2yz$

5.4 Different types of Algebraic Expressions

Look at the algebraic expressions $-7x^3yz, 3t+\frac{2}{5}st^2$ and $at^2 + 2hst + bs^2$

In section 7.1 you have already learnt that how to find the terms and their number in an expression. Can you tell 'How many terms are in $-7x^3yz$? You can easily say that number of terms in $-7x^3yz$ is one.

Algebraic Expression having one term is called Monomial.

Therefore $-7x^3yz$ will be called Monomial. -8, -3y, a are the examples of monomials.

Similarly, look at the number of terms of the expression $3t+\frac{2}{5}st^3$. You will say two.

First term is 3t and second is $\frac{2}{5}st^3$.

Expression $3t + \frac{2}{5}st^3$ is called Binomial.

Algebraic Expression having two terms is called Binomial.

$3x^2 - 5$, $p + q$, $u^2v + 9v^3$, $a^3 - 9b^3$, are the examples of Binomials.

In third algebraic expression $at^2 + 2hst + bs^2$ there are three terms. These terms are at^2, $2hst$, bs^2. This is called Trinomial.

Algebraic Expression having three terms is called Trinomial.

$a^2 + 2ab + b^2$, $a^3 - b^3 - 2abc$, $3p - qr + s$, $m^2 + m + 2$, are the examples of Trinomials.

Note that in general Algebraic Expressions with two or more terms are called multimonials.

$x^3 + y^3$, $t^3 + 2t^2 + 3t$, $l^3 - 3l^2 + 5l - 4$, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$, are the examples of multimonials.

Remark: In all the examples listed above, no term is having a variable in the denominator.

Example 5.3: Write which of the following are monomial, binomial or trinomial. Give reason in support of your answer.

(i) -1 (ii) $3t^2p + 5$ (iii) $x^2 + y^2 - z^2$ (iv) $4x^4y^3 + 3z^5$

Solution:

(i) In '-1' number of terms is one. Therefore it is a monomial.

(ii) In $3t^2p + 5$ number of terms is two, Therefore it is a binomial.

(iii) In $x^2 + y^2 - z^2$ number of terms is three. Therefore it is a trinomial.

(iv) This is binomial, because there are two terms in it.

5.5 Degree of an Algebraic Expression

In a term sum of the exponents of variables is called its degree.

For example:

Degree of $4x^2y$ is 3, because sum of the exponents of x and y is $2 + 1 = 3$.

Similarly degree of $2x^2$ is 2. Degree of anon-zero constant, say 8 is 0, since $8 = 8 \times 1 = 8 \times x^0$, where $x^0 = 1$. An Algebraic expression has many terms, which are separated by '+' or '–'. Degree of an algebraic expression is the highest of degrees of different terms of the expression having non-zero coefficient.

Note

For Example:

Powers of different terms of the algebraic expression $3x^2y + 7xy - 5x + 6$ are 3, 2, 1 and 0 respectively, out of which 3 is greatest. Therefore degree of this expression is 3.

5.6 Value of Algebraic Expression

Recall that in Section 4.3, we found perimeter 4L of a square of length L. We also found that if side of the square be 1 km then perimeter will be (4×1) km = 4 km , if this length be 2km then perimeter will be 8 km and if length of square be 5 km then perimeter will be 20 km. It applies to the perimeter of all the squares whose length of sides are different 1km, 2km and 5km respectively.

We can also say that value of 4L for L=1 km is 4km, for L= 2 km this value is 8 km and for L= 5 km value of 4L is 20 km.

For finding the value of an algebraic expression we must know the numerical value of the variables of the expression. For finding the value of the expression, we replace the variables by their values. This idea stands true for the expressions with more than one variable also.

Look at the algebraic expression $yx^2 - \frac{1}{3}xy^2 + 3$, suppose you want to find the value of this expression for x = 1, y = -1.

For this you will have to follow the following process:

Placing x = 1, y = -1 in $yx^2 - \frac{1}{3}xy^2 + 3$, we get

$$yx^2 - \frac{1}{3}xy^2 + 3 = (-1)(1)^2 - \frac{1}{3}(1)(-1)^2 + 3$$

$$= -1 - \frac{1}{3} \times 1 \times 1 + 3$$

$$= -1 - \frac{1}{3} + 3$$

$$= \frac{5}{3}$$

To understand the concept of finding the values of expressions for given values of the variables we are dealing with a few examples.

Example 5.4: For given values of the variables find the value of the expression:

(i) Value of 5y - z for y = 0, z = -1.

(ii) Value of $-xy + yz + zx$ for $x = 2, y = -1, z = -2$

Solution:

(i) Putting $y = 0, z = -1$ in $5y - z$, we get:

$$5y - z = 5.0 - (-1)$$

$$= 0 + 1$$

$$= 1$$

(ii) Putting $x = 2, y = -1, z = -2$ in $-xy + yz + zx$, we get:

$$-xy + yz + zx = -(2)(-1) + (-1)(-2) + (-2)\,2$$

$$= -(-2) + 2 + (-4)$$

$$= 2 + 2 - 4 = 0$$

Intext Questions 5.3

1. State which of the following expressions are monomial, binomial and trinomial. Write number of terms also in each of these:

   (i) 0 (ii) $3z + 7$ (iii) $t^2 - 5t + 2$

   (iv) $x^3 - 3xy + y^3$ (v) $p - 3q$

2. For given values of the variables find the values of the expressions:

   (i) Value of $\frac{2a}{b}$ for $a = 1, b = 2$

   (ii) Value of $(a + b)(a - b)$ for $a = 3, b = 2$

   (iii) Value of $xyz + yzx + zxy$ for $x = 3, y = -1, z = -3$

   (iv) Value of $2x^3y - 3xy^2 + z$ for $x = -1, y = -1, z = 2$

   (v) Value of $a^2 + b^2 + c^2 - 3d^2$ for $a = b = c = d = 2$

5.7 Addition of Algebraic Expressions

Now let us see what happens on collecting algebraic expressions. For example can we add $7x + 2y$ and $3x$? How can you do it? For understanding it, first you presume the expression as 7 oranges and 2 bananas, to which we are adding 3 oranges. What do we get? We get (7 +3) oranges and 2 bananas. Thus we collected alike items.

In other words, we combine the like terms-

$$7x + 2y + 3x$$

$= (7x + 3x) + 2y$ (grouping like terms)

$= (7 + 3)x + 2y$ (sum of the coefficients of like terms)

$= 10x + 2y$

This method is called the Grouping method.

For adding expressions, one easy technique is 'writing alike terms under each other. For example for adding 4a + 3b – 12 c and 2a - 5c, we write like this:

$$\begin{array}{lrrr} & 4a & + 3b & - 12c \\ & +2a & & - 5c \\ \hline \text{Sum} & = 6a & + 3b & - 17c \end{array}$$

This technique of adding expressions is called 'Column Method'. Let us take another example. Suppose 7 - 14x and 5x -a - 3 are to be added.

$(7 - 14x) + (5x - a - 3)$

$= (7 - 3) + (-14x + 5x) - a$ (grouping like terms with proper sign)

$= 4 + (5 - 14)\, x - a$ (Distributive Law)

$= 4 - 9x - a$

We can solve this problem by the following method also:

$$\begin{array}{rrr} 7 & - 14x & \\ +(-3) & + 5x & - a \\ \hline 4 & - 9x & - a \end{array}$$

Thus, in both ways, simple form of the expression, in which number of terms is least, is given below:

$4 - 9x - a$

Now let us see what happens on adding three algebraic expressions. This process is similar to the process of adding three numbers. We first add any two and add the third to the sum thus obtained. As an example, let us add:

$(4a + 3b - 12c) + (b + 2c) + (6a - c)$

We can do it by two methods- either adding any two at a time or collecting the coefficients of all the like terms.

$(4a + 3b - 12c) + [(b + 2c) + (6a - c)]$

$= (4a + 3b - 12c) + (6a + b + c)$

$= 10a + 4b - 11c$

or $(4a + 3b - 12c) + (b + 2c) + (6a - c)$

$= (4a + 6a) + (3b + b) + (-12c + 2c - c)$

$= 10a + 4b - 11c$

You can simplify it by writing in columns and using column method also.

Intext Questions 5.4

Simplify the following expressions. Write the number of terms in the expression thus obtained:

(i) $2\,[x + 5(x + 2)] - 6$

(ii) $(2x^3 + 7x^2y^2 + 9xy^3) + (6 + x^2y^2 - 3xy^3)$

(iii) $2[4x + 3\,\{2 + (x + 1)\} + x]$

5.8 Subtraction of Algebraic Expressions

Now let us discuss the process of subtracting one algebraic expression from the other expression. Do you think this is similar to the addition process? Your knowledge of subtracting numbers motivates you to say so. For example, if you are to subtract 3x from 7x + 2y, what will you get?

Once again you will make the groups of like terms. Thus:

$$
\begin{aligned}
(7x + 2y) - 3x &= (7x - 3x) + 2y \\
&= (7 - 3)\,x + 2y \\
&= 4x + 2y
\end{aligned}
$$

Similarly, how willyou simplify (4a + 3b - 12c) - (2a - 5c)?

We can do so by adding negative of (2a - 5c) to 4a +3b -12c. Thus by column method

$$
\begin{array}{r}
4a + 3b - 12c \\
+ (-2)a \qquad + 5c \\
\hline
2a + 3b \;\; - 7c
\end{array}
$$

Recall that negative of any expression is obtained by changing the signs of all terms. What will be (2a - 5c) - (4a +3b -12c)?

You can simplify it by applying the method used in above example. Do it yourself. You will get -2a -3b + 7c, which is trinomial which is negative expression of 2a +3b - 7c.

Note

Now we shall look at the problem of magical game, which you took in the beginning of this chapter. Have you understood that how it happened? Suppose to start with you took number x and other number to be added is y. Then according to question

Think of a number	x
Multiply by 2	$2x$
Add the double of the second number	$2x + 2y$
Subtract 4	$2x + 2y - 4$
Divide by 2	$\dfrac{2x + 2y - 4}{2}$
	$= \left(\dfrac{2}{2}\right)x + \left(\dfrac{2}{2}\right)y - \dfrac{4}{2}$
	$= x + y - 2$
Subtract the second number thought of	$= (x + y - 2) - y = x - 2$
Add 2	$= x - 2 + 2$
	$= x$

This is the same number, which was taken in the beginning.

Intext Questions 5.5

1. Simplify the following expressions. Out of these which are binomials?

   (i) $3a + [3(a - b) - c]$

   (ii) $2(b - c) - (bc + 3ab)$

   (iii) $10 - 4[3x - (1 - x)]$

2. Think of a number. Multiply it by 3. In it add a number which is 1 more than the original number. Add 7 to it. Divide by 4 and then subtract 2.Check if it is the same number thought in the beginning?

Till now you have seen algebraic expressions andyou simplified them by adding or subtracting them. Now we shall learn about another fundamental operation which is called multiplication.

5.9 Multiplication of Algebraic Expressions

Recall, to start with, how you found the value of 20 (19 + 11). You will write it either as

$20(19 + 11) = 20(19) + 20(11) = 380 + 220 = 600$

or you will first find 19 + 11 = 30 and then 20(19 + 11) = 20(30) = 600.

We can use second method only if wecan combine the numbers written with in brackets, but in most of the cases it is not possible. In such cases we perform the multiplication process on each term written in the bracket (Distributive Property). To understand it let us considerthe following examples:

Example 5.5: Multiply (s + 2) by 5.

Solution: $5(s + 2) = 5s + 5(2)$

$= 5s + 10$

We can do it like this also:

$(s + 2)\ 5 = (s)\ 5 + 2\ (5) = 5s + 10$

Remark: (s) 5 is written as 5s, not as s5.

Example 5.6: Multiply 2x and $x^2 - 2x + 1$

Solution: $2x\ (x^2 - 2x + 1) = 2x\ (x^2) - 2x\ (2x) + 2x\ (1)$

$= 2x^3 - 4x^2 + 2x$

Example 5.7: Multiply 2x +5 and 2x + 3

Solution: $(2x + 5)\ (2x + 3)$

$= 2x\ (2x + 3) + 5(2x + 3)$

$= 2x(2x) + 2x(3) + 5(2x) + 5(3)$

$= 4x^2 + 6x + 10x + 15$

$= 4x^2 + 16\ x + 15$

Intext Questions 5.6

1. Multiply the following and express the product in simplified form:
   (i) $y\ (3y^2 + 5y - 6)$
   (ii) $(a^2 - b^2)\ ab$

Let us Revise

- Numbers and variables connected by four fundamental operations are called algebraic expressions.
- Parts separated by the symbols '+' or '-' are called terms of the expressions.
- The expression having no sign indicate '+' sign. For example 3t means +3 x t
- In -5pqr; -5, p, q, r are the factors of - 5pqr. Out of these any one or more are called the coefficients of product of other factors (along with sign) .

- In an algebraic expression term having no variable is called the constant. For example in the expression $x2 + xy + 5$ '5' is a constant term.
- Algebraic expression having one term is called monomial.
- Algebraic expression having two terms is called binomial.
- Algebraic expression having three terms is called trinomial.
- For finding the value of an expression, variables are replaced by their numerical values and simplified
- For adding or subtracting two expressions, their like terms are added or subtracted.
- For multiplying algebraic expressions distributive property is used.

Exercise

1. Which of the following expressions are monomial, binomial or trinomial:

   (i) $ut + \frac{1}{2}gt^2$ (ii) 0 (iii) $x^2 + y^2 - a^2$

   (iii) $ax^2 + 2hxy + by^2$ (v) $p^2 + 2pq + q^2$ (vi) $v^2 - u^2$

2. Find the value of following expressions for given numerical values of the variables:

   (i) Value of $x^2 + y^2 - 169$ for $x = 5, y = 12$

   (ii) Value of $(s + t)(s - t)$ for $s = 5, t = 3$

3 $yz^2 - (7x - 2y) - 10yz^2$

4 $7 - 2[4x - (1 - 3x)]$

5 $-2(3x - z) - (z - y) + 5(x + 2y)$

6 $(x - 1)(x + 1)(x - 2)(x + 2)$

Answers

Intext Questions 5.1

1 (i) $3t$; 1 (ii) $x^2, 3xy$; 2

(iii) $t^2, 3t, \frac{1}{t^2}$; 3 (iv) $a^3, -b^3, 3$; 3

(v) ab, bc, –ca ; 3 (vi) $x^2, y^2, z^2, 2hxy$; 4

Note

Intext Questions 5.2

1 (i) y^2z (ii) $\frac{3}{7}s^2t^2$ (iii) $-3qr^2t$ (iv) $7x^2yz^2$

2 (i) 3 ; 5 (ii) 5 ; –3 (iii) 2 ; $-\frac{1}{7}$ (iv) $-\frac{3}{7}$; $-\frac{5}{7}$

3. (i) Unlike (ii) Unlike (iii) Unlike (iv) Like (v) Like

4. (i) x^2, $3x^2$ are like terms

$x^2, -y^2$; $x^2, -4xy$; $-y^2, 3x^2$; $-y^2, -4xy$; $3x^2, -4xy$ are unlike

$5x, \frac{3}{5}x$ are like terms

$\frac{3}{5}x, 5$; $5x, -3y$; $5x, 5$; $-3y, \frac{3}{5}x$; $-3y, 5$ are unlike terms.

xyz, –yzx, zyx are like terms

xyz, x^2yz ; –yxz, x^2yz ; zxy, x^2yz are unlike terms

Intext Questions 5.3

1. (i) Monomial,1 (ii) Binomial, 2 (iii) Trinomial, 3

   (iv) Binomial, 2 (v) Binomial, 2

2. (i) 1 (ii) 5 (iii) 27 (iv) 7 (v) 0

Intext Questions 5.4

1 (i) 12x + 14, binomial, number of terms is 2.

  (ii) 8x2y2 + 6 xy3 + 2x3 + 6, number of terms is 4.

  (iii) 16x + 18, number of terms is 2, binomial

Intext Questions 5.5

1. (i) 6a – 3b – c

   (ii) – bc – 3ab + 2b – 2c

   (iii) 14 – 16x, a binomial.

2. Yes

Solution: Think the number n

Multiply by 3 3n

In it add a number	
which is 1 more than the	$3n + (n + 1) = 4n + 1$
original number	$4n + 1 + 7 = 4n + 8$
Add 7	$\frac{4n+8}{4} = n+2$
Divide by 4	$n + 2 - 2 = n$
Subtract 2	$n + 2 - 2 = n$

Exercise

1. (i) Binomial (ii) Monomial
   (iii) Trinomial (iv) Trinomial
   (v) Trinomial (vi) Binomial
2. (i) 0 (ii) 16

3 $-9yz^2 - 7x + 2y$

4 $9 - 14x$

5 $-x + 11y + z$

6 $x^4 - 5x^2 + 4$

6

LINEAR EQUATIONS IN ONE VARIABLE

In this curriculum you have read at number of places that Mathematics helps us in understanding universe around us. How does it happen? Suppose you have some problem. First of all we try to translate it in the language of mathematics. Doing so is not always easy. But in case of some problems we can do so. For example, your problem is, 10 people are to be invited for meal. Out of these, 6 need 4 chapattis each and remaining needs 2 chapattis each. You want to know that how many chapattis you need to cook. If required number of chapattis be p, then

$$p = 6 \times 4 + 4 \times 2$$

If you do not know that how many need 4 chapattis then this number can be taken as x (say). If, required number of chapattis be p, then

$$p = 4x + 2(10 - x)$$

If you have 28 chapattis in all, then in mathematical form this problem will be written as:

$$28 = 4x + 2(10 - x)$$

Actually, mathematics helps us to solve our all problems. In this unit, we shall take those problems, which can be converted to special forms. These forms are known as 'Linear Equations in one variable'.

From this lesson, you will learn:

- To convert word problems to Algebraic Equations.
- $ax + b = 0$ is a linear equation in one variable.
- To understand to solve linear equations
- Solving linear equations in one variable
  - (i) By adding an expression on both sides
  - (ii) By subtracting an expression from both sides

Note

(iii) By dividing or multiplying by some non-zero numberto both sides

- Solving daily life problems with simple language through linear equations.

6.1 Concept of Equation

You have learnt about algebraic expressions like x + 10, y - 2, 10x + 2y, 6a + 3b-17c, xyz, x^3y etc. If we equate one algebraic expression to other expression then we get an equation. y - 2 = 0 is an equation. For getting other examples, let us convert following statements to mathematical statements:

(i) Three times a number is same as adding 2 to the number.

(ii) Sum of two consecutive numbers is 37.

(iii) Width of a rectangular garden is half of its length and perimeter of the garden is 600 meter.

How will you write these statements in Algebra?

In (i) if that number be y, which is not known to us, then we can write

$$3y = y + 2 \quad \ldots\ldots.. \ (1)$$

This is an equation.

In (ii) Let us take one number to be a, then other number will be a +1 or a - 1.

Thus statement becomes: a + (a + 1) = 37 ; or a + (a - 1) = 37 …….(2)

Again it is an equation.

Now can you write (iii)?

Suppose width is w meter, then length will be 2w.

You know that perimeter is the sum of the four sides.

So, from the statement,

$$w + 2w + w + 2w = 600 \qquad \ldots\ldots..(3)$$

This is another example of equation.

Intext Questions 6.1

1. Give three examples of equation.

6.2 Linear Equations

Now you have some knowledge about an equation. Think of a special type of equation. From your previous knowledge you know that those equations are the ones which are given below

Note

$$\frac{2}{3}y = y + 2 \qquad ...(1)$$

$$2a + 1 = 37 \qquad ...(2)$$

$$6w = 600 \qquad ...(3)$$

Do you find some similarity in these three equations? For example, how many variables are there in each? You will find that each equation is having one variable. y in the first, a in thesecond and w in thethird.

What is their Degree?

You can check that Degree of each equation is 1. Three equations which we considered are called linear equations in one variable. These equations are called linear since if we represent these equations geometrically in a plane then in each case we get a straight line.

Definition: Linear Equation in one variable is such an equation which has one variable and degree 1.

Now we will discuss different life situations in which we need to solve linear equations. In the next section we will learn the method to solve these.

Example 6.1: Express the following statements as Linear Equation.

(a) Cost of a magazine and a newspaper is ₹ 15. Cost of magazine is 4 times the cost of newspaper.

(b) You started from your home by cycle at 10 O'clock at the speed of 10 km per hour. At 11 o'clock in the afternoon your sister started from thesame place following you by same route at a speed of 20 km per hour. At what time will she cross you?

Mathematical Constructions

(a) Suppose cost of newspaper is n, since cost of magazine is 4 times the cost of the newspaper

$\therefore$ Cost of magazine = 4n

We know that total cost = 15

$\therefore$ $n + 4n = 15$

or $5n = 15$ .... (4)

This linear equation is representing the given problem.

Remark: You could have converted the problem to mathematical problem by presuming the cost of the newspaper as m also.

(b) Suppose your sister crossed you after x hours. So, in x hours, you covered 10x km distance. Since your sister started 1 hour later than you, so she took (x - 1) hours to cross you.

She covered a distance of 20 (x - 1) km

Both of you meet at a particular point

$\therefore \quad 10x = 20\,(x - 1)$

This linear equation is representing given problem.

Intext Questions 6.2

1. Present the following statements through linear equations:

   (i) Multiplying number 4 gives the same result; it gives on adding 6 to the number.

   (ii) Three new tailors Akram, Bano and Charu joined a Tailor's shop temporarily for trial. In one week, Charu stitched 10 blouses more than Akram and by the same time Bano stitched three times blouses more than Akram. In one week three together stitched 50 blouses.

   (iii) In a particular mixture, ratio of sand and cement is 4:1. Keeping in mind this ratio 25 kg of mixture is to be prepared.

   (iv) At the same time two trains started at the same time from two stations towards each other to cover a distance of 426 km. If there be a difference of 8 km per hour in their speeds, and they reach same place after 3 hours.

6.3 Solving Linear Equations

By now you have learnt what Linear Equation is ? You have discussed some real life problems, which we can present in the form of linear equations. So if we are to solve problems, then we need to be able to solve corresponding linear equations. Only in that case we can get help from mathematical constructions. Now we will see how it can be done? First we try to understand the meaning of solving linear equations. We start with an example. Consider the equation $x + 1 = 2$. In words this statement is "What added to 1 gives 2?"

Thus if we presume x to be 1, then equation $x + 1 = 2$ becomes a true statement. Does some other value of x can make the true statement? Check it by taking some other value (say 3) of x. You will get a false statement $3 + 1 = 2$. Thus only one value of x i.e. $x = 1$ makes both sides of the equation equal.

We say that $x = 1$ satisfies the equation or makes a true statement. This value of x is called the solution of the equation.

Hence solving an equation means finding all possible values of the variable which make the equation true. These values are called the solutions of the equation.

Definition: That value of the variable which makes the equation a 'True Statement' is called the solution of the equation.

Further note that a linear equation in one variable has only one solution.

Let us see how this unique solution is found.

Suppose we consider the equation $x + 5 = 27$. Now we want to find the value of x. So we want to get rid of + 5 from the left side, so that we are left with only x on this side. How we do it ? As we do in numbers, we subtract 5 from both sides.

By doing so, we get:

$$x + 5 - 5 = 27 - 5$$

Or $x = 22$

This is the solution of the given equation.

Remember that an equation is like a balance.

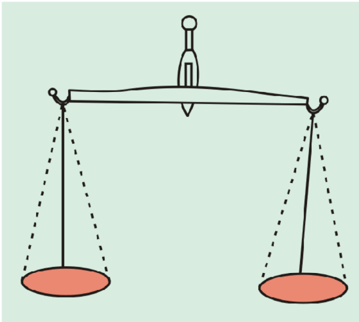


Figure 9.1

What so ever operation we perform on one side, we will be bound to do it on the other side too,so that equation is maintained. Now we consider equation(1) of section 6.1.

This equation is $3y = y + 2$

How will you solve this equation?

You see y on both sides of the equation. So can you bring both on same side of the equation? How?

Suppose you subtract y from both sides.

You get $3y - y = y + 2 - y$

Thus equation becomes

$$3y - y = 2$$

or $2y = 2$

or $\frac{2y}{2} = \frac{2}{2}$

$\therefore$ $y = 1$

On putting $\quad y = 1$ in $3y = y + 2$

$3 \times 1 = 1 + 2$

$3 = 3$

This is a true statement.

$y = 1$ is the required solution,

You have seen that for finding the solution, we change the equation in every step, but we do not allow any change in its equality. We follow thesteps as under once or more number of times:

Step 1: Adding or subtracting same expression on both sides of the equation.

Step 2: Multiplying or dividing with a non-zero number on both sides of the equation.

Let us see how we apply above steps to solve equation (4) i.e. $5n = 15$.

Using step 2, divide by 5 on both sides,

$$\frac{5n}{5} = \frac{15}{5} = 3 \qquad \Rightarrow \qquad n = 3$$

This is the required solution (Verify it)

Let us put this value of n in equation (4) of real life problem of section 6.2.We note that cost of Newspaper is 3 and cost of Magazine is ₹ 12.

Now look at Equation (5) of section 6.2.

Equation is $\quad 10x = 20\,(x - 1)$

or $\quad 10x = 20x - 20$

or $\quad 20 = 20x - 10x$

or $\quad 20 = 10x$

or $\quad \frac{20}{10} = x$

i.e $\quad x = 2$

Thus solution of the equation is $x = 2$

You can verify that solution of equation (5) of section 6.2 is $x = 2$

Now let us take example 1(b) of Section 6.2, in which we got this equation. We said that two will meet eachother after x hours. So sister will cross you after 2 hours of your starting. You started journey at 10 O'clock. At what time will you be behind your sister? 12 noon. You two must be feeling tired after cycling so far.

Note

Intext Questions 6.3

1. Solve the questions of 'Check your Progress 6.2'. Verify your answers.

Let us Revise

- An equation is the relation of equality between two algebraic expressions.
- A linear equation is that equation in which there is one variable and is of degree 1.
- That value of a variable which makes the equation a true statement is called its solution.
- A linear equation in one variable has one and only one (unique) solution.

Exercise

1. Solve the following linear equations and verify your answer:

   (a) $8 = 2 - 3x$

   (b) $7y - 9 = 6y - 10$

   (c) $[x - (x + 5) - (x - 5)] = 3x + 7$

2. Translate the following word problems to linear equations and solve them

   (a) Subtracting 5 from a number becomes equal to double of the original number. Find the number.

   (b) Length of a rectangle is three times its breadth. If its perimeter be 64 meter, then find its area (i.e. Length x breadth).

   (c) An item whose weight on earth is 48 kg, weighs 8 kg on moon. If a man's weight on earth be 54 kg , then what will be his weight on moon?

   (d) Second angle of a triangle 20° more than its first angle and the third angle is equal to the second angle. Find the three angles of the triangle.

Answers

Intext Questions 6.1

1. There can be infinite examples. Some are given below:

   $x = y$, $x = 2y$, $3xy = x^2y + 9$

2. There can be infinite examples. Some are given below:

   (i) $3x^2 - 5x = 2x + 7$ (Degree 2)

   (ii) $y - 3y^3 + 2y^2 = 4y^2 + 5$ (Degree 3)

   (iii) $3x - 4 = -2x + 5$ (Degree 1)

Intext Questions 6.2

(i) If number is n, then

$$4n = 6 + n$$

(ii) Suppose Akram stitched x blouses, then Bano prepared 3x and Charu x+10

$\therefore$ Equation is $x + 3x + (x + 10) = 50$

(iii) Suppose amount of cement and sand is x and 4x.

Then $x + 4x = 25$

Or $5x = 25$

(iv) If speed of slow train be x kmper hour then speed of second train will be $(x + 8)$ km per hour.

$3x + 3(x + 8) = 426$

Intext Questions 6.3

1. (i) $n = 2$

   (ii) $x = 8$, So Akram prepared 8, Bano 24 and Charu 18 blouses.

   (iii) $C = \frac{25}{4}$

   $\therefore$ Required quantity is $= 6\frac{1}{4}$ kg

   (iv) $x = 57$

   $\therefore$ Speeds of the trains are 57 km per hour and 65 km per hour.

Note

Exercise

1. (a) $x = -2$ (b) $y = -1$ (c) $x = -\frac{7}{4}$

2. (a) Mathematical statement is $x - 5 = 2x$, where x is the number.

   Solution $x = -5$

   (b) Suppose length = x meter, width = 3x meter

   $2(x + 3x) = 64$

   $8x = 64$

   Or $x = 8$

   $\therefore$ length of rectangle = 24 meter

   and breadth = 8 meter

   Area of rectangle = 192 square meter

   (c) Equation $\frac{w}{54} = \frac{8}{48}$, is when w is required weight.

   $\therefore w = 9$

   Man's weight on moon = 9 kg.

   (d) Equation is :

   $x + (x + 20) + 2(x + 20) = 180$

   Where $x°$ is the measure of first angle.

   $\therefore x = 30$

   Measures of three angles are 30°, 50° and 100°.

Module - III

Commercial Maths

Suppose Ram has ₹200.00 and Ahmed has ₹50.00 we can say that Ram has ₹(200-50) say ₹150.00 more than Ahmed. Similarly if zully has 10 Toys and Sheela has 5 toys then we can say that zully has 5 toys more than Sheela. In this way, we see that one way of comparing numbers is to find their difference.

Is this method always suitable? Let us take one more example. Suppose there are 300000. Books in library A and 5000 Books in library B. Here we can say that there are 295000 more books in library A than library B. This is more comfortable to say that library has $\left(\frac{300000}{5000}\right)$ or 60 times more books than library B. Similalry we can say for the above example in para 1, that zulley has toys two times the toys of sheela. Thus we observe that comparison of any two numbers is by division method which we express by ratio.

7

RATIO AND PROPORTION

From this lesson, you will learn

- Definition of Ratio
- Definition of Proportion Definition of simple and compound Ratio-III
- Solving some problems using Ratio and proportion
- Solving problems related to "Time Work" and "Time Distance" using Ratio & Proportion

7.1 Ratio

Definition : The relation between two similar quantities where one quantity is how many times or what part of the other quantity is, called Ratio

Ratio is represented by putting (:) symbol between them. So, we can also say (in the example on previous page) That the ratio of books is 60 : 1 in library A & Library B. The two quantities or numbers which are compared are called the terms of Ratio First term is Antecedent and second terms is called consequent similarly in 12 : 5, 12 is 'anticedent' and 'consequent' ? Please remember that 'Ratio' is always between similar quantities or things with the same unit of measure. It looks strange when we compare 'Toys' & books. Sometime we write ratio in the term of a fraction 4:1 is written as $\frac{4}{1}$ and 15:7 same as $\frac{15}{7}$.

Let us see now 12:8, we write it as $\frac{12}{8}$. A fraction is always written in it's lowest term i.e $\frac{12}{8} = \frac{3}{2}$. Hence 12:8 can also be written as 3:2, similarly 18:42 is written as 3:7. When we multiply the two terms of a ratio by any number (except o), The value of ratio will not change.

$\therefore \frac{12}{8} = \frac{3}{2}$ Hence 12:8 = 3:2

$\frac{3}{7} = \frac{18}{42}$ Hence 3:7 = 18:42. Remeber the ratio between two quantities is written without unit. Hence the ratio of 12 litre and 18 litre is $\frac{12}{18}$ or $\frac{2}{3}$ or 2 :3

Ratio of 25cm and 40cm is $\frac{25}{40}$ or $\frac{5}{8}$ or 5 : 8

Let us take some examples to explain the above

Example 7.1: Find the ratio of the following

(i) 38 and 114

(ii) 165cm and 220cm

(iii) ₹17.20 and ₹86.00

(iv) 2kg and 500gm

Sol (i) We know that 'Ratio is written in the form of a fraction

$\therefore$ 38 : 114 = $\frac{38}{114}$, writing this fraction in lowest term

38 = 19×2, 114 = 19×2×3

$\therefore \frac{38}{114} = \frac{19 \times 2}{19 \times 2 \times 3} = \frac{1}{3}$ Hence 38 : 114 = 1:3

(ii) 165cm and 220cm is written in the form of Ratio as 165:220 (no unit)

$= \frac{165}{220}$

55 is the H.C.F of 165 and 220

$\therefore$ 165 = 55×3, 220 = 55×4

Hence $\frac{165}{220} = \frac{55 \times 3}{55 \times 4} = \frac{3}{4}$ $\therefore$ 165 : 220 = 3:4

Remarks : In the first example, we make factors and in the second example we used H.C.F. Any one method can be used.

(iii) ₹17.20 and ₹86.00 is written as ratio 17.20 : 86.00 or $\frac{1720}{8600} = \frac{1}{5}$

Hence the ratio of ₹17.20 and ₹86.00 is 1:5

(iv) 2kg = 200gm [Ratio is between same units]

$\therefore$ Ratio of 2kg and 500g = 2000 : 500 or 4 : 1

Hence required ratio is 4 : 1

Example 7.2

There are 100 boys and 80 girls in a school find the following ratios

(i) Ratio of boys and total students

(ii) Ratio of girls and boys

Sol. (i) Total students = Boys + Girls = 100+80 = 180

$\therefore$ Ratio of boys to total students = 100 : 180

or $\frac{100}{180} = \frac{5}{9}$ $\therefore$ Required ratio is 5 : 9

(ii) Ratio of Girls to boys = 80 : 100 or 4 : 5 [Dividing both by 20]

Intext Questions 7.1

1. Find the ratio between following

   (i) 8 and 168 (ii) 2.5 and 7.5 (iii) ₹11.50 and 115

   (iv) 25 paise and ₹75.00 (v) 15m and 250cm

2. The average speed of a Train is 45km/hr and that of the other is 75km/hr. Find the ratio between the two average speeds.

3. Out of 50 people working in a company, 22 are make and rest all female. Find the ratio of the number of males and females.

4. The monthly income of a family is ₹15000. If the family saves ₹3000 per month, find the following ratios.

   (i) Ratio of income and expenditure

   (ii) Ratio of income and saving

   (iii) Ratio of savings and expenditure

5. 260 students appeared in an examination. Out of this 130 were declared pass. Find the following ratios

   (i) Total students appeared to pass students

   (ii) No of pass students to no failed

7.2 Proportion

We have learnt above that multiplying and dividing the terms of ratio by the same quantity (Expect 0) does not change the value of ratio

Hence 3 : 9 = 1 : 3 = 7 : 21 and soon

Similarly 9 : 270 = 1 : 30 = 5 : 150

Do remember first write a ratio in to lowest form then multiply the two terms by the same number.

When we get two equal ratios, we call that there are in proportion or these form a proportion.

In this way the equality of two ratios is called proportion

Hence we can say four quantities forms proportion if the ratio of first and second is same as the ratio of 3rd & 4th. Hence four quantities a, b, c, d form a "proportion" if

a:b = c:d

or $\frac{a}{b} = \frac{c}{d} \Leftrightarrow a \times d = b \times c$... (1)

a,b,c,d are the terms of "proportion" first temo 'a' last term 'd' are called extremes, second term 'b' and third term 'c' are called means/middle terms. From (1) above we see the product of 'extremes' is equal to the product of "means"/middle terms. When the two middle terms are same - a : b = c : b or $\frac{a}{b} = \frac{c}{d}$ here 'b' is called the mean proportion of a & c

Let us explains this by an example

Example 7.3 Which are true statements in the following

(i) 3 : 4 = 45 : 60 (ii) 3 : 2 = 9 : 8 (iii) 39 : 117 = 1 : 3

(iv) 15 : 7 = 45 : 24

Sol. (i) Product of middle terms 4 × 45 = 180

Product of extremes = 3×60 = 180

$\therefore$ 3 : 4 = 45:60 is true

(ii) 3 : 2 = 9 : 8

Product of middle terms = 2×9 = 18

Product of extermes = 3×8 = 24

$\therefore$ 18 ≠ 24 so 3:2 = 9:8 is not true

(iii) 39:117 = 1:3 Product of middle terms = 117 × 1 = 117

Product of extermes = 39×3 = 117

$\therefore$ 117 = 117 Hence 39 : 117 = 1 : 3 is True

(iv) 15:7 = 45:24

Product of middle terms = 7×45 = 315

Product of extremes = 15×24 = 360

315 $\neq$ 360 $\therefore$ 15:7 = 45:24 is not true

Example 7.4

First three terms of a proportion are 4,12 & 18.

Find the 4th term if the proportion.

Suppore the 4th term = x

$\therefore$ 4 : 12 = 18: x

Hence 12×18 = 4xx

or $\frac{{}^{3}\cancel{12}\times 18}{\cancel{4}} = x$

= 54 = x

Hence fourth form of the proportion is 54.

Example 7.5

Find the mean proportion of 3 and 27 suppose the terms of mean proportion are x.

Sol $\therefore$ 3 : x = x : 27

$\Rightarrow \frac{3}{x} = \frac{x}{27} \Rightarrow x^2 = 81 = 9^2$

$\Rightarrow x = 9$

Intext Questions 7.2

1. In the following, which statements are true?

(i) 4:5 = 28:35 (ii) 7:9 = 42:27 (iii) $\frac{15}{2} = \frac{15}{2}$

2. Find the value of x in the following proportions

(i) 24:36 = 36:x (ii) 5:7 = 15:x (iii) $\frac{34}{3}$:12 = 17:x

3. Find the mean proportion for the following extrements

(i) 2 and 8 (ii) 4 and 16 (iii) 6 and 216

(iv) 5 and 125

7.3 Types of Proportions

Proportions are of two types:

(a) Direct proportion/Direct variation

(b) Inverse proportion/Inverse variation

Let us learn about these

When two quantities of a proportion are related in a way that an increase/decrease in the terms of one proportion, it is called direct proportion/direct variation. More toys require more money, less no of oranges require less money etc.

When the quantities of a proportion are related in a way that increase in one may bring a decrease in the second portion then it is called inverse proportion/veriation. If the no. of laboures are **increased**. Then the no of days required to finish the work will be **decreased**.

Let us take example to explain the above

Example 7.6

The cost of 5 dozen oranges is ₹120. Find the cost of 7 dozen oranges. In this example when the number of oranges are <u>increased</u> the cost will also <u>increase</u>. This is direct variation

Oragnes	Cost
5 dozens	₹120
7 dozens	₹x

$\therefore\ 5:7 = 120 : x \quad \Rightarrow \quad 5\times x = 120\times 7$

$$\therefore\ x = \frac{\overset{24}{\cancel{120}} \times 7}{\cancel{5}} = 7\times 24 = 168$$

Hence the cost of 7 dozens oranges is ₹168.00

Remarks : Do remember that when x and y are directly proportional then $x;y$ or $\frac{x}{y}$ remains <u>constant</u>

In the above example

$x = 5$ dozens or $\frac{x}{y} = \frac{5}{120} = \frac{1}{24} = \frac{7}{168} = \frac{1}{24}$

Hence if we take this constant as k.

Then $\frac{x}{y} = \text{k}$ or $x = \text{ky}$, here 'k' is constant

Here we call x & y are in direct proportion

Example 7.7

A cyclist covers a distance in 3½ hrs with an average speed of 10km/hr. If he increases his average speed from 10km/hr to 14km/h, then how much time will be need to cover the same distance?

Sol. It is clear that when the average speed is increased time taken will be decreased, hence this is the case of inverse proportion.

Average speed km/hr	Time (in hrs)
10	3½
14	x

$\therefore$ 10:14 = x : 3½

Hence $10\times\frac{7}{2} = x \times 14$ or $x = \frac{10}{\cancel{14}\,2}\times\frac{\cancel{7}}{2} = \frac{10}{4} = 2.5$

Hence the time required will be 2½hrs or 2.5hrs. or 2hrs 30min.

Remarks : Do remeber when x & y are inverses proportional the $x.y$ is constant

Hence x = 10km/hr. $\therefore$ x.y = $10\times\frac{7}{2}$ = 35 constant y = 3½ hrs

$\therefore\ 14 \times x = 35 \Rightarrow x = \frac{35}{14} = 2½$

Similarly $x.y = \text{k}$ or $x = \frac{\text{k}}{\text{y}}$, hence k is constant.

Hence, we say x & y are incverses proportional or inverse variation.

Intext Questions 7.3

1. A person purchases 3kg honey in ₹ 360. How much honey will he purchase in ₹810.00?

2. The cost of 20 cold drink bottles is ₹800.00 How much will be the cost of 35 such written?

3. If 50 people can construct a wall in 15 days, Then in how many days. 75 people will construct the similar well?

4. If 25 people can reap the field crop in 9 days then how much time will be taken by 15 people to reep the same field crop?

Note

5. A houseful grain is sufficient for 40 days for 500 people. The same will last for how many days for 800 people?

6 The shadow of a 10m high hill is 8 meterat any point of time in the day. At the same time the shadow of another hill top is 12 meters. What is the height of the second hill top?

7. The cost of 24 coconut is ₹480. Find the cost of 120 such coconuts

8. 10 people can construct a road in 6 days. How much time will be taken to construct the same road by 15 people?

7.4 Unitary method

When the cost of some objects is given and to find the cost of a different number of such objects, Then

(i) First method using proportion

For ecample, if the cost of 5kg wheat is ₹100. Then what will be the cost of 16kg. Wheat? More wheat more cost hence this is direct proportion. Suppose required value is ₹x then

$5 : 100 = 16 : x$ or $5x = 16 \times 100$ or $x = \dfrac{16 \times 100}{5} = 320.00$

$\therefore$ The cost of 16kg. wheat is ₹ 320.00

(ii) Another method – First of all we find the cost of one object. Then multiply by the number of such objects/Things.

As we first find the cost of one/unit so it is called unitary method.

Let us solve the above example/problem by unitary method.

Cost of 5kg wheat = ₹100

Cost of 1kg wheat = ₹ $\left(\dfrac{100}{5}\right)$ or ₹20

$\therefore$ Cost of 16kg wheat $= (16 \times 20) =$ ₹320

Example 7.8

If the cost of 10 soap pieces is ₹150 and the cost of 4 tooth pastes is ₹60, Then find the cost of 6 soap pieces and 2 tooth pastes.

Sol. Cost of 10 soap piece = ₹150

Cost of 1 soap piece = $\dfrac{150}{10}$ = ₹15

Hence cost of 6 soap pieces = 6×15 = ₹90

Note

Cost of 4 toothpastes = ₹60

Cost of 1 toothpaste $= ₹\,\dfrac{60}{4} = ₹15$

$\therefore$ Cost of 2 toothpastes = 15×2 = ₹30

$\therefore$ Total cost of 6 soap pieces and two toothpastes = ₹ (90+30) = ₹120

Intext Questions 7.4

1. It the cost of 4kg sugar is ₹120. Find the cost of 1 quintal sugar.
2. If the cost of 25 copies of a book is ₹525, then find the cost of 10 copies of the same book
3. If the cost of one dozen medicine bottles is ₹612, Then how much will be the cost of 4 such medicine bottles?
4. If the cost of are dozen oil bottles is ₹720 and 6 pickle boxes cost ₹240, find the cost of 4 oil bottles and 3 pickles boxes.
5. 15 people can construct wall in 10 days, how much time will take for 10 people to construct the same wall?
6. Five people together can soap a filled in 8 days, how many people will be required to soap the same field in 2 days?

7.5 Time and work

Some times we have to take decisions to complete a particular work in a specific time. For example, if I work 3 hours daily, how many days will be required to complete the work and also how many persons to be engaged to complete the flooring of my house in 2 days? These type of questions are related to "Time & work", you will learn in detail in the following examples:

Example 7.9

Ram can finish a particular work in 15 days and Shyam in 10 days. Both together how much time will take to complete the work?

Sol. Ram completes the whole work in 15 days.

In one day he will complete $\dfrac{1}{15}$th work.

In one day Shyam will complete $\dfrac{1}{10}$th work.

$\therefore$ In one day both together will complete $\left(\dfrac{1}{15}+\dfrac{1}{10}\right)$th work $= \dfrac{2+3}{30} = \dfrac{1}{6}$

$\therefore$ The whole work will be completed in 6 days.

Note

Example 7.10

Lata and Sona together complete a work in 20 days. If Lata alone can complete the work in 30 days.

How much Sona alone will take to complete the work?

Sol Late and Sona together complete the work in 20 days.

In one day both will complete the work = $\frac{1}{20}$ th part of the work

In one day Lata completes the work = $\frac{1}{30}$ th part of the work

The work left to be done in one day by sona = $\frac{1}{20}-\frac{1}{30}=\frac{3-2}{60}=\frac{1}{60}$ part of the work

$\therefore$ Total work will be completed by Sona in 60 days.

Example 7.11

Karan and Ahmed can complete a work in 15 & 20 days respectively. Both worked together for six days then Karan left the work. How much time Ahmed will take to complete the work?

Sol. Karan's one day work = $\frac{1}{15}$

Ahmed one day work = $\frac{1}{20}$

Karan and Ahmed together

will complite the work in one day $=\frac{1}{15}+\frac{1}{20}=\frac{4+3}{60}=\frac{7}{60}$

Karan and Ahmed's 6 days work $=\frac{7}{60}\times 6=\frac{7}{10}$

Remaining work $=1-\frac{7}{10}=\frac{3}{10}$

Ahmed will complete $=\frac{3}{10}$

$\therefore$ Ahmed completes full work in 20 days

$\frac{3}{10}$ work in $=\left(\frac{3}{10}\times 20\right)=6$ days

$\therefore$ $\frac{3}{10}$ work will be completed in $=\left(\frac{3}{\cancel{10}}\times \cancel{20}^{\,2}\right)=6$ days

Exmaple 7.12

Rama and Kewal complete a work in 12 &18 days respectively. Both of them agreed to complete the work in ₹1720. How much each of them will receive out of ₹1720?

The ratio of their capacity to do the work in one day $\frac{1}{12} : \frac{1}{18}$

or $\frac{3}{36} : \frac{2}{36}$ or 3:2

Hence the contract amount will be divided in the ratio of 3:2

$\therefore$ Rama's share = ₹ $\left(\frac{3}{5} \times \cancel{1720}^{344}\right)$ = ₹1032

Kewal's share : ₹ $\left(\frac{2}{5} \times \cancel{1720}^{344}\right)$ = ₹688

Example 7.13

Joseph & Reena completed a work in 12 & 24 days respectively. Both started the work together. Joseph left the work 3 days before the completion of work, the remaing work was done by Reena along. In how many days the work was completed?

Sol. Joseph & Reena's one day work $= \frac{1}{12} + \frac{1}{24} = \frac{2+1}{24} = \frac{3}{24} = \frac{1}{8}$

The work finished by reena in 3 days $= \frac{1}{24} \times 3 = \frac{1}{8}$

The work completed by both Joseph & Reena $= 1 - \frac{1}{8} = \frac{7}{8}$

Both completed the work = 7days

Reena worked along for = 3 days

$\therefore$ Total time = 10 days

Intext Questions 7.5

1. A & B can finish a work in 10 days & 15 days respectively. How many days will they take together in finishing the work?

2. Rama, Sheela & Zube days complete a work respectively in 8, 12, 24 days. How many days will They require to finish the work together?

3. A, B & C together finish a work in 20 days. If A & B individually complete the work in 40 & 60 days respectively. How many days will 'C' require to complete the work?

4. Karam Singh & Ramila together can complete a work in 24 days. If Karam Singh day can do the work in 36 days, then how many days Ramila will take to finish the work?

5. A & B together finish a work in 8 days. Both worked together for 6 days. The remaining work was completed by B in 6 days. Find their individual capacity to finish the work.

6. Raman and Vikas can finish a work in 16 & 24 days respectively. Both worked together for 6 days then Raman left. In how many days will Vikash finish the work?

7.6. Time & Distance

There is a special importance of time and distance in our day to day activities. We all are careful in reaching at the right time for our work and accordingly decide the time to leave the house. This is only possible when we have an estimate of our speed. If 10km distance is to be covered and with a speed of 4km/hr.,

The time will be taken – $\frac{10}{4}$ hrs = 2½hrs.

Hence we can say time $= \frac{\text{Distance}}{\text{Speed}}$ or Distance = Time × Speed (i)

From (i) we can also write Time $= \frac{\text{Distance}}{\text{Speed}}$ (ii)

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{(iii)}$$

Before taking examples, let us know some rules. If two vehicles with speeds a & b are moving.

(i) in apposite directions, then their relative speed will be (a+b)

(ii) If in the same direction then relative speed is (a – b) when a>b

Let us take some examples

Example 7.14

A train is moving with a speed of 60km/hr. Find the speed, per second, of the train?

Sol. In one hour the train moves = 60km = 6000m

In one second the train will move = $\frac{{}^{50}\cancel{60000}}{{}_{3}\cancel{3600}} = \frac{50}{3}$ m

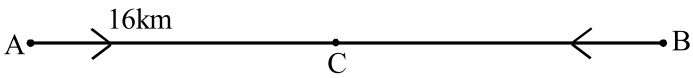
$\therefore$ Speed in m/s = $\frac{50}{3}$ or $16\frac{2}{3}$ m/s

Example 7.15

A & B are moving from their houses 40km a part, each other with speeds 4km/h & 6km/h respectively. After how much time & where will they meet each other?

Sol. When A & B are moving towards each other i.e opposite direction

$\therefore$ Relative speed = (6+4) = 10km/hr



Total time taken to cover 40km distance = $\frac{40}{10}$ = 4hrs.

After 4 hrs. A will travel $4 \times 4 = 16$km

$\therefore$ A & B will meet after 4 hrs and 16km from A's house to words B's house.

Example 7.16

A train is moving with a speed of 60km/hr

If the length of the Train is 200m. How much time will the train take to cross a pole?

Sol. To cross a pole, the train will have to cover it's own length (as the length of the pole is neglegible)

(60×1000) m distance is covered in = 3600 sec

1 m distance will be coverd = $\frac{{}^{6}\cancel{0}\ 360\cancel{0}\ \cancel{0}}{60\times100\cancel{0}}$ See = $\frac{6}{100}$ sec.

200m distance will be coverd - $\frac{6}{100} \times 200 = \frac{1200}{100}$ = 12 sec

Train will cross the poll in 12 sec.

Example 7.17

A trains is moving with a speed of 80km/hr. If the length of the train is 120m, then how much time will be taken by the train to cross a plateform 180m long?

Sol. Total distance to be coverd by the train = (120+180) = 300 meters

Note

(80 x 1000) m distance is covered by the train in 6.3600 sec.

1m distance will be covered in see $\frac{3600}{80 \times 1000}$

300 m distance will be covered in = $\frac{\overset{9}{\cancel{36}}00 \times 3\cancel{00}}{{}_{2}\cancel{80} \times 1\cancel{000}} = \frac{27}{2}$ see

Hence Train will cover/cross the platform in $\frac{27}{2}$ sees $= 13\frac{1}{2}$ sees

Example 7.18

A train moving with a speed of 60km/hr crosses a 220m bridge in 24 seconds. Find the length of the train.

Sol. Let the length of train = xm

Total length of train + Bridge = (x+220m)

Time taken = 24 sec.

In 3600 sec distance covered = (60×1000) meter

In 24 sec distance covered $= \frac{60 \times 1000}{3600} \times 24 = 400$ meter

$\therefore$ 220 + x = 400

$x = 400 - 220 = 180$m

$\therefore$ length of Train = 180m

Example 7.19

Two racers take part in a race. Racer A starts running when racer B has gone 100m ahead. If the racer A takes 6 minute and racer B takes 10 minutes in running distance of one km. In how much time racer A will cross racer B?

Sol. Racer A runs in 6 minute 1 km

Then in 10 minutes $= \frac{1 \times 10}{6}$

$= \frac{5}{3}$ km

$\therefore$ Racer A runs $\left(\frac{5}{3} - 1\right) = \frac{2}{3}$ km more than racer B in 10 minutes

Note

or racer A runs $\left(\frac{2}{3}\times 1000\right)$ runs more in 10 minutes

So, A runs 100 meter in $\left(\frac{10\times 3\times 100}{2\times 1000}\right)$ minutes

$= 1\frac{1}{2}$ minute

Hence race A will meet race B in $1\frac{1}{2}$ minute

Intext Questions 7.6

1. A train is running at a constant speed of 721km/hr. How much distance it will cover in 15 second?
2. A train covers 60km distance in one hour and a car covers 300 meter in 15 seconds. Which vehicle is running faster?
3. A train, whose length is 300 meter, is running at a speed of 120km/hr. How much time will it take to cross a pole?
4. A train , where length is 200 meter, crosses a pole in 12 seconds. Find the speed of the train.
5. A train, whose length in 200 meter, is running at a speed of 72km/hr. How much time will it take to cross a 280m plot form?
6. A train of length 360 meter crosses a 40m long bridge in 20 seconds. Find the speed of the train.
7. Two racers P & Q take part in a competetum. Racer Q starts running when P has cover one km and Q takes 12 minutes to cover 1km. Then how much time Q will take to cross?

Let us Revise

The ratio of two quantities x & y is $x : y$ or $\frac{x}{y}$

- When the units of x & y are same

When two ratio are equal they form a proportion. As-

x : y = a : b is a proportion

- In $x : y$ = a : b, x & b are extreme terms and y & a are means. If the middle terms are same then they are call mean proportion.

As In $x : y = y : b$, y is the means proportions of x & b. or $y^2 = x$-b

- If there is an increase or decrease in one quantity this also affects second quantity to increase or decrease in the same ratio, then these are in direct proportion.

  When x & y are indirect proportion then $x : y = \frac{x}{y} = k$ or $x = ky$.

- If increase / decrease in one quantity affects to decrease/increase in the second quantity, then there are in inverse proportion. When x & y are in inverse proportion then $x.y = k$ or $x = \frac{k}{y}$.

- In place of using propotion, unitary method can also be used to solve problems.

Exercise

1. Find the ratio in the following

   (i) 16 & 72 (ii) 2.5 and 7.5

   (iii) ½ and ¾ (iv) ₹8½ and ₹34

   (v) 15cm and 1.5 meter (vi) 10 ℓ and 78 ℓ

   (vii) 9.5 and 7.6 (viii) 6.64 and 0.096

   (ix) 134 meter and 201 meter (x) 27 paisa and ₹1.08

2. In a class of 60 students, 35 are boys and rest are girls. Find the following ratio:

   (i) No. of boys to the total students

   (ii) No. of girls to the totals students

   (iii) No. of girls to no of boys

3. One person covers 10 km distance in 2½ hours and the second person covers 15km distance in 4 hours. Find the ratio of their average speed.

4. Which are the true statements in the following?

   (i) 9:7 = 63:39 (ii) $\frac{15}{2} : \frac{7}{4}$ = kg : 3½ kg

   (iii) 5. 75 : 23 = 8 : 40 (iv) 8 : 9 = 4 : 5

   (v) 15 : 4½ = 5 : 2

5. Find the value of x from the following:

   (i) 15 : 13 = 1.95 : x

   (ii) 2 : 3 = 2½ : x

   (iii) 0.15 : 7.5 = 8cm: xcm

(iv) $8 : 25 = 4 : x$

(v) $15 : 10 = 3 : x$

6. Form the proportion from the following
   (i) 13, 18, 90, 65
   (ii) 5, 12, 15, 4
   (iii) 12, 9, 13½, 18
7. The cost of 3 dozens copies is ₹180. Find the cost of 10 dozen similar copies of the same size.
8. The cost of 30 kg sick sack is ₹960. Find the cost of one quintal rice (100 kg)
9. 2½ litre petrol is used in a scooter for 80km distance. How much petrol will be used in 128 km distance.
10. If 75 person can finish a work in 3days, then how much time will 15 person take to finish the work?
11. If the cost of 3 books is ₹ 180 and cost of 4 copies is ₹84 then find the cost of similar one dozen books and 6 copies.
12. The cost of a 15kg apple box is ₹930. Find the cost of 5kg apples, if the cost of wooden box is ₹30.
13. A, B & C finish a work in 6, 8 & 12 days respectively they together take a contract to finish the work for ₹10400, how much amount will each get?
14. Kewal Ram and Malik settled a contract for ₹ 2700 to finish a work. If they have the capacity to do the work in the ratio 8 : 7 Find their share in the contract.
15. A & B complete a work in 12 & 18 days respectively. Both started the work together. B left 8 days before the work finished and the remaing work was finished by A. How much time was taken to finish the work?
16. Kala and vimla starts walking from their houses, which are 2km apart, towards each other. Kala's speed is 4km/hr and Vimla's speed is 6km/hr. After how much time, from the start, they will meet each other?
17. A goods train and a car started running to words eachother from the two cities 160km apart. Speed of Goods train is 48km/hr and the speed of cal is 72km/hr. After how much time & where they will meet each other?

Answers

Intext Questions 7.1

1. (i) 1:21 (ii) 1 :3 (iii) 1:10 (iv) 1 : 300 (v) 6 : 1
2. 3 : 5 3. 11 : 14

Note

4. (i) 5 : 4 (ii) 1:1 (iii) 1:4

5. (i) 2 : 1 (ii) 1 : 1

Intext Questions 7.2

1. (i) True (ii) False (iii) True

2. (i) 54 (ii) 21 (iii) 18

3. (i) 4 (ii) 8 (iii) 36 (iv) 25

Intext Questions 7.3

1.	6.75kg	2.	₹ 1400	3.	10 days
4.	15 days	5.	25 days	6.	15 m
7.	2400	8.	4 days		

Intext Questions 7.4

1.	₹3000	2.	₹210	3.	₹204
4.	₹360	5.	15 days	6.	20 persons

Intext Questions 7.5

1.	6 days	2.	4 days	3.	120 days
4.	72 days	5.	A : 12 days B : 24 days		
6.	9 days				

Intext Questions 7.6

1.	300meter	2.	Car 12 km/hr more speed		
3.	9 secs	4.	60 km/hr	5.	24 secs
6.	72 km/hr	7.	7 minute, 12 secs		

Exercise

1. (i) 2 : 9 (ii) 1 : 3 (iii) 2 : 3
   (iv) 1 : 4 (v) 1 : 10 (vi) 5 : 39
   (vii) 5 : 4 (viii) 20 : 3 (ix) 2 : 3
   (x) 1 : 4

2. (i) 7 : 12 (ii) 5 : 12 (iii) 5 : 7

3. 16 : 15

4. True statement (i), (ii)

Note

5. (i) 1.69 (ii) $\frac{15}{4}$ (iii) 400

   (iv) $\frac{25}{2}$ (v) 2

6. (i) 13 : 65 :: 18 : 90

   (ii) 5 : 15 : : 4 : 12

   (iii) $9 : 13\frac{1}{2}$: : 12 : 18

7. ₹ 600

8. ₹ 3200

9. 4ℓ

10. 15 days

11. ₹ 846

12. ₹ 300

13. A : ₹ 2400, B : ₹3200, C = ₹ 4800

14. Kewelram : ₹ 1440, malik, ₹1260

15. 12 Days

16. 12 Minutes

17. After 1hr. 20 minutes from the starting and when the train travelled 64km and car 96km.

8

PERCENTAGE AND IT'S APPLICATIONS

You are already acquainted with the fractions. For example, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{32}{35}$, . . . etc. are fractions. You have also learnt that for comparing two or more fractions. we express them in the form of equivalent fractions, so that their denominators are equal. A fraction whose denominator is 100 is called percent.

From this lesson, you will learn:

- Writing percent in the form of fraction
- Writing a fraction in the form of percent
- Solving questions based on percentage
- Solving profit and loss questions through the application of percentage
- Solving Discount related questions based on percentage

8.1 Fractional form of percent

Let us learn how to write a fraction in the percent form and write the percent into fractions. To understand this let us take some examples:

Example 8.1

Compare $\frac{3}{8}$ and $\frac{2}{7}$

We have to write $\frac{3}{8}$ & $\frac{2}{7}$ in the form of equivalent fractions such that the denominator is the LCM of their denominators

LCM of 8 & 7 is 56

Hence

$$\frac{3}{8} = \frac{3\times 7}{8\times 7} = \frac{21}{56}$$

Note

$$\frac{2}{7} = \frac{2\times 8}{7\times 8} = \frac{16}{56}$$

Here we see that $\frac{21}{56} > \frac{16}{56}$

$\therefore \frac{3}{8} > \frac{2}{7}$

Example 8.2

Compare $\frac{9}{10}$ and $\frac{4}{50}$

Fractions are $\frac{9}{10}$ & $\frac{41}{50}$

$$\frac{9}{10} = \frac{9\times 5}{10\times 5} = \frac{45}{50}$$

$$\frac{41}{50} = \frac{41}{50}$$

Now $\frac{45}{50} > \frac{41}{50}$ $\quad \therefore \frac{9}{10} > \frac{41}{50}$

For converting the fractions into percent we need to make denominator as 100

$$\therefore \frac{9}{10} = \frac{9\times 10}{10\times 10} = \frac{90}{100}, \text{ also } \frac{41}{50} = \frac{41\times 2}{50\times 2} = \frac{82}{100}$$

Here also $\frac{90}{100} > \frac{82}{100}$ $\therefore \frac{9}{10} > \frac{41}{50}$

90% and 82% 90% is more them 82%

8.1.1 To convert fractions into percent

In the above example you have seen that $\frac{90}{100}$ is 90% we write $\frac{90}{100}$ as $90\times\frac{1}{100}$

$\left[\frac{1}{100}\text{represents}\%\right]$

$$\therefore 90\times\left(\frac{1}{100}\right) = 90\%$$

Similarly $\frac{82}{100} = 82\times\left(\frac{1}{100}\right) = 82\% \left(\frac{1}{100}\text{ is }\%\right)$

To convert a fraction into percent, we have to make the denominator as 100.

Note

Hence we multiply numerator & denominator by the same number so that denominator becomes 100. In case we donot get 100, then we multiply the denominator and numerator by 100 but denominator should remain, 100 numerator can be simplified.

Example 8.3

Write the following fractions into percent

(i) $\frac{3}{5}$ (ii) $1\frac{7}{15}$ (iii) 0.7

Sol. (i) $\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60 \times \frac{1}{100} = 60\%$

(ii) $1\frac{7}{15} = \frac{22 \times 20}{15 \times 20} = \frac{440}{300} = \frac{440}{3} \times \frac{1}{100} = \frac{440}{3}\%$

or $146\frac{2}{3}\%$

(iii) $0.7 = \frac{7}{10} = \frac{7 \times 10}{10 \times 10} = \frac{70}{100} = 70 \times \frac{1}{100} = 70\%$

Example 8.4

Write the following % in to fraction

(i) 45% (ii) $16\frac{2}{3}$ (iii) 11.5%

$45\% = 45 \times \frac{1}{100} = \frac{45}{100} = \frac{9}{20}$ (Dividing numerator and denominator by 5)

(ii) $16\frac{2}{3}\% = \frac{50}{3}\% = \frac{50}{3} \times \frac{1}{100} = \frac{50}{300} = \frac{1}{6}$

(iii) $11.5\% = 11.5 \times \frac{1}{100} = \frac{23}{2} \times \frac{1}{100}$

$= \frac{23}{200}$

Example 8.5

(i) 160 is what percent of 200?

(ii) 3.5 kg is what percent of 25 kg

Sol. (i) Required% $= \left(\frac{\cancel{160}^{80}}{\cancel{200}} \times \cancel{100}\right) = 80\%$

Another method 160 is out of 200 then out of 100 will be half of 160 or 80

(ii) Required percent of 25kg for 3.5kg

$\left(\frac{3.5}{25}\times 100\right)$ % {Here we are multiplying by 100, as we are to find how much out of hundred}

$= \frac{\overset{7}{\cancel{35}}}{\underset{5}{\cancel{5}}\cancel{250}}\times\overset{2}{\cancel{100}}$

$= 14\%$

Intext Questions 8.1

1. Write the following fractions into percent

(i) $\frac{15}{20}$ (ii) $\frac{5}{6}$ (iii) 0.68 (iv) $1\frac{1}{4}$ (v) $\frac{3}{25}$

2. Write the following percent into lowest form fraction

(i) 15% (ii) $66\frac{2}{3}$ (iii) $13\frac{1}{3}\%$

(iv) 35% (v) $23\frac{3}{4}\%$

3. (i) What percent of 180 is 90?

(ii) What percent of ₹75 is ₹45?

(iii) What percent of 50 is 15 litre?

8.2 Find out a specific percent of a given amount

Let us take some examples to explain how we find the given percent of an amount

Example 8.6

(i) Find 15% of ₹1500

(ii) Find 45% of 250kg

Sol: (i) 15% of ₹ 1500

$= \left(15\cancel{00}\times 15\times\frac{1}{\cancel{100}}\right) = ₹\ 225$

(ii) 45% of 250kg

$= \left(\overset{5}{\cancel{250}}\times 45\times\frac{1}{\underset{2}{\cancel{100}}}\right)$ kg

Note

$$= \frac{225}{2} \text{ kg}$$

$$= 112\frac{1}{2} \text{ kg}$$

Example 8.7

If 25% of the length of a line segment is 6 meter, then find out the length of the line segment.

Sol. Out of 100 is = 25

Out of 1 is $= \frac{25}{100}$

Out of x is $= \frac{25}{100}x$

$\therefore \frac{25x}{100} = 6$

$\therefore x = \frac{6 \times \cancel{100}^{4}}{\cancel{25}}$

$\therefore$ Required length = 24 meter

or

say if 25m length of line segment then total length = 100

If 1m length then the total length $= \frac{100}{25}$

If 6m length then the total length $= \frac{100}{25} \times 6 = 24\text{m}$

Intext Questions 8.2

1. Find the value of the following

   (i) 26% of 25 litre (ii) 75% of 40kg (iii) 20% of ₹ 1900

2. Find the value of x in the following

   (i) 16% of x is 260

   (ii) 1.5% of x is ₹ 108

   (iii) 90% of x is 216km

8.2.1 Some word problems based on percentage

Example 8.8

There are 1300 trees in a garden. Out of them 26% are Guava trees. What is the number of rest of the trees?

Sol. Total no. of trees = 1300

No. of Guava trees $= \left(1300 \times \frac{26}{100}\right) = 338$

No. of rest of trees = 1300 – 338 = 962 trees

Note

Example 8.9

The monthly income of a person is ₹16,000, out of this he spends ₹12000. What percent of his income does he save?

Sol. The amount of saving = Total income –expenditure

= ₹ (16000–12000)

= ₹ 4000

Saving in the form of percent

$$= \left(\frac{4000}{16000}\times 100\right)\%$$

= 25%

Example 8.10

Find the amount which becomes ₹1331 after 10% increase.

Sol. Let the required amount = ₹x

Hence x + (10% of x)

$$= x + \frac{x}{10} = \frac{11x}{10}$$

Give $\frac{11x}{10} = 1331 \Rightarrow x = \frac{1331\times 10}{11} \Rightarrow x = ₹1210$

$\therefore$ Required amount = ₹1210

Example 8.11

In a particular year there is a 150% increase in the enrolment. If in the begining there were 1500 students, then find the students after enrolment.

Sol. No of students in the begining = 1500

Increase $= 1500 \times \frac{15}{100}$

$$= \frac{15}{100} \times \overset{15}{1500}$$

= 225

Hence the no of students ofter admission/enrolment = 1500 + 225

= 1725

Note

Intext Questions 8.3

1. Sunita secured 76% marks, in an examination, out of total of 800. Find the marks obtained by Sunita.

2. An employee received ₹15000 as bonus from the company. If the bonus is 20% of the total annual income, then find his annual income.

3. 60% of a number is 48. Find that number.

4. Reena secured some marks in an examination. In the same examination Seema secured 20% more marks. If the maximum marks of the examination were 600. Total marks secured by them were 720, find the marks secured by each one of them.

8.3 Profit and Loss

Every day we make purchases from the market mostly we purchase these items from the retailers. The retailer makes purchases from the whole seller. This amount is called the cost price of the retailer. The retailer sells goods to the customer, This is called selling price of that thing. This is clear if the selling price is more than the cost price, then whether there is a profit for the retailer or loss.

$\therefore$ Profit = Selling Price – Cost Price, Loss = Cost Price – Selling Price

Sometimes the retailer spends some amount on cartage and salary to the employees engaged with him. These are called over head charges and the retailer adds this into his cost price. Example. Cost Price of a TV is ₹16000 and ₹100 spent as cartage for bringing the TV then CP of TV becomes ₹16100 unless it is made clear overhead charge are added to cost price.

Percent Profit/loss

Do remember percent profit/loss is always calculated on cost price. Let us take an example.

Example 8.12

A shopkeeper purchased an object for ₹ 1400 and sold it for ₹1512. Find the profit percent.

Sol. Cost Price = ₹1400

Selling Price = ₹1512

Profit = ₹ (1512–1400) = ₹112

$\therefore$ Profit on ₹1400 = ₹112

Note

$\therefore$ Profit on ₹100 = ₹ $\frac{112}{1400} \times 100$ = ₹8

$\therefore$ Percent profit = ₹8%

$$\text{Percent profit} = \left(\frac{\text{Total Profit} \times 100}{\text{Cost Price}}\right)\%$$

$$\text{and Percent loss} = \left(\frac{\text{Total loss} \times 100}{\text{Cost Price}}\right)\%$$

Intext Questions 8.4

1. Find the percent profit or loss in the following questions.

	S.P	CP	Over Head Charges
(i)	₹550	₹450	__________
(ii)	₹1440	₹1500	__________
(iii)	₹300	₹225	₹25
(iv)	₹210	₹190	₹10
(v)	₹190	₹180	₹20

2. Ramesh purchased a table for ₹3000 and sold it for ₹2950. Find his percent loss or profit

3. Kamini purchased a cycle for ₹1500 and sold it for ₹1800. Find his percent profit or loss.

4. Ahmed purchased a moter cycle for ₹1200. He spent ₹1300 on it's repair and sold it for ₹19000 Find the % profit or loss of Ahmed.

5. Ahmed purchased oranges at the rate of ₹30 per dozen and sold them at the rate of ₹40 per dozen. Find the % loss/profit of Ahmed.

> S.P, C.P, % loss / profit, out of these there if any two are given then the third can be calculated.

Let us take some examples to explain this

Example 8.13

A horse whose cost price ₹1,35,000 was sold at a profit of 10%. What is the SP of the horse?

Note

Sol. Cost price of horse $= ₹1,35000$

% profit $= 10$

$$\therefore \text{S.P} = \frac{1,35,000 \times (100+10)}{100}$$

$$= \frac{1,35,0\cancel{00} \times 110}{1\cancel{00}} = ₹148500$$

Example 8.14

A watch was sold for ₹3290 and there was a loss of 6%. Find the cost price of watch.

Sol. S.P $= ₹3290$, Loss $= 6\%$

$$\text{S.P} = \frac{\text{CP} \times (100-6)}{100}$$

$$3290 = \frac{\text{CP} \times 94}{100} \quad \therefore \ \text{CP} = \frac{3290 \times 100}{94} = ₹3500$$

Intext Questions 8.5

1. Find the unknown x from the following:

	S.P	CP	Loss%	Profit%
(i)	x	₹650	5	______
(ii)	₹243	x	______	$12\frac{1}{2}\%$
(iii)	x	₹500	______	5%
(iv)	₹250	x	$16\frac{2}{3}\%$	______
(v)	x	₹40	______	15%

2. A table was sold for ₹1920 with a loss of 4%. Find the C.P of the table.

3. A shop keeper earns 40% profit on selling an object for ₹910. Find the C.P of that object.

4. Suresh spent ₹250 on the repair of a plough whose cost price is ₹550. He sold it at a profit of $12\frac{1}{2}\%$. Find the S.P. of the plough.

Some other Examples

Example 8.15

If x is 20% more than y then find what % less of x is y?

Sol. Let the value of y be = 100

Then x will be 120 $\{\because$ 20% more than y$\}$

Now when x is 120 then y = 100

When x is 100 then $y = \frac{100}{120} \times 100 = \frac{1000^{0}}{120} = \frac{250}{3} = 83\frac{1}{3}$

$\therefore$ y from x is less % $\left\{100 - \frac{1}{3}\right\}\%$ $\left[100 - 83\frac{1}{3}\right]\%$ or $16\frac{2}{3}\%$ less

Example 8.16:

Ali secured 434 marks in an examination with 62%. In the same examination Ram secured 350 marks. What % marks did Ram score?

Sol. Let maximum marks = x

$\therefore$ 62% x = 434 $\Rightarrow \frac{62}{100} \times x = 434$

or $x = \frac{434 \times 100}{62} = 700$

Ram secured marks = 350

$\therefore$ % marks of Ram $= \left(\frac{350}{700} \times 100\right)\% = 50\%$

$\therefore$ Ram secured 50% marks in the examination.

Example 8.17

A man purchased eggs at the rate of ₹48 per dozen. At what rate per egg should he sell to receive 15% profit

Sol. Cost of 12 eggs = ₹48

Cost of 1 egg $= \frac{48}{12} =$ ₹4

$\therefore$ Cost of 100 eggs = 100 × 4 = ₹400

Profit = 15%

$\therefore$ Profit on 100 eggs $= \frac{15}{\cancel{100}} \times \cancel{400} = ₹60$

$\therefore$ S.P. of 100 eggs $= ₹(400+60) = ₹460$

Example 8.18

Ram kumar sold a radio to Dutt at a profit of 8%. Dutt spent ₹58 on it's repair and sold it to Seema for ₹836. Dutt got 10% profit in this transaction. At what price did Ram kumar sell this radio?

Sol. S.P. of Dutt = ₹836

Profit = 10%

$\therefore$ Cost Price of Dutt $= \frac{836 \times 100}{(100+10)} = \frac{836 \times 100}{110} = ₹760$

This includes ₹58 of repair

$\therefore$ Dutt's actual cost price = ₹ (760–58) = ₹702

Ram Kumar's C.P. $= \frac{702 \times 100}{108} = ₹650$

$\therefore$ Cost Price of Ram Kumar = ₹650

Intext Questions 8.6

1. If A's value is 20% less than B's value then what percent B's value is more than A's value?

2. After 10% reduction in the price of rice, a person can purchase 10 kg more rice in ₹1400. Find the original price of rice and the reduced price.

3. Rama obtained 204 marks in an examination, her percent marks are 34%. If Sophia obtained 212 marks in the same examination, find the % of marks obtained by Sophia.

4. A man purchased oranges at the rate of ₹72 per dozen. At what rate per 100 she should sell them to get 20% profit.

5. Ali sold a car to Ahmed for ₹2,50,000. Ahmed spent ₹50000 on it's repair and then sold it at a profit of 8%. Find the S.P. of the car.

8.4 Discount

To increase the sale or to sell the old articles business people given advertisement like "prices are 30% reduced"; special sale offer at "20% off/discount" This is sold on a special counter. The amount reduced is called "Discount". The amount which is printed on article is called its 'market price' & the amount is reduced is called 'Discount'. The amount paid by the customer is called the S.P. of the article. Discount is, often, some percent of marked price. Let us take some examples.

Example 8.19

A business man gives 15% discount on the blankets prepared by him. If the marked price of a blanket in ₹1200, Then how much will the customer pay?

Sol. Marked price = ₹1200, Discount = 15%

Discount on ₹1200 = $1200 \times \frac{15}{100}$ = ₹180

Hence the customer will pay ₹1020 for the blanket

Example 8.20

The marked price of a pair of shoes is ₹1150 and the same is sold for ₹950 during sale. Find the rate of discount on the pair of shoes.

Sol. Marked price : ₹1150

Selling Price : ₹950

Total discount = ₹ (1150–950) = ₹200

$$\therefore \% \text{ discount} = \left(\frac{200}{1150}\times 100\right)\% = \frac{2000}{115} = 17.4\% \text{ (Approx)}$$

Intext Questions 8.7

1. Find the discount for the following

	Marked Price	Discount
(i)	₹54	10
(ii)	₹480	6
(iii)	₹350	8
(iv)	₹150	10
(v)	₹160	5

2. A fan with marked price ₹2000 is sold at a discount of 15%. Find the selling price of the fan.

3. Find the percent discount for the following

	Marked Price	Selling Price
(i)	₹65.00	₹ 50.00
(ii)	₹80.00	₹ 65.00
(iii)	₹120.00	₹105.00

Let us Revise

- Percent is that fraction whose denominator is 100.
- Profit or loss percent is always caculated on the cost price.
- S.P. – CP = Profit {S.P $\rightarrow$ Selling Price}
- C.P – SP = Loss { C.P $\rightarrow$ Cost Price}
- $\text{C.P} = \dfrac{\text{S.P} \times 100}{(100 + \%\text{Profit})}$ or $\dfrac{\text{S.P} \times 100}{(100 - \%\text{loss})}$
- $\text{S.P} = \dfrac{\text{C.P} \times (100 + \%\ \text{profit})}{100}$ or $\dfrac{\text{C.P} \times (100 - \%\ \text{loss})}{100}$
- Discount is calculated as a percent of the marked price.

Exercise

1. Write the following in percent form

(i) $\dfrac{7}{10}$ (ii) $\dfrac{2}{25}$ (iii) 0.75

(iv) 0.28 (v) 2.8

2. Write the following in the lowest form of a fraction

(i) 12% (ii) 8.2% (iii) 32%

(iv) 0.9%

3. (a) Find the value of the following

(i) 5 % of 150 (ii) 18% of 5 liter

(iii) 40% of 112kg (iv) 40% of 8 cm

(b) (i) What percent of 150 is 96?

(ii) What percent of 40 is 14?

4. Find the value of x

 (i) 12% of $x = 135$

 (ii) 80% of $x = 26$ liter

 (iii) 4% of $8x = 36$

5. Find the value of x in the following

	C.P	S.P	% Profit	% Loss	Overhead Charges
(i)	₹400	₹500	x%	____	____
(ii)	₹400	₹ x	40%	____	____
(iii)	₹150	₹ x	____	20%	____
(iv)	₹x	₹400	10%	____	₹50
(v)	₹900	₹x	____	10%	____

6. Sunita obtained 70% marks is an examination. If the maximum marks are 800, find the marks obtained by Sunita.

7. Areina obtained 60 marks out of 80 in a mathematics question paper. Find her % marks.

8. A cycle was sold at a 10% loss after purchase for ₹2400. Find the selling price of the cycle.

9. Three articles are sold at the rate of marked price of 4 such articles. Find the % profit.

10. Marked price of an article is ₹1600. The shopkeeper gives 20% discount. How much will the customer pay for it?

11. The price of an article is ₹1800 after 10% discount. Find the marked price of the article.

12. The price of an article after 35% discount is the same as that of another article of ₹1300 after 10% discount. Find the marked price of the first article.

13. A man purchased 2 oranges for ₹10 and sold them at the rate of ₹4 per orange, find the profit or loss%

14. A man marks 3.0%. more price on an article, also he gives 20% discount on the new marked price. Find his profit/loss percent.

Intext Questions 8.1

1. (i) 75% (ii) $83\frac{1}{10}\%$ (iii) 68% (iv) 125% (v) 12%

2. (i) $\frac{3}{20}$ (ii) $\frac{2}{3}$ (iii) $\frac{2}{15}$ (iv) $\frac{7}{20}$ (v) $\frac{19}{80}$

3. (i) 50% (ii) 60% (iii) 30%

Intext Questions 8.2

1. (i) 6.5 ℓ (ii) 30 kg (iii) ₹380

2. (i) 1625 (ii) ₹7200 (iii) 240 km

Intext Questions 8.3

1. 608 2. ₹75000 3. 80 4. 300, 420

Intext Questions 8.4

1. (i) $22\frac{2}{9}\%$ profit (ii) 4% loss

   (iii) 20% profit (iv) 5% profit (v) 5% loss

2. $1\frac{2}{3}\%$ loss

3. 20% profit

4. $42\frac{6}{7}\%$ profit

5. $33\frac{1}{3}\%$ profit

Intext Questions 8.5

1. (i) ₹ 617.50 (ii) ₹216 (iii) ₹525

   (iv) ₹300 (v) ₹ 46

2. ₹2000

3. ₹650

4. ₹900

Intext Questions 8.6

1. 25% 2. ₹ $17\frac{7}{9}\%$, 14 3. 52%

4. ₹720 5. 3,24,000

Intext Questions 8.7

1. (i) ₹5.40 (ii) ₹28.80 (iii) ₹28 (iv) ₹15 (v) ₹8

2. ₹1700

3. (i) $23\frac{1}{13}\%$ (ii) $18\frac{3}{4}\%$ (iii) $12\frac{1}{2}\%$

Exercise

1. (i) 70% (ii) 8% (iii) 75% (iv) 28% (v) 280%

2. (i) $\frac{3}{25}$ (ii) $\frac{41}{500}$ (iii) $\frac{8}{25}$ (iv) $\frac{9}{1000}$

3. (a) (i) 7.5 (ii) $\frac{9}{10}\ell$ (iii) 4.8kg (iv) 3.2cm

   (b) (i) 64% (ii) 35%

4. (i) 1125 (ii) 32.5ℓ (iii) $\frac{900}{8}$

5. (i) $12\frac{1}{2}\%$ (ii) ₹560 (iii) ₹120 (iv) ₹350 (v) ₹810

6. 560 7. 75% 8. 2160 9. $33\frac{1}{3}\%$

10. ₹1280 11. ₹2000 12. ₹1800

13. 4% loss 14. 4% profit

9

SIMPLE AND COMPOUND INTEREST

When we borrow some money from a person, bank or cooperative society for a specific period, we pay back the money along with some additional amount for using that money for a certain period. This additional amount is called Interest. To calculate Interest we need to know the amount and the period for which it is borrowed and rate of interest. The borrowed amount is called principal, time period is number of years/ months. The sum including interest & principal is called amount.

Amount = Principal + Interest

Interest is calculated for a particular period and the rate percent of principal. The rate per annum is called the rate of Interest. 10% rate of Interest means ₹ 10 is interest for ₹ 100 for 1 year period.

From this lesson, you will learn:

- How Interest is calculated.
- The information required to calculate the Interest.

9.1 Simple Interest

When Interest is calculated for the whole period on the initial principal borrowed, it is called simple Interest

The formula for calculating similar Interest.

$$I = \frac{Prt}{100}$$

Where I is simple Interest, P = Principal, r is the annual rate of interest, t is time in years for which the money was borrowed.

Let us take some questions related to this formula. Before start solving the question, we should know three questions out of the four (P, I, r & t) to find out the fourth.

Example 9.1 A person borrowed ₹1200 from Bank for a period of 2 years at the rate of Interest 10% annual, calculate the Interest

Sol. P = ₹1200, r =10% or ₹ $\frac{10}{10}$, t = 2 years

$$\therefore \ I = 12\cancel{00} \times \frac{10}{1\cancel{00}} \times 2 = ₹240$$

Hence the person will pay ₹240 as interest to the Bank

Example 9.2 Calculate the I interest for a period of 219 days on a sum of ₹1800 at 6% annual rate of interest and also find the amount

Sol. P = ₹1800, $r = 6\% = \frac{6}{100}$, $t = 219 \text{ days} = \frac{219}{365}$ years $= \frac{3}{5}$ years.

$$\therefore \ I = 18\cancel{00} \times \frac{6}{1\cancel{00}} \times \frac{3}{5} = \frac{324}{5} \text{ or } ₹64.80$$

Amount = ₹ (1800 + 64.80) = ₹1864.80

Example 9.3 How much amount should I deposit in a Bank so that after 2 years at a rate of 8% annual, I should get ₹ 128 as Interest

Sol. Here we want to find the Principal.

$$r = 8\% = \frac{8}{100}, \quad t = 2 \text{ years}, \quad I = ₹1280$$

$$\therefore \ I = prt \qquad I = \frac{prt}{100}$$

$$1280 = P \times \frac{8}{100} \times 2$$

$$\therefore \ P = ₹\left(\frac{1280 \times 100}{8 \times 2}\right) = ₹8000$$

Example 9.4 At the rate of 8% annual, after how much period on ₹1600 the interest will be ₹128?

Sol. I = ₹128, P = ₹1600, $r = \frac{8}{100} = \frac{8}{100}$ time = ?

$$128 = 1600 \times \frac{8}{100} \times t$$

$$\Rightarrow t = \frac{\cancel{128} \times \cancel{100}}{\cancel{1600}_{16} \times \cancel{8}} = 1 \text{ year}$$

Note

Example 9.5 At what rate of annual interest will ₹500 be the interest on a sum of money ₹2500 after 4 years?

Sol. P = ₹2500, I = ₹500, t = 4 years, r = ?

$$500 = \frac{25\cancel{00} \times 4 \times r}{\cancel{100}}$$

$$r = \frac{500}{100} = 5$$

∴ Rate = 5%

Example 9.6 At the rate of interest 8% annual the difference in interest on a certain sum of money is ₹360. Find the principal.

Sol. Let the principal = ₹100

$$\text{Interest for 3yrs} = ₹\left(\frac{100 \times 8 \times 3}{100}\right) = ₹24$$

$$\text{Interest for 5yrs} = ₹\left(\frac{100 \times 8 \times 5}{100}\right) = ₹40$$

Difference in interest = ₹(40 – 24) = ₹16

If the difference is ₹16 then principal = ₹100

$$\text{If the difference is ₹1 then principal} = \frac{100}{16}$$

$$\text{If the difference is ₹360 then principal} = \left(\frac{100}{16} \times 360\right)$$

$$= ₹2250$$

∴ The required principal = ₹2250

Intext Questions 9.1

(i) P = ₹1200, t = 5 years, r = 6%, I = ________

(ii) P = ₹1600, t = 3 years, r = 10%, A = ________

(iii) P = ______ , t = 4 years, r = $3\frac{1}{2}$%, I = ₹112

(iv) P = ₹2800, t = ______, r = 10%, I = ₹560

(v) P = ₹5000, t = 4 years, r = _______, I = ₹1600

Note

2. At the rate of Interest 5%, Find the interest on ₹3500 for 146 days (1 year = 365 days)

3. Find the prinapal which at 5% annual rate of interest becomes ₹720 in 4 years.

4. Find the principal for which interest is ₹ 1920 in 4 years at the rate of annual interest 10%

5. At 8% annual simple interest in how much period will the interest be ₹1920 on a sum of ₹400?

6. At what annual rate of interest on a sum of money ₹900, will the interest be ₹324 in 9 years?

7. At what rate percent annually, on a sum of money ₹1000, will the interest be ₹450 in $4\frac{1}{2}$ years?

8. In how much time, the interest on sum of money ₹800 at 8% annual rate of will the interest be ₹1056?

9.2 Compound Interest

Till now we have seen such cases where principal is constant for the whole period but this is not always necessary. In some situations, after a certain interval the interest due is added into the principal this becomes the new principal for the next period. The difference of the amount at the end of the last interval and the initial principal is called the compound interest. The period after which every time the interest is added to the principal to make the new principal for the next period, is called conversion period. When interest is added to the principal after each year, then we say that interest is compounded annually. Similarly the converssion period may be 6 monthly & three monthly. If the amount is ₹'A', principal 'P', rate = r% per conversion period, if number of periods is n then $\boxed{A = P\ \{1+r\}n}$

If we denote compound interest by C then $\boxed{C=A-P}$

$\therefore\ C = A - P \Rightarrow C = P\ \{1+r\}^n - P \Rightarrow C = P\ [(1+r)^n - 1]$

Let us take some examples to explain the above.

Example 9.7 Find the compound interest and the amount of ₹1000 at the rate of 5% for 2 years. When interest is compounded annually.

Sol. $A = P\ [1+r]^n \ \therefore\ A = 1000\left[1+\frac{5}{100}\right]^n$, time = 2 years = n

$$A = 1000\left[\frac{21}{20}\right]^2 = \frac{10\cancel{00}\times 21\times 21}{4\cancel{00}}$$

Note

$$= ₹1102.50$$

$$\therefore\ C = A\text{-}P = (1102.50 - 1000) = ₹102.50$$

Example 9.8 Find the compound Interest of ₹4000 for one year at 10% annually, when interest is compound six months.

Sol. $P = ₹4000,\ r = \frac{10}{2}\%$ or 5% time = 1year = 2 six months

$$\therefore\ n = 2$$

$$C = 4000\left\{\left(1+\frac{5}{100}\right)-1\right\}^2$$

$$= 4000\left\{\frac{21}{20}\times\frac{21}{20}-1\right\} = 4000\left[\frac{441-400}{400}\right]$$

$$= \frac{{}^{10}\cancel{4000}\times 41}{\cancel{400}} = ₹410$$

Intext Questions 9.2

Find the compound interest and amount for the following

	Principal (P)	Rate % annual (r)	Time (t)	Conversion period
(i)	₹5000	10%	2 years	Annually
(ii)	₹7000	10%	1 years	Half Yearly
(iii)	₹2000	5%	1 years	Half Yearly
(iv)	₹500	20%	9 months	Quarterly
(v)	₹2500	20%	6 months	Quarterly

Till now we have learnt to calculate A & C when P, r & n are given. Now we shall learn that out of the four P, A, r, & n if any three are given then the fourth can be calculated. Let us see such situations in the following examples.

(a) A, r and n are given, we can calculated?

Example 9.9 Find the principal which becomes ₹3630 after 2 years with 10% annual rate of Interest, when interest is compounded annually.

Sol. A = ₹3630, P=?, r = 10%, n = 2

$$\therefore\ 3630\ P\left[1+\frac{10}{100}\right]^2 = P\times\frac{11}{10}\times\frac{11}{10}$$

$$3630 = \frac{21}{100}\text{P} \Rightarrow \text{P} = \frac{3630\times 100}{121} = ₹3000$$

$$\therefore\ \text{P} = ₹3000$$

Example 9.10 Find the amount for which compound interest is ₹408 at 8% for 1 year, when interest is compounded half yearly.

Sol. C = ₹408, r = 8% annual, $r = \frac{8}{2} = 4\%$

$x = 1$ year = 2 six months

$$\therefore 408 = \text{P}\left\{\left(1+\frac{4}{100}\right)^2 - 1\right\}$$

$$= \text{P}\left\{\frac{26}{25}\times\frac{26}{25} - 1\right\} \Rightarrow \text{P}\left\{\frac{676-625}{625}\right\} = \frac{\text{P}\times 51}{625}$$

$$\therefore\ \text{P} = \frac{\overset{8}{\cancel{408}}\times 625}{\cancel{51}} = 50000.00$$

$$\therefore\ \text{P} = ₹50000.00$$

Intext Questions 9.3

(a) Find the principal when

	A	r	t/n	c	Conversion period
(i)	₹ 2163.20	4%	24 years	____	yearly
(ii)	₹ 3528	10%	1 year	____	1 half yearly
(iii)	_______	8%	2 years	₹832	yearly
(iv)	_______	20%	6 months	₹820	quarterly
(v)	₹3025	10%	2 years	____	yearly

(b) Find 'r' when A, P_E and n are given:

Example 9.11 At a certain rate of interest the Principal ₹64 becomes ₹125 after 1½ years, when interest is compounded half yearly. Find the annual rate of interest.

Sol. A = ₹125, P = 64, n = 1½ years, r = ? = 3 months

$$\text{using } \text{A} = \text{P}\left[(1+r)^n\right]$$

$$125 = 64\left[1+\frac{r}{100}\right]^3$$

$$\frac{125}{64} = \left[1+\frac{r}{100}\right]^3$$

$$\left[\frac{5}{4}\right]^3 = \left[1+\frac{r}{100}\right]^3$$

$$\frac{5}{4} = 1+\frac{r}{100}$$

$$\frac{5}{4}-1 = \frac{r}{100} \quad \Rightarrow \frac{1}{4} = \frac{r}{100} \Rightarrow 100 = 4r$$

$$\Rightarrow r = \frac{100}{4} = 25\%$$

r = 25% per six month $\quad \therefore$ Annual rate is 50%

Example 9.12 At a certain rate of interest annually, the principal ₹400 becomes ₹441 in 6 months, when the interest is compound quarterly.

Sol. $A = ₹441,\ p = ₹400\ \ n = \frac{6}{3} = 2,\ r = ?$

$$\therefore\ 441 = 400\left[1+\frac{r}{100}\right]^2$$

$$\frac{441}{400} = \left[1+\frac{r}{100}\right]^2$$

$$\left[\frac{21}{20}\right]^2 = \left[1+\frac{r}{100}\right]^2 \Rightarrow \frac{21}{20} = 1+\frac{r}{100}$$

$$\Rightarrow \frac{21}{20}-1 = \frac{r}{100}$$

$$= \frac{1}{20} = \frac{r}{100} \Rightarrow 20\ r = 100 \Rightarrow r = \frac{100}{20} = 5\%$$

$\therefore$ Rate of interest quarterly is 5% $\therefore$ Annual rate of interest is 20%

Intext Questions 9.4

Find the unknown in the following

	A	P	r	n	Conversion period
(i)	₹4410	₹4000	----------	2 years	Annually/yearly
(ii)	₹9680	₹8000	----------	1 years	Half yearly

Note

(iii)	₹12100	₹10000	----------	2 years	Yearly
(iv)	₹6760	₹6250	----------	6 months	Quarterly
(v)	₹18522	₹16000	----------	9 months	Quarterly

(c) To find n, when A, p & r are given

Example 9.13 In what period at the rate of 10% yearly, the principal ₹800 becomes ₹926.10, when the interest is compounded half yearly?

Sol. A = ₹926.10, P = ₹800, $r = \frac{10}{2}\%\ 5\%$

$$\frac{9261}{10} = 800\left\{\left(1+\frac{5}{100}\right)^n\right\}$$

$$\frac{9261}{8000} = \left(1+\frac{5}{100}\right)^n$$

$$\left(\frac{21}{20}\right)^3 = \left(1+\frac{5}{100}\right)^n$$

$$\therefore \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n \Rightarrow n = 3$$

When the interest is compounded half yearly

$$\therefore n = 1\frac{1}{2} \text{ years}$$

Intext Questions 9.5

	A	P	r	Converssion period
(i)	₹9261	₹8000	5%	Annually
(ii)	₹3087	₹2800	10%	Half yearly
(iii)	₹3630	₹3000	20%	Half yearly
(iv)	₹9261	₹8000	20%	Quarterly
(v)	₹17576	₹15625	16%	Quarterly

Difference between SI & CI

Some times we need to take a decision, in which situation we shall get more interest

Note

Example 9.14 Find the difference between Compound interest and Simple interest for ₹48000 for 3 years with a rate of 5% yearly. When interest is compounded annually in the compound interest case.

Sol. P = ₹48000, r = 5%, n = 3 = t

$$SI = P\,r\,t = 480\cancel{00} \times \frac{5}{\cancel{100}} \times 3 = ₹7200$$

$$\text{Compound interest} = c = P\left\{\left(1+\frac{r}{100}\right)^n - 1\right\}$$

$$= ₹48000\left\{\left(1+\frac{5}{100}\right)^3 - 1\right\}$$

$$= ₹48000\left\{\left(\frac{21}{20}\right)^3 - 1\right\}$$

$$= ₹48000\left[\frac{9261-8000}{8000}\right]$$

$$= ₹\frac{{}^{6}\cancel{48000} \times 1261}{\cancel{8000}} = ₹7566$$

∴ Difference between CI & SI ₹(7566–7200)

= ₹366

Sometimes the difference is given we need to find some other quantity such as P, n or r etc.

Example 9.15 On a certain sum of money at the rate of 10% in $1\frac{1}{2}$ years, the difference is ₹ 183. Find the principal if compound interest is compounded six monthly.

Sol. For SI, P = ? r = 10% $t = \frac{3}{2}$ years

Let the principal be ₹100

$$\therefore \text{SI on } ₹100 = \cancel{100} \times \frac{{}^{5}\cancel{10}}{\cancel{100}} \times \frac{3}{\cancel{2}} = ₹15 \quad (1)$$

$$\text{CI on } ₹100 = 100\left\{\left(\frac{1+5}{100}\right)^3 - 1\right\}$$

$$= 100\left\{\left(\frac{21}{20}\right)^3 - 1\right\}$$

$$= ₹\left(\frac{9261-8000}{8000}\right) = ₹\ \frac{1\cancel{00} \times 1261}{80\cancel{00}}$$

$$= ₹\ \frac{1261}{80} \quad (2)$$

From 1 & 2 we get on subtraction

$$\frac{1261}{80} - \frac{15}{1} = \frac{1261-1200}{80} = ₹\ \frac{61}{80}$$

If the difference between CI & SI $\frac{61}{80}$, then principal = ₹100

If the diffference between CI & SI 1, then principal $= \frac{100 \times 80}{61}$

If the diffference between CI & SI 183, then principal $= \frac{100 \times 80 \times \cancel{183}^{3}}{61}$

$= ₹24000$

∴ Required principal = ₹24000

Intext Questions 9.6

1. Find the difference between compound and simple interest

	P	r	t/n	Conversion period
(i)	₹16000	10%	$1\frac{1}{2}$	Half yearly
(ii)	₹12000	20%	6 months	Half yearly
(iii)	₹5000	10%	2 years	yearly

2. On a certain sum of money the difference between CI & SI is ₹16 at 10% annual in 2 years. Find the principal

3. Find the unknown in the following

	P	r	t/n	conversion period	CI–SI
(i)	_______	10%	2 years	yearly	₹150
(ii)	_______	8%	1 year	Half yearly	₹10
(iii)	_______	20%	9 months	Quarterly	₹183

Let us Revise

- An additional money given, along with the borrowed money after a specific period, is called interest
- SI = P × r × t
- Out of the four quanties when any three are given, fourth can be calculated
- Having known CI, SI, rate & time, we can find principal, conversion period etc

Exercise

1. In the following, find the unknown using simple interest formula

	P	r	t	I
(i)	₹6000	5%	3 years	________
(ii)	₹5000	________	2 years	₹1000
(iii)	₹2000	8%	________	₹480
(iv)	₹25000	10%	2 years	________
(v)	________	8%	$1\frac{1}{2}$ years	₹1080

2. At 10% annual rate for simple interest, in how much period the principal will be

   (i) Double

   (ii) Tripple

3. At simple interest the principal of ₹750 become ₹810 in 2years. At the same rate of interest how much would be the amount after 5 years?

4. Find the unknown in the following

	A	P	CI	r	t/n	Conversion Period
(i)	______	₹5000	______	4% yearly	3 years	yearly
(ii)	______	₹6000	______	5% yearly	2 years	yearly
(iii)	______	₹8000	______	10% yearly	$1\frac{1}{2}$ years	yearly
(iv)	₹9261	______	______	20% yearly	9 months	Quarterly
(v)	₹17576	₹15625	______	16% yearly	______	Quarterly

(vi)	₹1331	₹1000	_______	20% yearly	_______	Half yearly
(vii)	₹1331	₹1000	_______	_______	3 yrs	yearly

5. On a certain sum of money at 5% yearly rate of interest, in 2 years, the difference of CI & SI is ₹60 Find the principal

6. At compound interest rate a sum of money becomes $\frac{729}{512}$ times of itself in 3years. Find the rate of interest.

7. For questoin numbers 4(i) & 4(ii) find the compound interest

8. Find the principal in the following

	CI - SI	Rate of interest	t/n	Conversion period	P
(i)	₹40	10%	1 year	Half yearly	_______
(ii)	₹10	20%	1 year	Half yearly	_______
(iii)	₹549	20%	9 months	Quarterly	_____
(iv)	₹122	10%	$1\frac{1}{2}$ months	Half yearly	_______
(v)	₹40	8%	1 year	Half yearly	_______

Answers

Intext Questions 9.1

1. (i) ₹360 (ii) ₹2080 (iii) ₹800 (iv) 2 years (v) 8%

2. ₹70 3. ₹600 4. ₹4800 5. 4 years

6. 4% 7. 10% 8. 4yearly

Intext Questions 9.2

	Amount	Compound Interest
(i)	₹6050	₹1050
(ii)	₹7717.50	₹717.50
(iii)	₹2101.25	₹101.25
(iv)	₹578-81	₹78.81
(v)	₹27562.50	₹2562.50

Note

Intext Questions 9.3

(i) ₹2000 (ii) ₹3200 (iii) ₹5000 (iv) ₹8000 (v) ₹2500

Intext Questions 9.4

(i) r = 5% (ii) 20% (iii) 10% (iv) 16% (v) 20%

Intext Questions 9.5

(i) 3 years (ii) 1 year (iii) 1 years (iv) 9 months (v) 9 months

Intext Questions 9.6

1. (i) ₹2522 , ₹2400 , ₹122

   (ii) ₹1230 , ₹1200 , ₹30

   (iii) ₹1050 , ₹1000 , ₹50

2. ₹1600

3. (i) ₹15000 (ii) ₹6250 (iii) ₹24000

Exercise

1. (i) ₹900 (ii) 10% (iii) 3 years

   (iv) ₹5000 (v) ₹9000

2. (i) 10 years (ii) 20 years

3. ₹900

4. (i) ₹5624.32, ₹624.32 (ii) ₹6615, ₹615

   (iii) ₹9261, ₹1261 (iv) ₹8000, ₹1261

   (v) ₹1951, 9 months (vi) ₹331, years

   (vii) ₹331, 10%

5. ₹24000

6. 12.5%

7. (i) ₹24.32 (ii) ₹15 (iii) ₹61 (iv) ₹61 (v) ₹76

8. (i) ₹16000 (ii) ₹1000 (iii) ₹72000 (iv) ₹16000

   (v) ₹25000

Module - IV

Geometry

Geometry is Greek (yunani) language word, in which Geo means "earth" and metre. means measurement. Hence "Geometry" means measurement of "Earth". It's beginning was in Egypt about 4–5 thousand years ago Babiloneans also contributed significantly in this area Euclid was a Greek mathematician (300 BC), When he was a professor of Mathematics in Alexandaria in Egypt, he consolidated all the results of geometry known by that time and arranged them in order to form a book known as "elements"

In ancient India वेदियां were constructed in different geometrical shapes. For this the knowledge of geometry was necessary. Many Indian mathematicians, Bodhain, Bhaskar, Aryabhatt and Brahmagupta are in the fore front of world mathematicians for bringing new geometrical facts.

The knowledge of geometry is used in measurment, graphical representation, Building consturction, Drawing maps, dams & road construction etc.

In this module you will learn about point, angle, triangle, quardilateral, circle and their properties. We need to understand that all these are plane shapes. Line segment and angle measurement will also included in this module.

10

FUNDAMENTAL GEOMETRICAL CONCEPTS

In your daily life you use different shapes objects. Carefully look at your Book and Geometrical box. Feel the corners, edges and surface of all these objects with your hand. Observe where their edges meet each other.

Where do their faces meet?

Where do the edges meet?

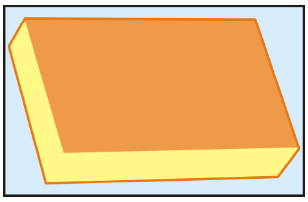


Figure 10.1

You have read about line, line segment and ray on a plane surface. For the construction of a building or drawing a map, we need exact measurements. All these instruments are available in your Geometry box. Drawing and measuring geometrical shapes following instruments are used.

(a) Scale/ruler (b) Divider (c) Compass

(d) Set squares (e) Protractor

From this lesson, you will learn

- About point, line and plane.
- About various parts of shapes.
- Intersecting, parallel and Concurrent lines.
- Geometrical instruments and their use.
- Methods to measure the line segment.
- Draw a line segment of a given measurement.

10.1 Point, Line and Plane

Point: Take a pencil with the sharp tip. Press this tip on a paper. What do you see? A symbol or mark on the paper as in fig (10.2)

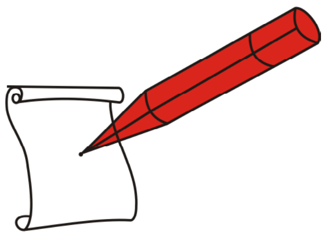


Figure 10.2

This symbol/mark in Geometry is called 'Point'. We use a point to represent the position of an object. This has no length & breadth. As much the sharp will be edge of pencil, The better will be the point. Different points indicate different positions, to represent a number, we use capital letters of English, alphabets and Hindi varnmalas. As shown in the impression on paper of a tip of the needle and the corner of your geometry box are examples of a point.

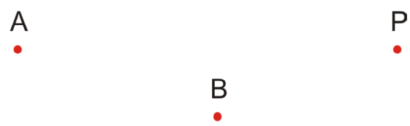


Figure 10.3

10.1.1 Line

Fold a piece of paper and press the fold, it will make a symbol of a line as in fig 10.4

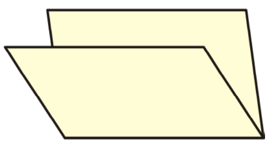


Figure 10.4

Figure 10.5

Ask two children to hold a thread and ask them to streteh it properly. This is another example of straight line as in fig. 10.5. Now try to expand the length of the thread, still this represent a line. The length of a line is infinite and this can be extended on both sides indefinity as shown in 10.6

Figure 10.6

For drawing a line (straight line) we make use of a scale or strip and mark arrow on both ends to show that this is infinite. Take a scale and place it on a paper/ note book using a sharp edge pencil along the edge of the scale, make as many points as possible.

Figure 10.7

What do you observe? You will see it looks like a line.

This shows that a straight line is the set of infinite points. All the points are on the line. There are some other points on the plane which are not on the line. In the figure 10.8.

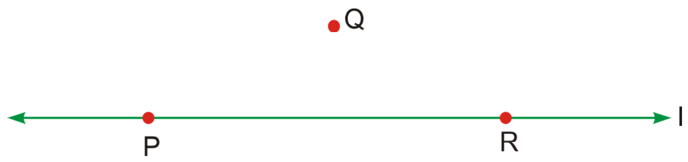


Figure 10.8

Points P & R are on the line but 'Q' is not on the line. The name of a line is represented by the two points on the line or a small alphabet say 'ℓ'. The above line is $\overline{PR}$ or $\overline{RP}$ or 'ℓ'

The line can also be understood the path of a variable point which in the some direction on either sides can go upto infinity.

10.1.2

Plane : Using your palm over the surface of your book, top of table, black board & the wall, you can feel the surface as even. This is an example of a plane. Football ground or surface of water (still) in waterpond are also examples of plane.

Figure 10.9

If we extend the length/breadth of a playing field, plane remains same. The page of a book/note book are examples of a plane.

A plane is extendable indefinitely in all directions

Mark two points on a plane paper. Are these on the plane of paper? Yes they are on the plane of the paper

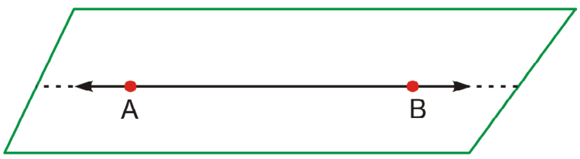


Figure 10.10

In the above, draw line $\overline{AB}$ by joining A & B and extend on both sides still AB is on the plane of paper. Point, line & plane are fundamental concepts of Geometry we can not define these but can explain.

Note

Intext Questions 10.1

1. Give two examples of the following

   (a) A Point (b) A Line (c) A Plane

2. Fill in the blanks with appropriate words

   (a) A dust particle is the example of a __________

   (b) The floor of a room is the example of a __________

   (c) The edge of the surface of a book is the example of a __________

   (d) There are __________ points on a line

   (e) The position/locatoin of a city is represented by a _______ on the map.

   (f) We need __________ points to name a line

   (g) The length of the line is __________

   (h) The line joining two points on a plane also __________ on the plane

3. Write three different names of the line draw below.

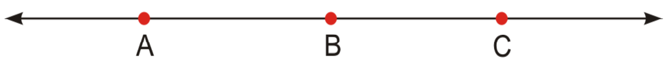


Figure 10.11

4. Write the names of lines drawn in fig. 10.1.2. Are these on the same plane?

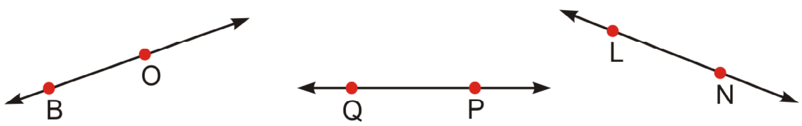


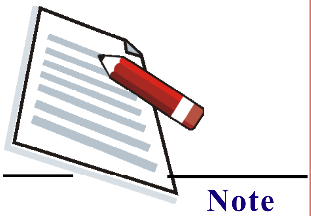
Figure 10.12

5. How many lines can be drawn on a plane?

10.2 Line Segment and Ray

10.2.1 Line Segment

Draw a line 'ℓ' and mark 3 points on it A, B & C. The length between A & B is a part of line ℓ, so it is called a line segment. Similarly AC & BC are also parts of line 'ℓ'. Hence these are also line segments



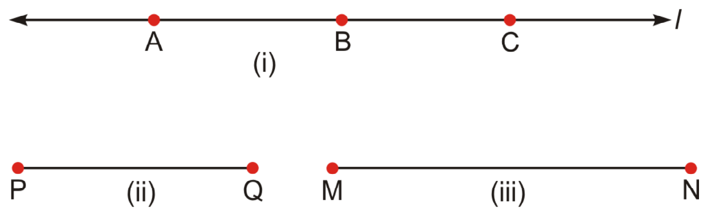


Figure 10.13

5. In fig 10.13 (iii) PQ is a line segment, whose end points are P & Q In figure 10.13 (iii) MN is a line segment with end points M & N. NO arrows are marked on line segment, as this is limited & a part only.

A part of a line, with two end points is called line segment

Remember

The edges of a box, floor and table are also limited and so there are line segments. Suppose P & Q are the locations of two houses. The distance between P & Q is the length of line segment PQ, which can be mesured with a scale a line segment has two end points and the minimum distance is the length of the segment and this length is fixed.

The comparission of two line segments is done with the help of a scale or divider.

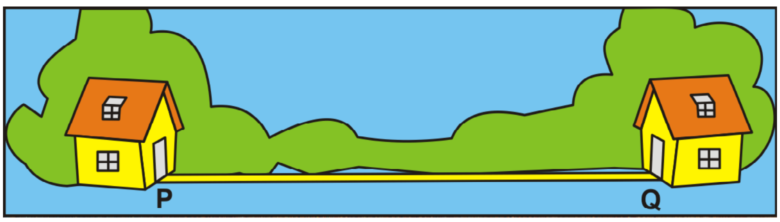


Figure 10.14

line segments AB & CD are equal because their lengths are equal ∴ AB = CD.

The length of line segment $\overline{PQ}$ is less than $\overline{RS}$

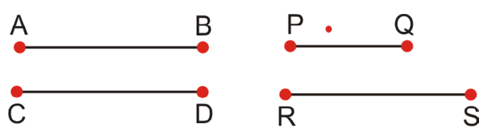


Figure 10.15

10.2.2 Ray

We are aware about the sum rays & rays from a torch. All these start from a source point.

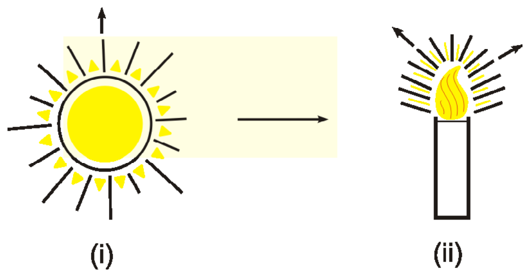


Figure 10.16

Let us mark a point 'O' on a paper, start from O and draw a line in the some direction. This is

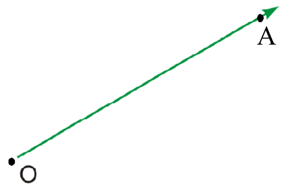


Figure 10.17

also a part of line, in which the starting point is there but there is no end point. This is called a Ray, to name this ray we mark another point 'A' on it and so $\overrightarrow{OA}$ is a ray.

There is a starting point on a ray and it goes upto infinity in the some direction

In the figure 10.18

1. There is only a starting point on a ray and the length is infinite Fig.10.18.
2. There are two end points on a line segment and it's length is finite fig 10.18 (ii).
3. There is no end point on line and it's length is inifinite Fig. 10.18 (iii).

To represent a ray we use two points one is the initial point and other on it on the increasing side.

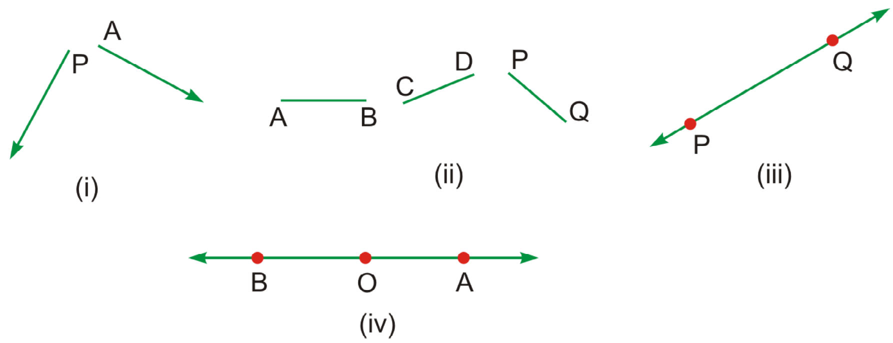


Figure 10.18

In fig 10.18 (iv) there are two rays $\overline{OA}$ and $\overline{OB}$

Intext Questions 10.2

1. Fill in the blanks
   (a) The length of a line segment is __________ .
   (b) There are __________ end points of a line segment.
   (c) There is one __________ point on a ray.
   (d) The length of a ray is __________ .
   (e) To represent a ray we mark an arrow only in __________ direction
2. Look at figure 10.19 and tell the names
   (a) Any two line segments
   (b) Two opposite rays

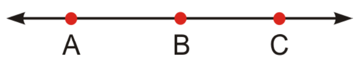


Figure 10.19

3. Mark a point P on a paper and starting from this
   (a) Draw two line segments on the same line their name be PQ & PR
   (b) Draw two opposite rays, name these as $\overrightarrow{PL} \times \overrightarrow{PM}$
4. Write the names of all line segments in the following

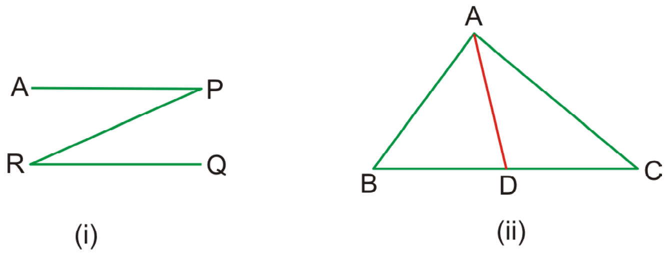


Figure 10.20

5. Write the names of opposite pairs of rays in figure 10.21

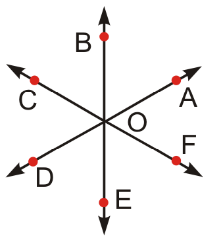


Figure 10.21

10.3 A line passing through two points and two lines in a plane

Mark a point 'A' on a paper. How many lines can you draw passing through this point? You can draw as many line as you want (Infinite number of lines can be

drawn) Take another point B away from A. Through this point also you can draw infinite number of lines.

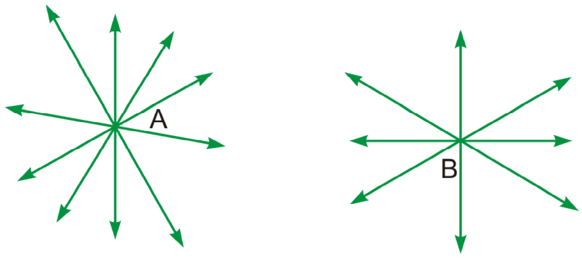


Figure 10.22

Hence we can say that

Through a given point on a plane you can draw infinite number of lines.
In the fig. 10.23, out of the line passing through A and B how many lines are passing through both the points.

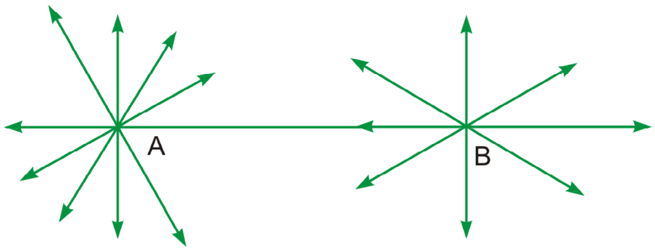


Figure 10.23

You can see that only one line AB or BA is passing through both points
In the figure given below 10.24, through C & D and L & M only one line can be drawn.

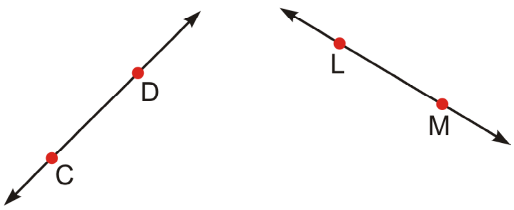


Figure 10.24

One and only one line can be drawn through two given points on a plane

Just think over, if there are three points then how many lines can be drawn

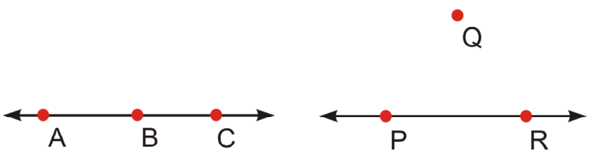


Figure 10.25

We see above that through A, B, & C only one line is drawn but there is no line passing through P, Q & R. Hence it is not always possible to drawn a line through three given points.

Now you see the following pair of lines.

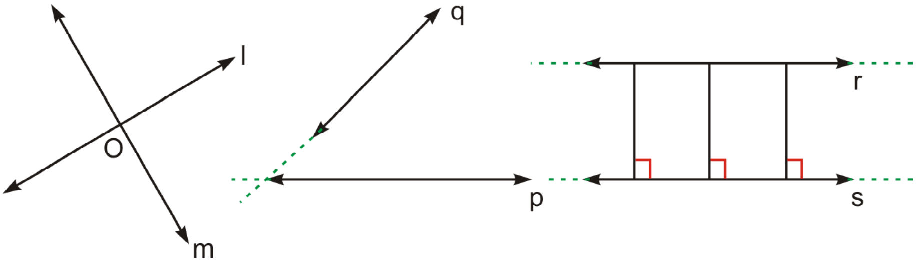


Figure 10.26

There is one common point O on lines ℓ & m. In other word one can say that these are intersecting lines which meet each other at point 0

lines p & a do not appear to be intersecting but other lines are infinite in length hence on extension they intersect. Hence, these are also interesecting lines. Lines r and s do not have any common point on extension both sides. They do not intersect, as the distance between them is constant everywhere. These are called parallel lines.

A train runs on two rails, this is an example of parallel lines because they never meet each other.

Figure 10.27

In our houses, on windons, doors etc. there are many designs where we can see such lines, which never meet or the distance between is same every where the opposite edges of scale, black board and the table are also parallel.

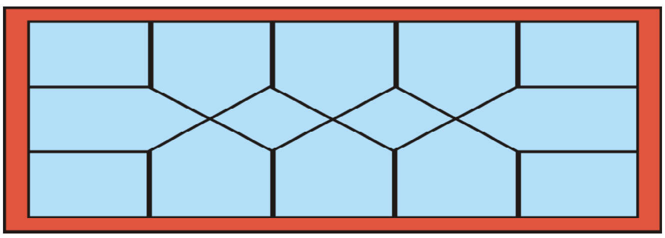


Figure 10.28

Note

Two lines drawn on a plane either intersect or are parallel

Look at the figure 10, 29 lines ' ℓ ' & 'm' are intersecting at point 0.

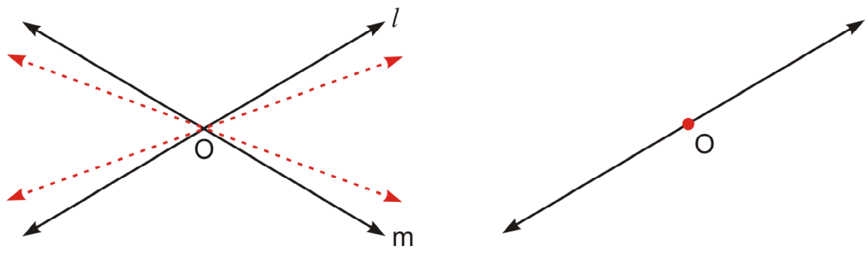


Figure 10.29

Now we move line 'm', fixing 0 point and slowly this line will cover line ' ℓ ' and both lines will look like and line. In such situation we call then concident line. In other words, when two lines are coincident their all points are common.

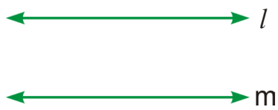


Figure 10.30

Now take another example 'l' and 'm' two parallel lines. Now we move line 'm' towards line ' ℓ ' in such a way that they remain parallel. After some time line 'm' may coincide with line ' ℓ '. These are also called coincident lines.

In two conincident lines all the points of one line are common to the other line

In this way we can say that

1. There is no common point in two parallel lines
2. There is only one common point in two intersecting lines
3. There are all the points common in two conincident lines

Let us see how much you have learnt 10.3

1. Fill in the blanks to mark the statement true:
   (a) Two parallel lines ___________ any point.
   (b) Two intersecting lines have __________ .
   (c) When there are two or more than two points common these are called___________ .
   (d) On a plane two different lines either parallel or ___________ lines.
2. Observe figure 10.31 and answer the following
   (a) Two pairs of parallel lines.
   (b) All the pairs of intersecting lines.

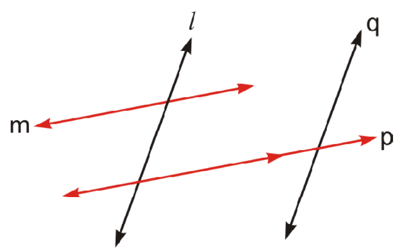


Figure 10.31

Note

3. Take a scale and draw two lines along it's two long edges. What type of lines are these ?

4. Observe fig 10.32 and answer the following:

   (a) One pair of parallel lines

   (b) Two pairs of intersecting lines

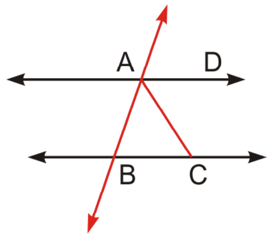


Figure 10.32

5. (a) How many lines can be drawn through a given point on a plane? Can you draw all there lines?

   (b) How many lines can be drawn from two different points on a plane?

   (c) How many lines can be drawn through the points?

10.4 Collinear points and concurrent lines

In fig. 10.33 look at the points A, B, & C can we draw a line which can pass through all of them? If we draw a line through 'A' & 'B', it does not pass through 'C' no line will pass all three points because these three points are not on a line.

Figure 10.33

Now look at the following three points P, Q, & R in fig. 10.34

Figure 10.34

We can draw a line through these point as shown in fig.10.35

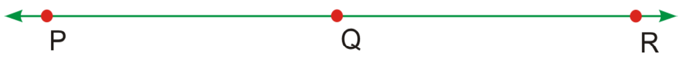


Figure 10.35

In other words we can say all these points are on a line or these are called collinear points

Similerly the below four points by L, M, N, & O are on a line, these are collinear points

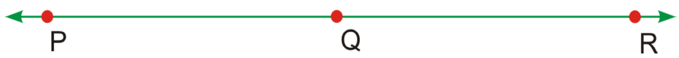


Figure 10.36

If a line is passing through three or more points, then they are collinear points.

If we cannot draw a line through all the points then they are non-collinear points

10.4.2 Marking Collinear points

Collinear points can be marked with the help of a scale. Take any two points on a paper make any two point A & B, draw a line through A & B and now

Figure 10.37

mark on this line other two points C & D. All these are collinear points

10.4.3 Concurrent lines

We have learnt that two lines on a plane are either parallel or they intersect. Fig 10.38

Figure 10.38

If there are three lines on the plane then these line will intersect at three or two point or at one point or will be parallel.

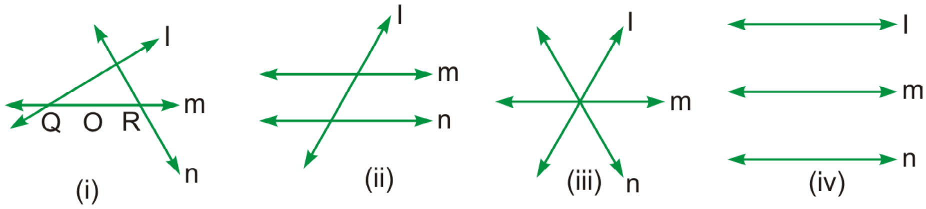


Fig. : 10.39

If three or more line intersect at one point or pass through one point, they are called cocurrent lines. This point is called the cocurrent point of these line

In the above figure 10.39 (iii) are concurrent lines. Below are shown some concurrent lines in fig. 10.40.

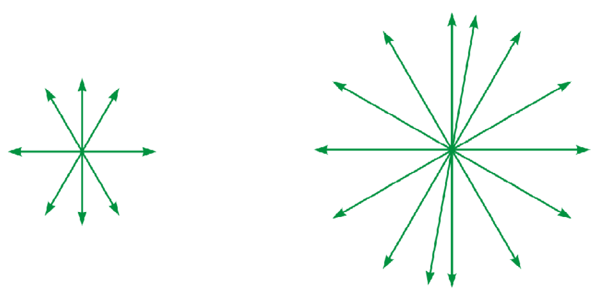
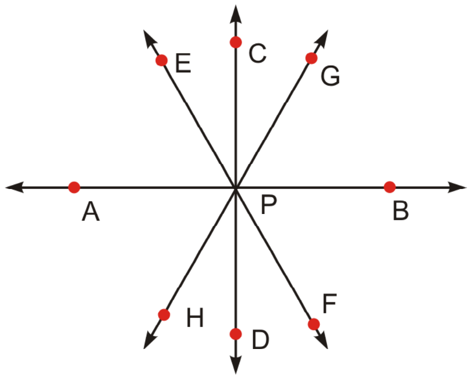


Figure 10.40

10.4.4 Drawing Concurrent lines



First we mark a point on the paper. Then with the help of a scale we can drawn many lines through the same point as shown in fig 10.41

In fig. 10.41, we marked a point 'P' then we draw lines through this point as $\overline{AB}, \overline{CD}$. These are all cocurrent lines and 'P' is the point of concurrence.

Intext Questions 10.4

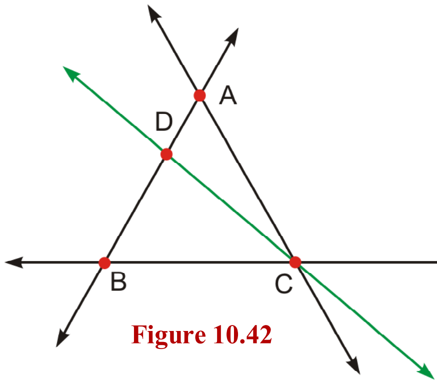


Figure 10.42

1. In the given figure 10.42
   (a) Name the collinear points
   (b) Name three concurrent lines

2. In the given fiugre 10.43
   (a) Write the names of three collinear points
   (b) Write the names of four collinear points

3. In figure 10.43, write the names of concurrent lines

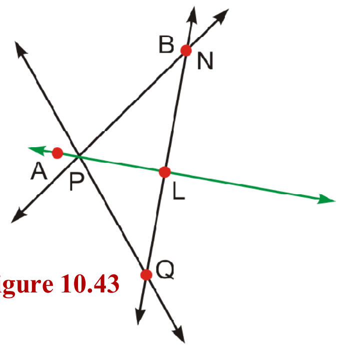
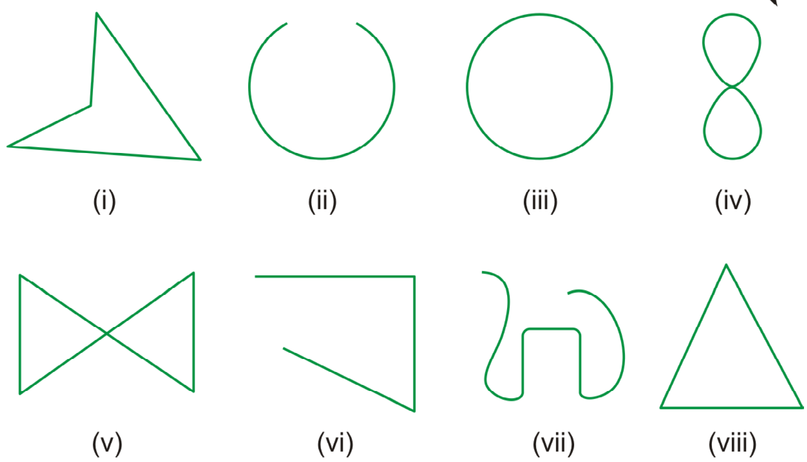


Figure 10.43

10.5Open and Closed Shapes

Observe the following figure



Place the tip of your pencil at any point in above figure (i) now move your pencil on the shape in any direction. Now observe that without changing the direction, can you reach the starting point? You will reach as the starting point, hence this is a closed curve (shape). Figure (ii) is not a closed curve, This is an open curve. Fig iii, iv, v, viii are also closed figure (curves), figure vi & vii are open figure/ curves.

10.5.1 Simple shapes/figures

Figures (i) (ii) (iii) (vi) (vii) and (viii) on the previous P & C never cross it's part any where, these are simple figures.

Figures IV & V are not simple figures.

Fig. (i), (iii) & (viii) are simple closed figures.

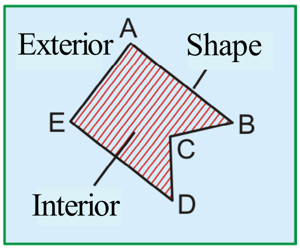
10.5.2 Inner and outer parts of simple closed figures

Each simple closed figure divides the plane into three parts

(i) Inner part of figure

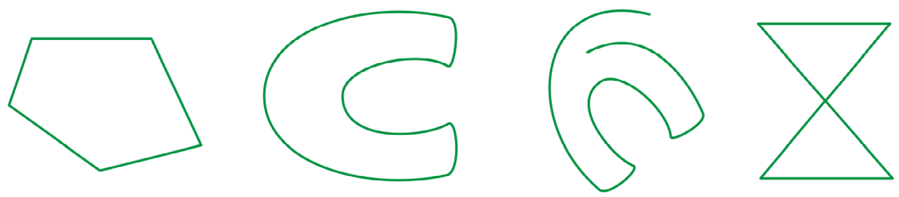
(ii) Outer part of figure

(iii) The figure self

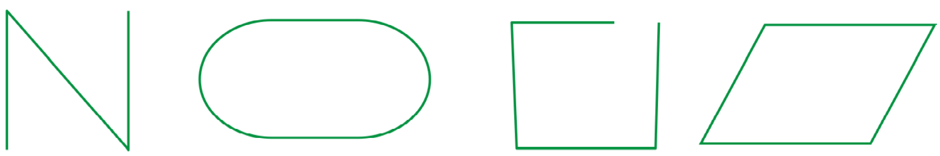


Intext Questions 10.5

1. Identify from the following open and closed figures



(i) (ii) (iii) (iv)



(v) (vi) (vii) (iv)

Also tell which are simple closed figures

10.6 Geometrical Instruments and their use

You might have seen a Geometrical Box. There are many things – like pencil, sharpener eraser, in addition there are following shapes to be used for drawing

geometrical shapes

(i) Scale (ii) Divider (iii) Compass

(iv) Setsquare (v) Protractor

Now we will study how to use these in drawing geometrical shapes.

10.6.1 Scale/ruler

There is one scale in the Geometry Box. Could be metal, wood or plastic. Its edges are parallel to each other. Normally this scale is of 6 inches (15.25 cm approx) or 121 inches (30.5cm approx) It's one edge shows inches the other, cm. Each inche compart is further divided in 10 smaller parts.

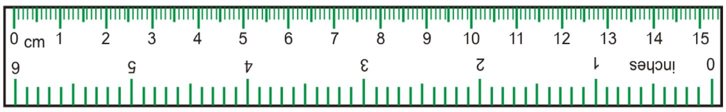


Figure. : 10.44

This is used in the following ways

(i) To draw a line segment between two given points

(ii) Drawing a line segment equal to the length of a given segment

(iii) To measure the line segment

(iv) To see if the line is straight or not

10.6.2 Dividers

A divider is used to measure the length of a given line segment. In the fig 10.45, to measure the length of segment AB, place the divider at point and open it till it reaches point B. Now measure thin length with the help of a scale as shown in fig.10.45 & 10.46.

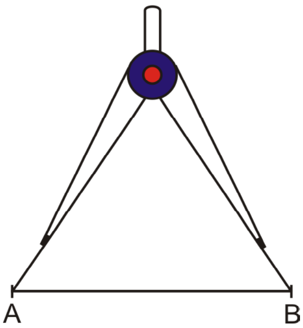


Figure 10.45

This distance will be length of line segment AB given in figure 10.45 and the length is shown in figure 10.46 i.e 3.6cm

$\therefore$ length of $\overline{AB} = 3.6$cm

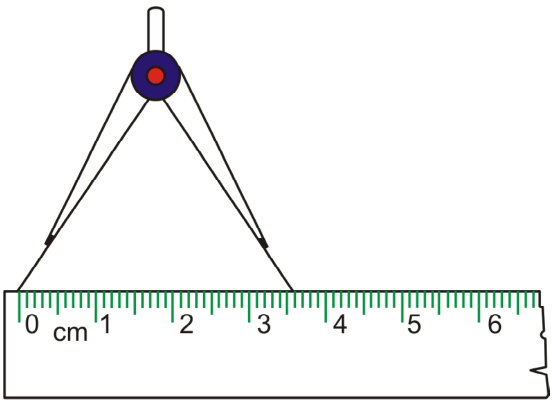


Figure 10.46

10.6.3 Compass

This is also like divider, one part is the some as that of divider and on the other side there is a space (round) to fix the pencil with a screw. To measure the length of any line segment with the help of compass, same procedure to be followed as for the divider, except at one end the pencil tip is placed. The use of compass is done in the following:

(i) To measure the length of a line segment

(ii) To construct a cirlce with a given radius

(iii) To construct different measures of angles also using a scale

10.6.4 Set-Square

In the geometry box, there are two triangular instruments (Fig 10.47) normally, these are also made of steel, wood or plastic. In fig 10.47 (a) the angles of the set square are 90°, 60°, & 30° in figure 10.47 (b), the angles are 90°, 45°, 45°. The perpendicular side of both are marked as in the case of scale. set square is also used for different purposes.

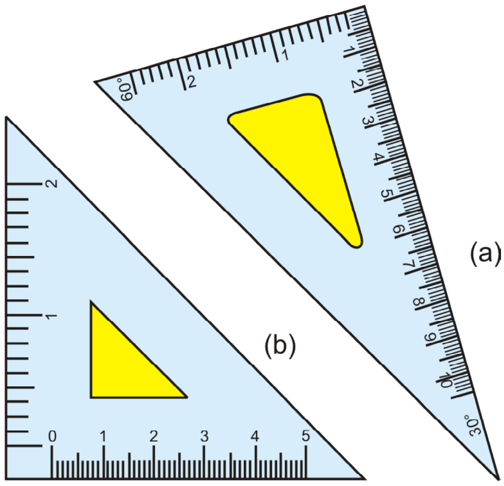


Figure 10.47

(i) To construct angles 30°, 45°, 60°, 75°, 90°, 105° etc.

(ii) To construct parallel and perpendicular lines

10.6.5 Protractor

This is used to make angles and to measure the angles it's shape is like a semicircle. It is made of plastic, steel or wood. There are marking on it's semicircular shape as shown in fig 10.48. The semicircular part is divided into 180 equal parts and marks are from 0° to 180°. The markings are from both sides (0° to 180°) (180° to 0°). Normally there are 10° apart markings. The center point of the straight edge is called the centre of protractor. The line passing through centre is called base line.

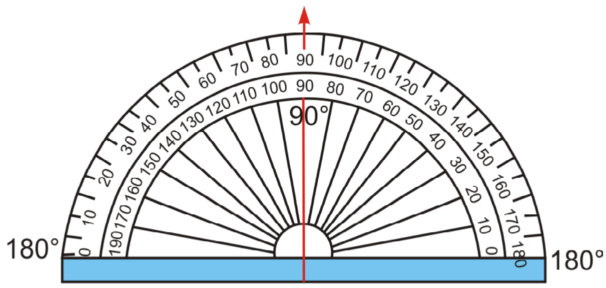


Figure 10.48

Note

To measure any angle, we place the centre of protractor at one point so that 0° mark is on the base line. Read the number on the semi circle, other side of angle is touching or appears to be touching. This number will give us the measure of angle. In the below given figure (10.49) the measure of angles is 45°.

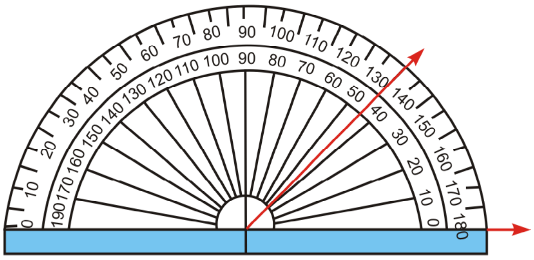


Figure 10.49

10.7 Measure of Line Segment

For moving from one place to another, we like to tread least; we want to take the shortest route.

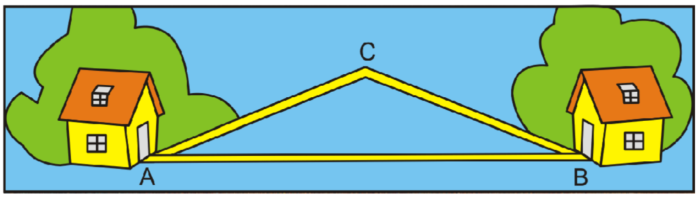


Figure 10.50

In the above figure, there are two routes/ways to more from A to B. One way is straight from A to B, other is from A to C then C to B. From A to B is the short route. The shortest distance between the two points is called the length of the line segment. This can be measured using an appropriate measuring scale.

For measuring a small length, we use the scale available in the Geometry box.

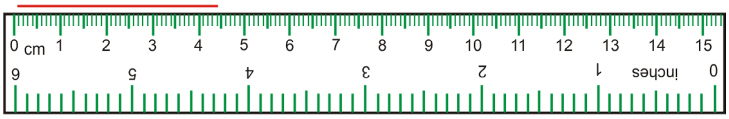


Figure 10.51

For measuring longer distances we use the big measuring tape of cloth, plastic or metal.

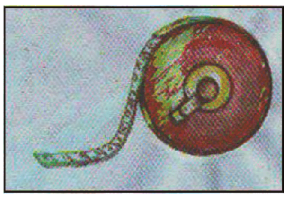


Figure 10.52

Note

We need a specific unit for any measure. To measure smaller lengths/distances we use cm or millimeter scales, where as for measuring long distances we use meter/kilometer.

Following table shows relationship between small & big units

Unit	Symbol	Relation
Kilometer	Km	
Hectometer	Hm	10 hm = 1 km
decameter	dam	10 dam = 1 hm
meter	m	10 m = 1 dem
Decimeter	dm	10 dm = 1 m
Centimeter	cm	10 cm = 1 dm
Milimeter	feeh	10 mm = 1 cm

Can you now tell, how many meters are there in a kilometer?

From the table we can see 1km = 10hm,

= 10×10 dm [1hm = 10dm]

= 100dm

=100×10m

1 km = 1000m

10.8 To measure a line segment

Example 10.1 : The length of a given line segment can be measured in following steps:

Steps 1 :

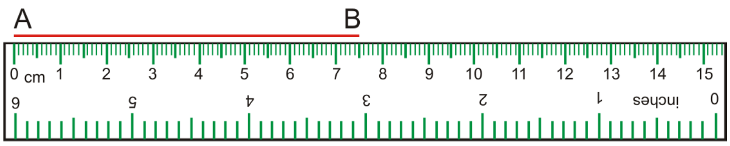


Figure 10.53

Place a scale along the line segment such that the 'o' mark of the scale is at point A of the line segment

Step 2 : Now we read the marking on the scale in cm and mm against the mark touching the other end B of the segment.

In this way in the above figure, the measure of AB is 7cm and 5mm or 7.5cm.

10.9 Using the scale draw a line segment of given measure

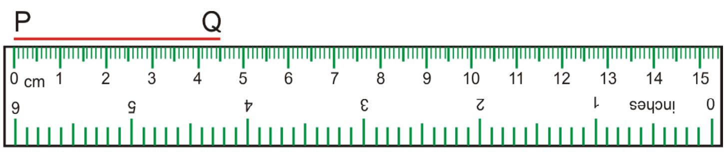


Figure 10.54

Step 1: Mark a point P on the paper, now place the scale in such way that 'O' on the scale should coincide point 'P'

Step 2 : Now read on the scale the required length 4cm 5mm or 4.5cm and mark point 'O' on the paper in front of 4.5cm mark.

Step 3: Now join P & Q with the help of pencil such that the pencil should be touching the scale all around.

Now $\overrightarrow{PQ}$ is the required line segment of length 4.5cm

Intext Questions 10.6

1. With the help of the scale, measure the following segements

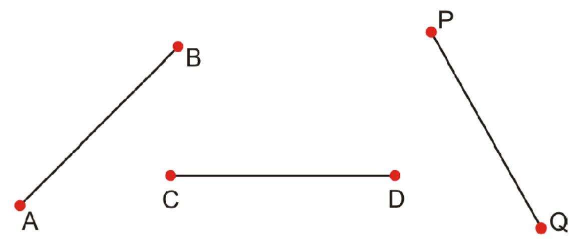


Figure 10.55

2. Draw on a paper two line segments PQ & RS. Find their lengths.
3. Measure the length & breadth of the top of your Maths book and write it using appropriate units
4. Construct the line segments of the following length

   (i) AB = 7.2cm (ii) PQ = 6cm (iii) 6cm 5mm
5. Using a measuring tape, measure the length and breadth of your classroom

Let us Revise

- Point, line and plane are the concepts, which have no definition. We can understand them with examples and their relationship.
- A line segment is a part of the line with two end points and has a fixed length.
- A ray is also a part of the line but it has one end point and is infinite on the side.
- A line is going to infinity on both sides and it's length is infinite.
- One and only one line can be drawn through two given points.
- Plane is an even surface, which extends in all directions.
- Two lines on the some plane

  (i) Intersecting or (ii) Parallel or (iii) Concident

Note

- If a line is drawn through 3 or more points then these are collinear points.
- In a plane if 3 or more than 3 lines pass through the same point these are concurrent lines.
- A simple closed figure divides the plane into in 3 parts, outer, inner and the figure it self.

Exercise

1. In the figure 10.56

   (i) Taking different pairs of points, how many line segments can be drawn? Write their names and write the cocurrent line.

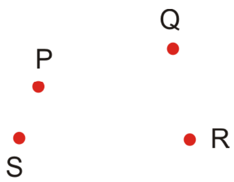


Figure 10.56

2. How many lines can be drawn through three points

   (i) When these are collinear

   (ii) When these are not collinear

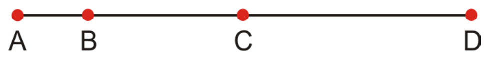


Figure 10.57

3. In figure 10.57 there are four points on the line A, B, C & D. Write the names of all the line segments through these points using in pairs

4. From the figure 10.58, write the names of the following

   (a) A pair of parallel lines

   (b) A pair of intersecting lines

   (c) Three concurrent lines

   (d) Three collinear points

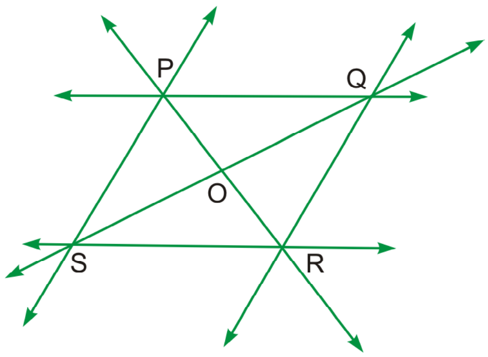


Figure 10.58

5. Mark a point 'L' on the paper and then

   (a) Take three more points M, N & O so that all these are collinear points

   (b) Draw four lines so that all are concurrent lines

6. Mark a point E on the paper and then

   (a) Draw three rays starting from E point

   (b) Through this point how many rays can be drawn?

7. In how many parts does a simple closed shape divide a plane?

Intext Questions 10.1

1. (a) Dustparticles and the corner of a book
   (b) The edge of the book and a streched wire
   (c) Floor of a room or top of table
2. (a) Point (b) Plane (c) Line
   (d) In finite (e) Point (f) Two
   (g) Infinite (h) that plane
3. AB, BC, AC, BA, CB and CA (Any Three)
4. BO or OB ; QP ; PQ or LN ; NL ; yes
5. Infinite

Intext Questions 10.2

1. (a) Fixed (b) Two (c) Initial
   (d) Infinite (e) One
2. (a) AB, BC or AC (b) BA and BC
3. (a)

Figure 10.59

(b)

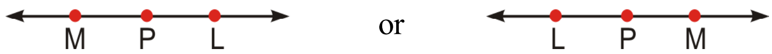


Figure 10.60

4. (a) AP, RP, RQ (b) AB, AD, AC, BC, BD and DC
5. OB and OE ; OC and ; OD and OA

Intext Questions 10.3

1. (a) No (b) One (c) Coincident (d) Intersecting
2. (a) (i) m and P (ii) ℓ and q
   (b) (i) m and ℓ (ii) p and q (iii) ℓ and p (iv) m and v
3. Parallel line
4. (a) AD and BC
   (b) AB and BC ; BC and AC, AD and AB, AC and AD (Any two)
5. (a) Infinite No (b) Only One (c) One or not any

Intext Questions 10.4

1. (a) A, D and B (b) AC, DC and BC
2. (a) A, P and B (b) B, N, L and Q
3. AL, NP and PQ

Intext Questions 10.5

1. Closed : (i), (ii), (iv), (vi) and (viii)

   Open : (iii), (v), and (vii)

   Simple closed : (i), (ii), (vi) and (viii)

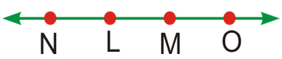
Exercise

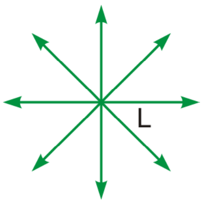
1. (a) Six PQ, QR, RS, SP, PR and SQ

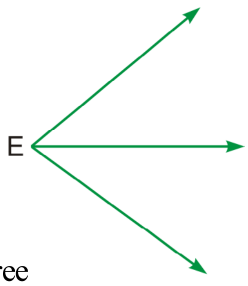
   (b) QP, QS and QR, SP, SQ and SR, PS, PQ and PR and RS, RQ and RP
2. (a) Only One (b) Not any
3. AB, AC, AD, BC, BD and CD
4. (a) PQ & SR ; SP & RQ

   (b) PQ & PS ; QP & QR ; QR & SQ etc.

   (c) PQ, PR & PS ; QP, QR & QS ; SP, SQ & SR ; RP, RQ & RS

   (d) S, O & Q ; P, O & R
5. (a) 

   (b) 

6. (a) 

   (a) Infinite
7. Three

Note

11

ANGLE AND PARALLEL LINES

Have you seen a clock which helps us to know the time throughout the day? How we mark hours in a clock?

In the last lesson, you learnt about geometrical shapes like Point, Surface and Lines etc. In this lesson, you will learn about Angles.

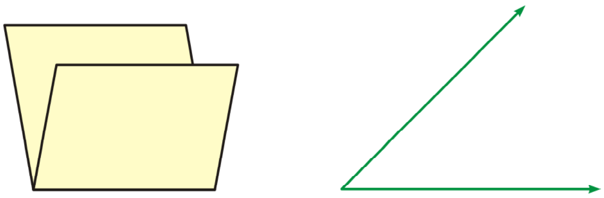


Figure 11.1 (i)

When two surfaces or lines intersect each other, figures thus formed are known as Angle. We have several types of angles in our surroundings. In order to draw different figures we need to learn about Angles.

Look at the opposite edges of a book, a notebook or a table. You will observe that for each one of these, perpendicular distance between the opposite edges remains the same.

Now place a ruler (or a scale) on a sheet of paper and draw lines on both sides of the long edges.

Figure 11.1 (ii)

You will observe that perpendicular distance between these lines is always same. Such lines are called Parallel Lines.

Straight railway tracks also are an example of **Parallel Lines**.

From this lesson, you will learn

- What is an angle?
- How to write and name an angle?
- About different types of angles
- Relation between different types of angles
- How to measure angles?
- Properties of parallel lines
- Drawing a line parallel or perpendicular to a given line

11.1 Rotation

Before understanding about angle, we need to learn about Rotation. You might have seen movement of hands of a clock, wheel of a pottery maker and a giant wheel in a village fare. All these are examples of Rotation.

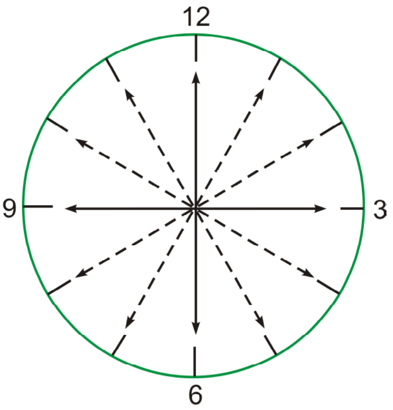


Figure 11.2

You concentrate on moving minute hand of a clock (see figure 11.2). With rotation pointed side of a minute hand reach at 3 from 12, then at 6, then at 9 and at 12 again in the end. We say that it has taken a complete revolution. It completed half revolution when it reached 6 and before that a quarter revolution when it reached 3.

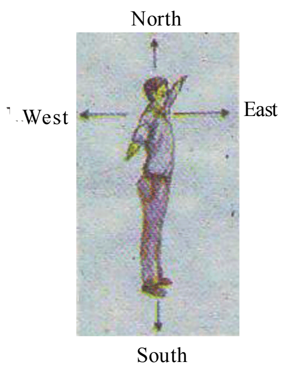


Figure 11.3

For another example (figure 11.3) assume that you are standing on ground for morning exercise facing towards east. If you take one fourth turn towards your right hand then you will be facing to which direction? Towards South, is it correct or not? How many more one fourth turns do you need to take so that you again face towards East? If you take half turn then you will be facing west. After this if you take a full turn you will not be facing towards east.

But concept of complete rotation, half rotation and one-fourth rotation is not sufficient to explain all types of rotation. For example, in figure 11.4 using minute hand we wish to show time between 4:00 o'clock and 4:25 o'clock. We will not be able to show this using the above mentioned rotation.

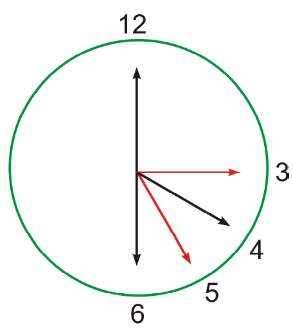


Figure 11.4

We need to learn some unit of rotations other than a complete rotation. But before this we need to understand angle as measure of a rotation.

11.2 Angle

Look at figure 11.5. There are two rays OA and OB which have the same initial point. You can have it by rotating ray OA around O, till it coincides with OB or by rotating ray OB around O and reaching OA. Figure shown in figure 11.5 is known as an Angle. We write it as $\angle AOB$ or $\angle BOA$ (sign $\angle$ is used to write angle AOB or angle BOA).

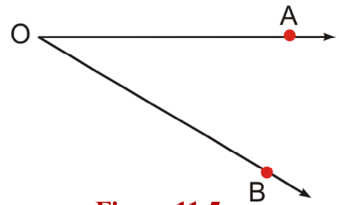


Figure 11.5

We use the symbol '$\angle$' to write an angle. For example we write angle $\angle AOB$ as in figure 11.5 there is a clockwise rotation and in figure 11.6 there is an anticlockwise rotation. In this way we can consider an angle as rotation.

If we take points C and D on the rays forming $\angle AOB$ (figure 11.6) then we can name this angle as $\angle COD$ also.

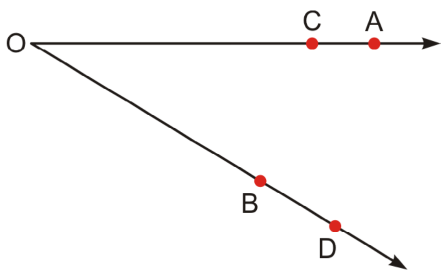


Figure 11.6

Therefore $\angle BOA$, $\angle DOC$, $\angle DOA$ and $\angle COB$ are names of the same angle. The same angle can be named as $\angle BOA$, $\angle DOC$, $\angle DOA$ and $\angle BOC$ also. Initial point 'O' of both the rays is called the vertex of the angle and rays OA and OB are called its arms.

To draw an angle we are to draw two rays which have the common initial point. For example, two rays OP and OQ have been drawn with a common initial point O (figure 11.7).

From this we got $\angle POQ$ or $\angle QOP$.

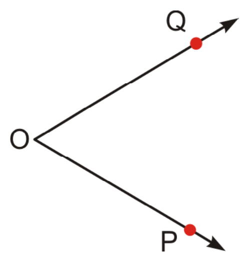


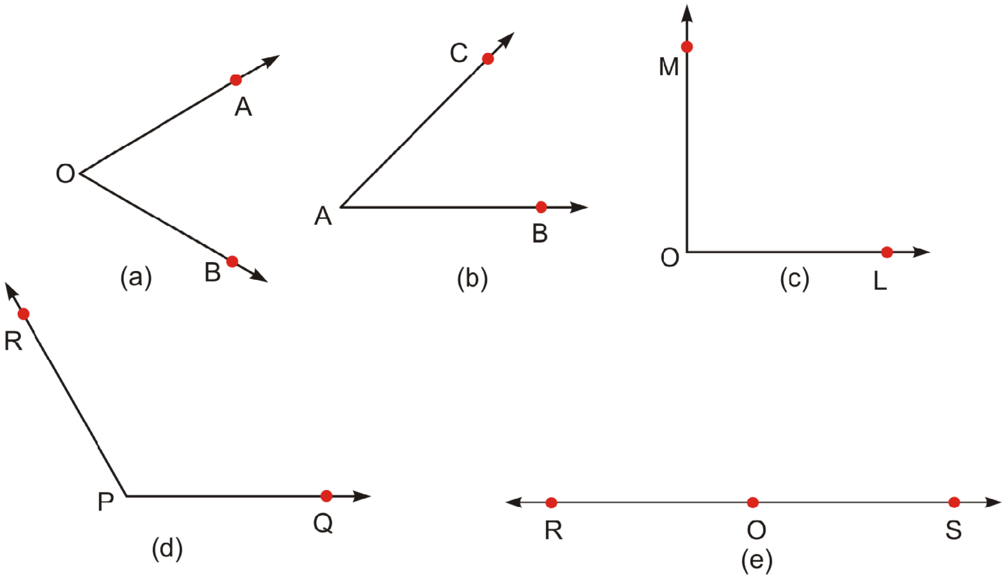
Figure 11.7

Intext Questions 11.1

1. Fill in the blanks in order to make the following statements true.
   a) Rotation from South to North is ....
   b) Rotation from North to South is ....
   c) Rotation from West to South is ....
   d) Rotation of minutes hand of a clock from 4 to 10 is ....
   e) Rotation of hour hand of a clock from 2:00 pm to 4:00 pm is ....

Note

2. Write the names of angles given in the following figures.



3. Write all the possible names for the following angle.

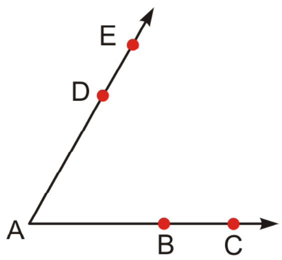


Figure 11.9

4. Write the vertex and names of the arms of $\angle POQ$, $\angle PQR$, $\angle LMN$ and $\angle RST$.

5. Draw angles with the vertex and arms given below.

	Vertex	**Arms (rays)**
a)	P	PO and PQ
b)	M	MA and MB
c)	O	OX and OY

6. Draw angles to show

a) Half rotation

b) One-fourth rotation

c) Less than one-fourth rotation

d) More than one-fourth rotation

Write their names also.

11.3 Measure of Angles

Recall that in introduction you learnt about half rotation and one-fourth rotation. You also noted that we need to take some other unit to measure an angle. This, we shall learn it in this section.

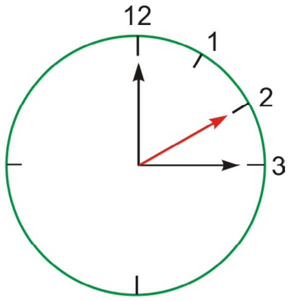


Figure 11.10

Let us observe the angle between arms of a clock at 2:00 pm. We cannot measure this angle in terms of one-fourth rotation; it will be angle between arms of a clock at 3:00 pm. We divide angle with one-fourth rotation in 90 equal parts and measure one part as a unit. We call it a Degree. Therefore one-fourth represents 90°. We read it as 90 degrees and write '°' (symbol of degree) above 90. Now can you guess the angle between the arms of a clock (in degrees) at 2:00 pm? Is it not 60°? Similarly this angle will be 30° at 1:00 pm.

Now you can easily observe that angle between the arms of a clock (in degrees) at 2 o'clock will be 150°. At 4 o'clock it will be 120° and at 6 o'clock it will be 180°. So degree measure of half rotation is 180°.

In your Geometry box, there is semi-circular disc for measuring angle, which is known as Protector. Half-circle is divided in 180 equal parts. On the protector these parts are marked from 0 to 180 on both sides-clockwise as well as anticlockwise (see figure 11.11). On protector every mark denotes 1°. In the drawing of protector thick line has been drawn which is called Base line and its mid-point O is called the centre.

For measuring an angle we place XOat the vertex of the angle and base line on a ray of the angle. Number on the protector to which the other ray of the angle points out will be the degree measure of the angle. Therefore in figure 11.11 measure of $\angle AOB$ is 75°.

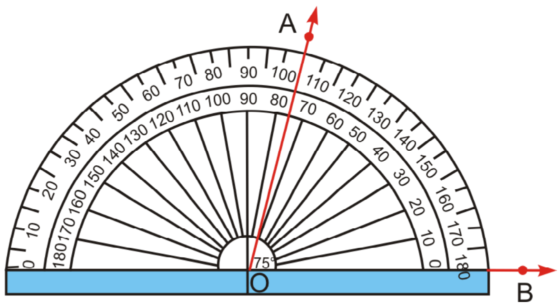


Figure 11.11

Similarly, if we want to draw an angle of 130° then we draw a ray OA. We place a protector in a way that Point X lie on Point O and base line coincide with OA. We join point O near the 130° mark on the protector with B. Now measure of $\angle AOB$ is130°.

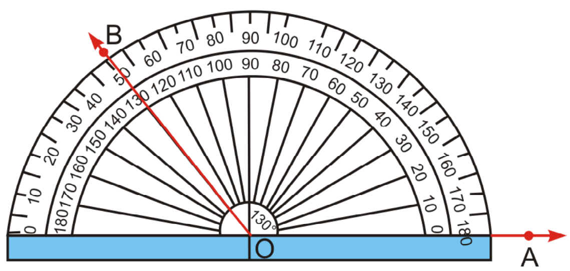


Figure 11.12

11.4 Drawing Angles

In the following section we will learn to draw angles of some specific measures-60°, 120°, 30° and 90°

11.4.1 Drawing Angle of 60°

Step 1: Using ruler draw a line AB (fig 11.13(i)).

Step 2: Mark a point O on it (Fig 11.13(ii)).

Step 3: Taking a convenient radius and centre O draw a semicircle which intersect line AB at P and Q (Fig 11.13(iii)).

Step 4: Treating Q as centre and taking the same radius draw an arc, which intersect the semicircle at R. (Fig 11.13(iv)).

Step 5: Join O and R and extend. $\angle QOR$ is the required angle of measure 60° (fig 11.13(v)).

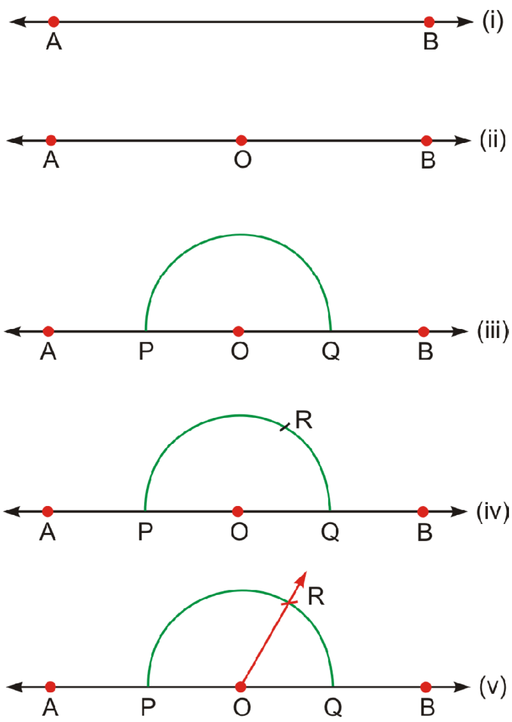


Figure 11.13

11.4.2 Drawing Angle of 120°

Step 1-4: Repeat steps 1-4 mentioned above.

Step 5: Treating R as centre and taking the same radius draw an arc, which intersect the semicircle at S.

(fig 11.14 (i)).

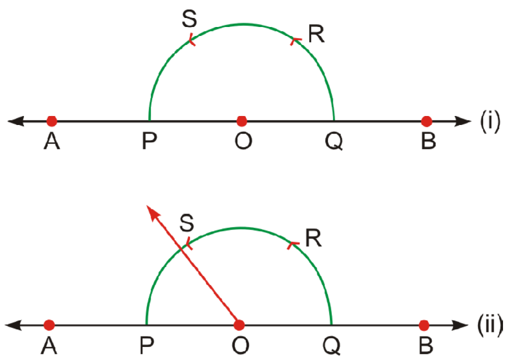


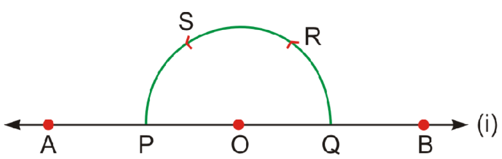
Figure 11.14

Step 6: Join O and S and extend.

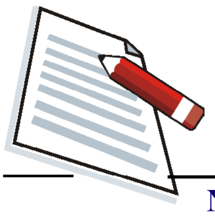
$\angle QOS$ is the required angle of measure 120° (fig 11.14 (ii)).

11.4.3 Drawing Angle of 90°

Step 1-5: Repeat steps 1-5 mentioned above for drawing angle of 120°.



Step 6: Treating R as centre and taking some convenient radius draw an arc.


Note

(fig 11.15(i)).

Step 7: Treating S as centre and taking the same radius draw another arc which intersect the earlier arc at T.

Step 8: Join O and T and extend.

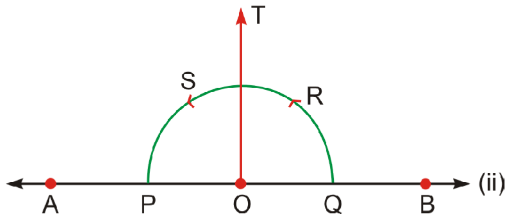


Figure 11.15

∠BOT is the required angle of measure 90° (fig 11.15(ii)).

11.4.4 Drawing Angle of 30°

Step 1-4: Repeat steps 1-4 mentioned above for drawing angle of 60°.

Step 5: Treating Q as centre and taking some convenient radius draw an arc.

Step 6: Treating R as centre and taking the same radius draw another arc which intersect the earlier arc at S.

Step 7: Join O and S and extend. ∠BOS is the required angle of measure 30°(fig 11.16).

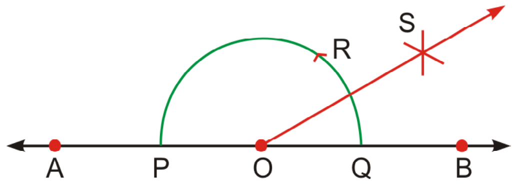


Figure 11.16

Intext Questions 11.2

1. Using protector draw angles representing the following:

   a) one-fourth rotation

   b) half rotation

2. Using protector draw the following angles:

   (a) 40° (b) 70° (c) 90°

   (d) 110° (e) 150° (f) 180°

3. Using protector measure the angles BAC, ABC and BCA given in figure 11.17

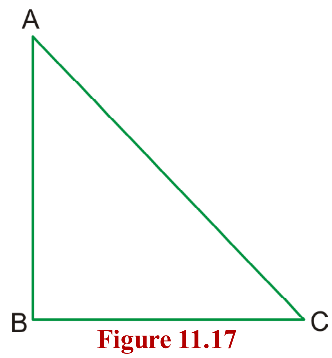


Figure 11.17

4. Using protector measure ∠DAB and ∠DCB given in figure 11.18. Also measure ∠ ABC and ∠ DCA.

5. Using ruler and compass draw angle of 150°.

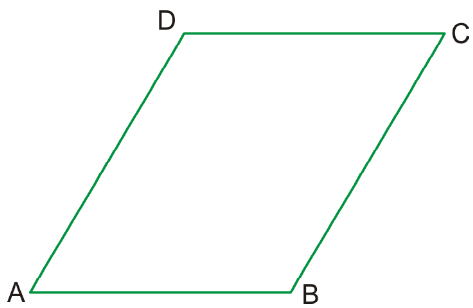


Figure 11.18

Note

11.4 Types of Angles

Right Angle: Recall that in a clock at 3 o'clock hour hand is at 3 and the minute hand is at 12. Minute hand stands straight over the hour hand. Similarly every wall of your room stands straight over the floor. Standing straight over the floor you make an angle of 90°. All these are examples of 90°. Such angle is called a Right angle.

An angle of 90°is called a Right angle.

Acute Angle: Any angle having measure less than 90° and more than 0° is called an Acute angle.

Obtuse Angle: Any angle having measure more than 90° and less than 180° is called an Obtuse angle.

Straight Angle: Any angle having measure equal to 180° is called a Straight angle.

You will observe that a straight angle represents a half rotation. Both the rays of a straight angle lie on a straight line but have opposite directions.

Reflex Angle: Any angle having measure more than 180° and less than 360° is called a Reflex angle.

Complete Angle: Any angle having measure equal to 360° is called a Complete angle.

Zero Angle: Any angle having measure equal to 0° is called a Zero angle. Observe that in case of Zero angle no rotation takes place by ray.

Intext Questions 11.3

1. From the angles given below categorise Right angle, Acute angle, Obtuse angle, and Straight angle:

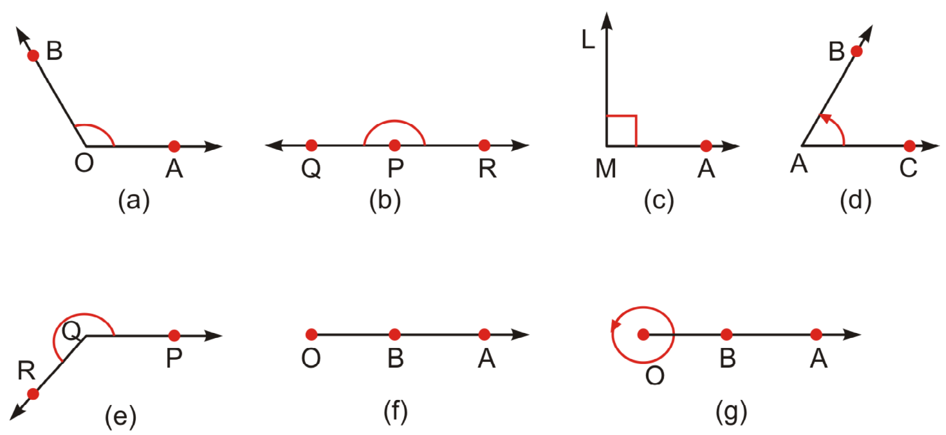


Figure 11.19

2. In figure 11.20 categorise $\angle DAB$, $\angle ABC$, $\angle ADC$, $\angle DCB$, $\angle BAP$ as Acute angle, Right angle, Obtuse angle and Straight angle.

3. Using protector measure the angles given in Question 1 and Question 2.

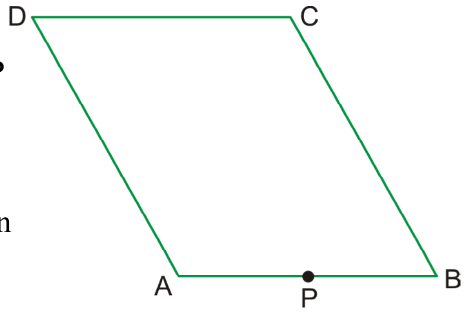


Figure 11.20

11.6 Pairs of Angles

Adjacent Angles: Two angles are known as Adjacent angles if they have a common vertex and one common arm and the other arms are on the opposite sides of the common arm. In figure 11.21 $\angle AOB$ and $\angle BOC$ are Adjacent angles. OB is their common arm and a common vertex is O and arms OA and OC lie on the opposite sides of arm OB. Note that $\angle AOC$ and $\angle AOB$ are not adjacent angles, because even though they have a common arm OA and a common vertex O but the other two arms lie on the same side of the common arm. Similarly $\angle AOC$ and $\angle BOC$ are not adjacent angles. Why?

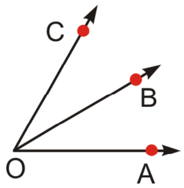


Figure 11.21

Example 11.1: In figure 11.22 given below find out that whether pairs of angles are adjacent or not. Give reasons also.

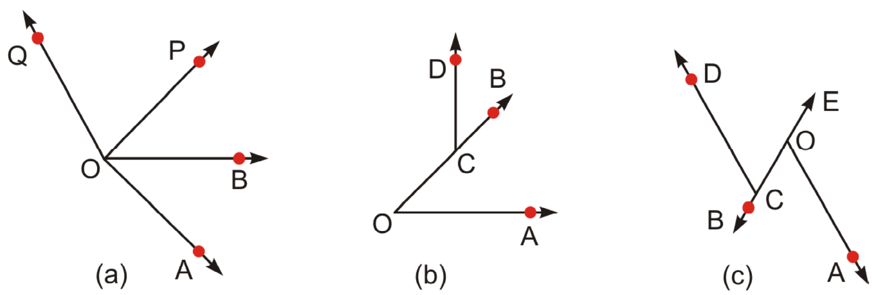


Figure 11.22

(a) $\angle AOB$ and $\angle POQ$; (b) $\angle AOB$ and $\angle BCD$; (c) $\angle BCD$ and $\angle OCD$

Solution:

(a) Both the angles have a common vertex O, but do not have any common arm. So these are not adjacent angles.

(b) Both the angles have one arm on the same line, but their vertices (O and C) are different. So these are not adjacent angles.

(c) Both the angles have a common vertex C and have a common arm CD. Their other two arms lie on different sides of the common arm. So pair of these angles is a pair of adjacent angles.

Example 11.2: An angle AOB is given. Draw one more angle such that the two angles are Adjacent angles.

Solution:

1. Take a point C on the other side of OB than the one in which A is situated.
2. Draw OC and extend it.

   $\angle$AOB and $\angle$BOC are Adjacent angles. Why?

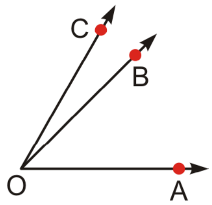


Figure 11.23

Complementary Angles: Two angles are known as Complementary angles if sum of their measures is 90°. For example angles of 30° and 60° are Complementary angles. Two angles may not be adjacent but if they are adjacent and complementary then they make a Right angle.

Look at figure 11.24 (a) and (b)

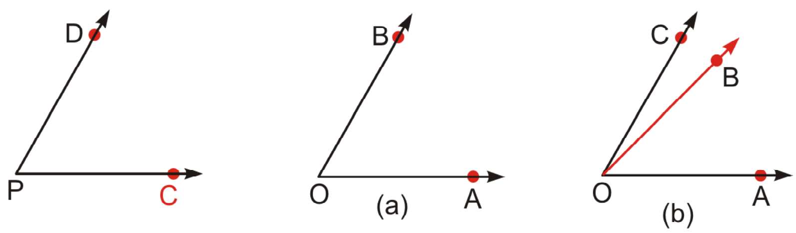


Figure 11.24

1. To draw the complementary angle of a given angle first of all we measure the angle.
2. We subtract its measure from 90°.
3. Using protector we draw an angle of newly found measure.

Example 11.3: Find out if the pairs of angles given below are Complementary.

a) Angles of measures 30° and 60°

b) Angles of measures 37° and 53°

c) Angles of measures 45° and 55°

d) Angles of measures 45° and 45°

Solution: (a), (b) and (d) are complementary angles, but (c) are not complementary angles.

Example 11.4

Solution:

1. Using compasses draw $\angle$AOC of 90°.
2. Draw angle BOC as shown in figure 11.25.

Note

$\angle AOB$ and $\angle BOC$ are complementary angles. These are adjacent angles also.

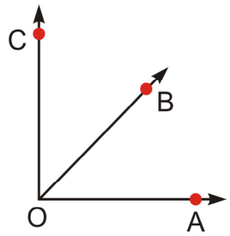


Figure 11.25

Supplementary Angles: Two angles are known as Supplementary angles if sum of their measures is 180°.

For example angles of 60° and 120° are Supplementary angles. Two right angles are also Supplementary angles. You will see that two adjacent supplementary angles form a Straight angle. Such angles are called linear pair.

Example 11.5:

(a) Find the measure of an angle supplementary to angle of 40°.

(b) Find if the pairs of angles given below are supplementary or not?

(i) Angles of 30° and 60°

(ii) Angles of 60° and 120°

(iii) Angles of 70° and 90

(iv) Angles of 80° and 100°

Solution:

(a) Measure of angle supplementary to angle of 40° = (180° – 40°) = 140°.

(b) (i) and (iii) are not supplementary angles, (ii) and (iv) are pairs of supplementary angles.

Example 11.6: (a) For a given angle, draw its supplementary angle.

(b) Draw an angle such that it forms a linear pair with the given angle.

Solution: (a) is the given angle (see figure 11.26 (i))

Steps of construction:

1. Using protector measure $\angle AOB$.
2. Subtract this degree measure from 180° and find out the remaining degree measure.

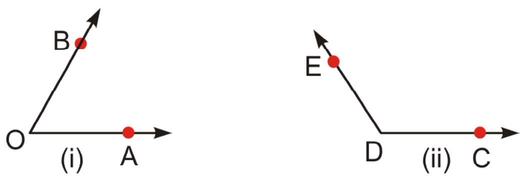


Figure 11.26

3. Using protector draw angle CDE having measure found in step 2. $\angle AOB$ and $\angle CDE$ are supplementary angles (see figure 11.26 (ii)).

(b) ∠AOB is given (see figure 11.27)

Steps of construction:

1. Extend AO upto D.
2. Get ∠BOD.
3. ∠AOB and ∠BOD form a linear pair.

You may note that ∠AOD. Therefore ∠AOB and ∠BOD are adjacent and supplementary angles

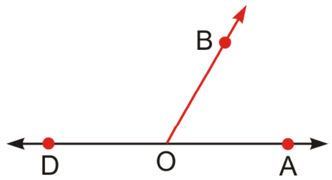


Figure 11.27

Vertically opposite Angles

Two lines AB and CD intersect at O (figure 11.28). Following angles are formed at O.

(i) ∠AOB (ii) ∠COD
(iii) ∠AOD (iv) ∠BOC
(iv) ∠AOC (vi) ∠BOD

First two of these are straight angles. Pairs of angles in (iii) and (iv) are called **Vertically opposite Angles**.

Similarly (v) and (vi) are pairs of vertically opposite angles.

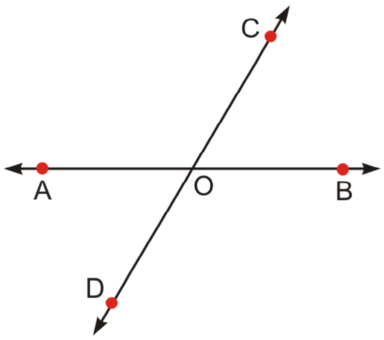


Figure 11.28

Measure ∠AOD and ∠BOC. You will observe that their measures are same. Similarly measures of ∠AOC and ∠BOD will be same.

Vertically opposite angles are always equal.

Example 11.7: Look at the figures given below and identify vertically opposite angles.

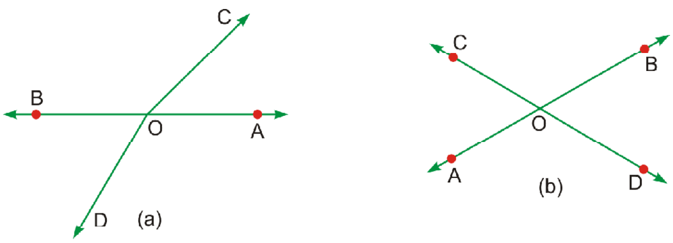
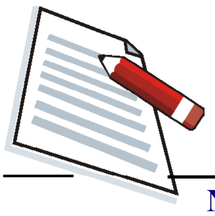


Figure 11.29

Solution: In figure 11.29 (a) ∠AOC and ∠BOD are not vertically opposite angles, because COD is not a straight line.

Similarly, ∠AOD and ∠COB are not vertically opposite angles.



Note

In figure 11.29 (b) $\angle AOD$ and $\angle COB$ are vertically opposite angles. Similarly, $\angle$ AOC and $\angle$ BOD are vertically opposite angles.

Intext Questions 11.4

1. In figure 11.30, write the names of all the

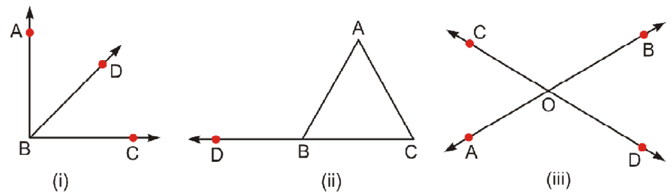


Figure 11.30

(a) supplementary angles

(b) complementary angles

(c) vertically opposite angles

(d) linear pairs

(e) pairs of adjacent angles

2. (a) In the figure 11.30 (i), measure $\angle ABD$ and $\angle CBD$, and find the sum of their measures.

(b) In the figure 11.30 (ii), measure $\angle ABC$ and $\angle ABD$, and find the sum of their measures.

(c) In the figure 11.30 (iii), measure $\angle AOC$ and $\angle ABD$, and find the sum of their measures.

(d) In the figure 11.30 (iii), measure $\angle AOC$ and $\angle BOD$. Are their measures equal?

(e) In the figure 11.30 (iii), measure $\angle AOC$ and $\angle AOD$, and find the sum of their measures.

11.7 Non-parallel and Parallel Lines

You have already studied that two lines drawn in a plane either intersect or do not intersect.

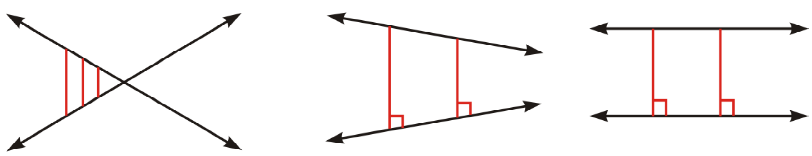


Figure 11.31

Perpendicular distance between two lines which do not intersect, always remains the same. Therefore neither do they meet nor do they intersect and do not have any point

Note

common with each other. On the other hand mutually intersecting lines intersect at one point, which is common to both of them. Therefore we say

Two lines in a plane are parallel, if perpendicular distance between them always remains the same and they do not intersect.

For finding the perpendicular distance between two lines ℓ_1 and ℓ_2 we place one edge of the set-square on ℓ_1 and read the distance of the other line ℓ_2 on the other edge of the set-square (see figure 11.32). By sliding the set-square on ℓ_1 we can observe that perpendicular distance on other points is same or not.

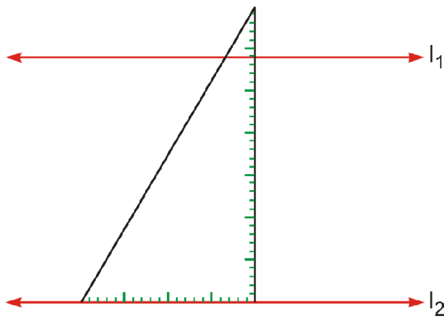


Figure 11.32

In figure 11.33 look at the line segments AB and CD. These do not intersect each other. Can we say that these are parallel? No, not at all, because these line segments are parts of lines ℓ_1 and ℓ_2 which not parallel.

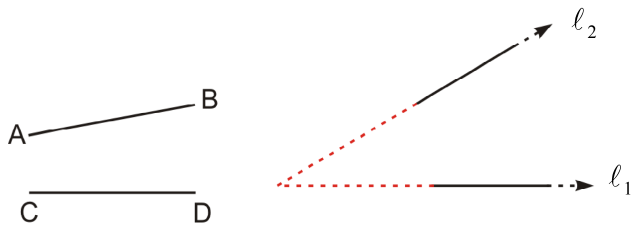


Figure 11.33

Same is true for rays OA and QB (see figure 11.34). These do not intersect each other and they are not parallel, because the lines whose parts are they, are not parallel.

To examine that two given lines are parallel or not, it is not possible to measure perpendicular distance of every point on one line from the other line, because lines are extendable on both sides.

So we will study about angles formed by parallel lines. These will help us in examining that two lines are parallel or not.

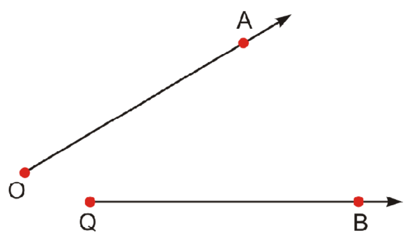


Figure 11.34

Transversal

If a line intersects two or more than two lines at different points then it is called a Transversal.

In figure 11.35 (i), PQ is a transversal which intersects l_1 and l_2.

In figure 11.35 (ii), AB is a transversal which intersects l_1, l_2 and l_3 at different points C, D and E.

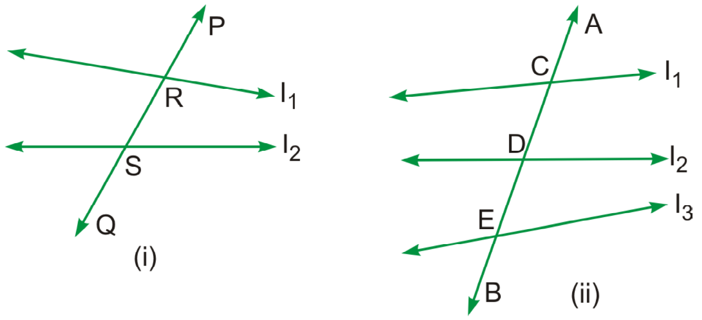


Figure 11.35

When a transversal intersect two straight lines, it forms eight angles with them as shown in figure 11.36.

Out of these some pairs of angles will be very useful for learning about parallel lines.

In figure 11.37 look at the angles marked by 1 and 5. Both the angles are formed on the same side of the transversal and with the two lines. These are called a pair of Corresponding angles. Angles marked by 2 and 6 are also of the same type and form another pair of Corresponding angles.

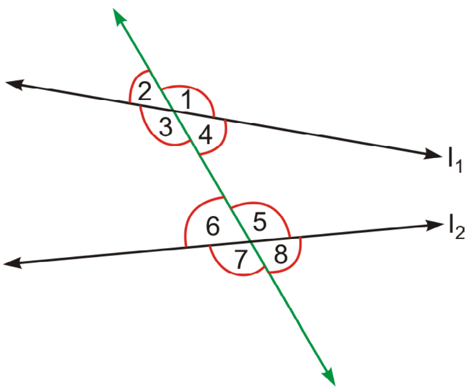


Figure 11.36

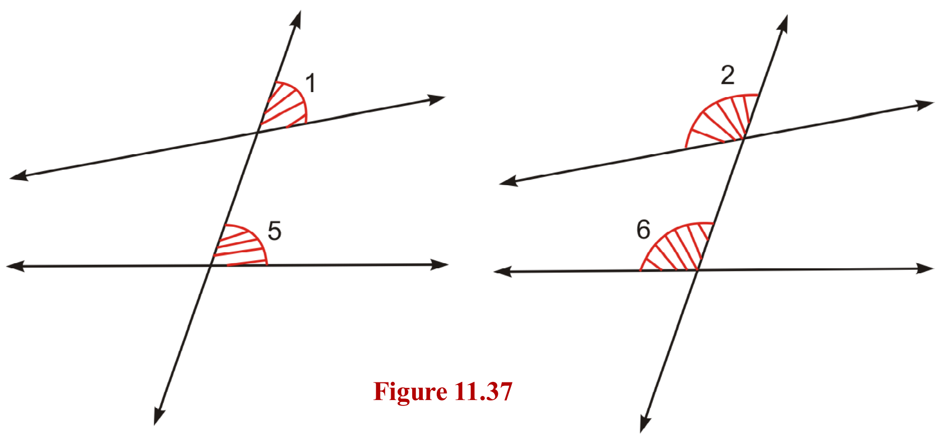


Figure 11.37

Note

Looking at all the angles in Figure 11.38 we get two more pairs of Corresponding angles; pair of angles marked by 3 and 7, and pair of angles marked by 4 and 8.

In this way in figure 11.38 we have four pairs of corresponding angles.

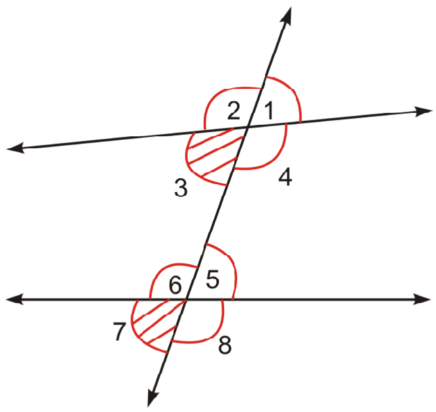


Figure 11.38

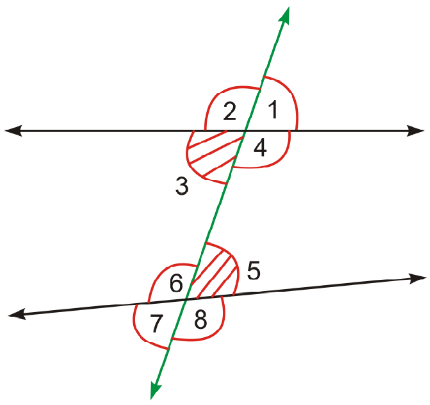


Figure 11.39

Look at the angles marked by 3 and 5 in the figure 11.39. These are formed on different sides of the transversal and are inside the two lines. These form a pair of Alternate angles.

Similarly angles marked by 4 and 6 form another pair of Alternate angles.

Look at the angles marked by 4 and 5 in the figure 11.39. These are formed on the same side of the transversal and are inside the two lines. These form a pair of Interior angles. Similarly angles marked by 3 and 6 form another pair of Interior angles.

Angles marked by 1, 2, 7 and 8 are called Exterior angles.

Intext Questions 11.5

1. Fill in the blanks:

   (a) Parallel lines do not mutually ....

   (b) Distance between two parallel lines remains ....

   (c) A line which intersects two other lines at different points is called a ....

   (d) A pair of Alternate angles lie on .... sides of the transversal and are .... the parallel lines.

   (e) Corresponding angles are formed on the .... side of the transversal and .... the two lines.

   (f) Pairs of Interior angles are formed on the .... side of the transversal and .... the parallel lines.

11.8 Pairs of Parallel Lines

(a) Corresponding angles

AB and CD are two intersecting lines and EF is a transversal which intersects these lines at G and H.

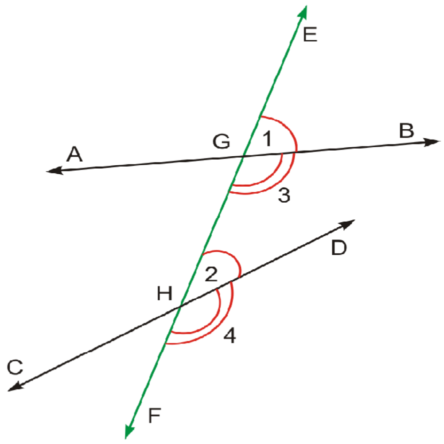


Figure 11.40

Look at the angles marked by 1 and 2 in figure 11.40. Are you able to identify which type of angles are these? It is a pair of corresponding angles. Upon measuring we get that $\angle 2$ is smaller than $\angle 1$.

Now look at the pair of angles marked by $\angle 3$ and $\angle 4$. This also is a pair of corresponding angles. Upon measuring we see that these angles also are not equal. $\angle 3$ is smaller than $\angle 4$.

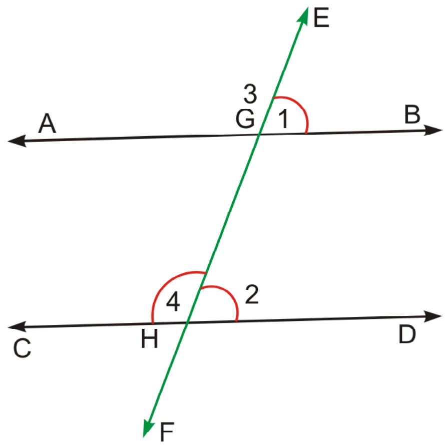


Figure 11.41

Let us now consider a pair of parallel lines AB and CD. Transversal EF intersects these parallel lines at G and H. In figure 11.41, identify angles marked by 1 and 2. This also is a pair of corresponding angles.

Let us measure these angles.

$\angle 1 = 65°, \angle 2 = 65°$

We will get that $\angle 1 = \angle 2$.

Angles marked by 3 and 4 also form a pair of corresponding angles. Upon measuring we will get that $\angle 3 = \angle 4$.

We may measure other pairs of corresponding angles or draw any other figure like the above then always we will get that

If a transversal intersects two parallel lines then pairs of corresponding angles thus formed are equal.

(b) Alternate Angles

Let us again consider two parallel lines AB and CD which are intersected by a transversal EF. In figure 11.42, identify angles marked by 2 and 5. They form a pair of alternate angles. Upon measuring these angles we get that

$\angle 2 = 60°, \ \angle 5 = 60°$

So $\angle 2 = \angle 5$.

If we measure other pair of alternate angles marked 4 and 6, we get $\angle 4 = \angle 6$.

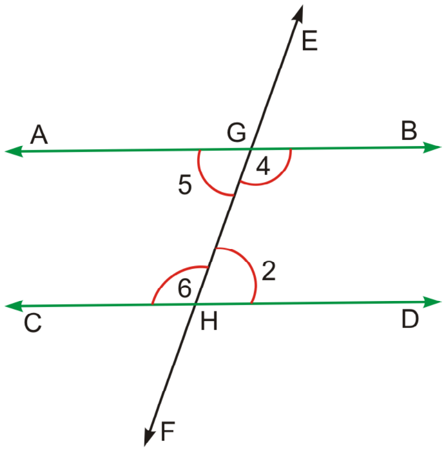


Figure 11.42

If we draw any other figure like the above and measure pairs of alternate angles we will always get that

If a transversal intersects two parallel lines then pairs of alternate angles thus formed are equal.

(c) Interior Angles

Let us again look at the pair of parallel lines AB and CD which are intersected by a transversal EF. In figure 11.43, identify angles marked by 4 and 2. Which type of pair is it? It is a pair of interior angles on the same side of the transversal. Upon measuring these angles we get that

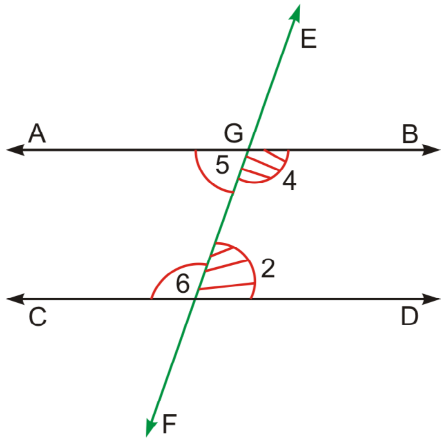


Figure 11.43

$\angle 4 = 110°, \ \angle 2 = 70°$

Are these two angles equal? No, but if we find the sum of their measures we get $\angle 4 + \angle 2 = 180°$

If we measure other pair of interior angles which are marked 5 and 6,

we get $\angle 5 = 70°, \ \angle 6 = 110°$

and $\angle 5 + \angle 6 = 180°$

We get the conclusion that

If a transversal intersects two parallel lines then sum of the pairs of interior angles on the same side of the transversal is 180°.

In this way we learnt three properties of pair of parallel lines intersected by a transversal. These in short are as under:

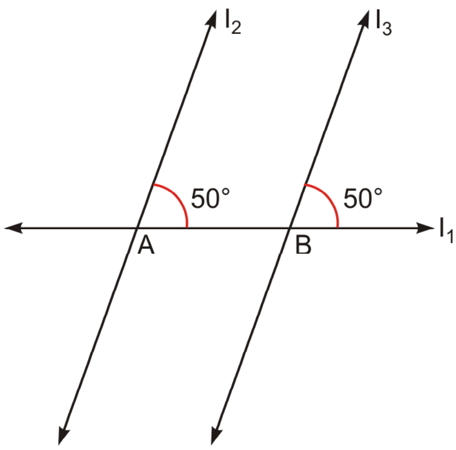


Figure 11.44

(a) Corresponding angles are equal.

(b) Alternate angles are equal.

(c) Sum of the interior angles on the same side of the transversal is 180°.

Let us verify the converse of each of these properties.

(a) Verification for Corresponding angles

Let us take a line l_1 and take points A and B on it. Using protector we draw two lines l_2 and l_3, both of which make angle of 50° with l_1. Observe that these form a pair of corresponding angles.

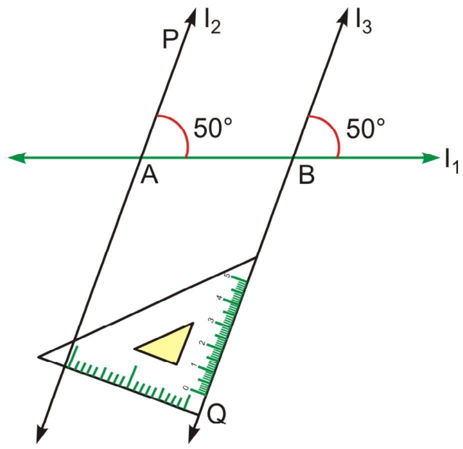


Figure 11.45

Now by placing a set-square along one line and sliding, we measure the perpendicular distance between l_2 and l_3. We get that distance remains same. So we say that lines l_2 and l_3 are parallel. If we repeat the activity by taking different line and different angle then we will always get that

If a transversal intersects other two lines in such a way that pair of corresponding angles are equal then the lines are parallel.

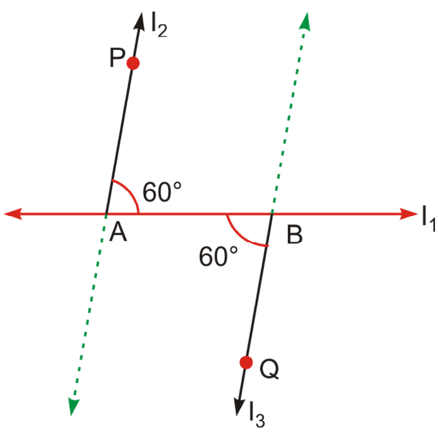


Figure 11.46

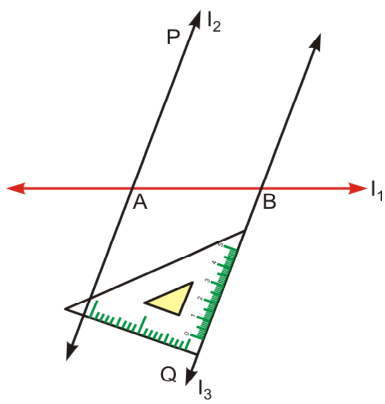


Figure 11.47

Even if we repeat the activity we will always get that

If a transversal intersects other two lines in such a way that pair of alternate angles are equal then the lines are parallel.

(c) Verification for Interior angles

Let us draw a line l_1 again and take points A and B on it. Using protector and drawing $\angle PAB = 70°$ and $\angle QBA = 110°$ we get two lines l_2 and l_3.

Observe that $\angle PAB + \angle QBA = 70° + 110° = 180°$ and these two form a pair of interior angles on the same side of the transversal. Upon measuring distance between lines l_2 and l_3 we get that lines are parallel.

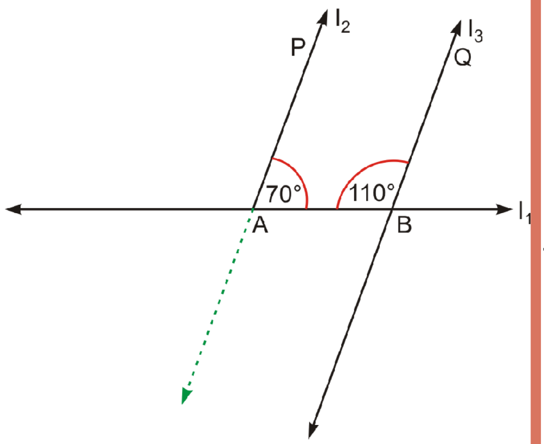


Figure 11.48

Even if we repeat the activity we will always get that

If a transversal intersects two lines in such a way that sum of interior angles on the same side of the transversal is 180°, then the lines are parallel.

We can verify the above properties by drawing using compass also.

(a) By drawing equal alternate angles

Step 1: Draw a line l_1 and take two points A and B on it.

Step 2: Draw line l_2 by drawing angle PAB at A.

Step 3: Using compasses draw $\angle QBR$ at B so that $\angle QBR = \angle PAB$. (figure 11.49)

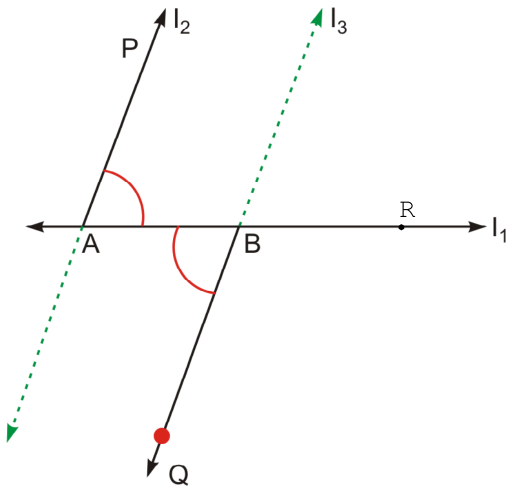


Figure 11.49

It is a pair of alternate angles.

Step 4: Using set-square verify that l_2 and l_3 are parallel lines.

(b) By drawing equal corresponding angles

Step 1: Draw a line l_1 and take two points A and B on it.

Step 2: Draw line l_2 by drawing $\angle PAB$ at A.

Step 3: Using compass draw $\angle QBR = \angle PAB$ so that both the angles are on the same side of l_1 (figure 11.50).

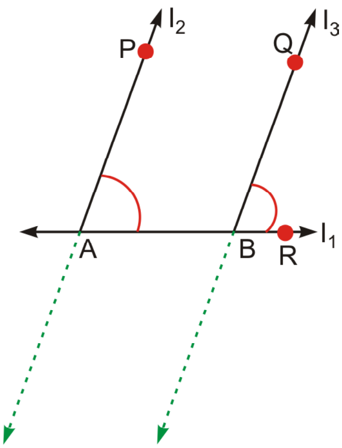
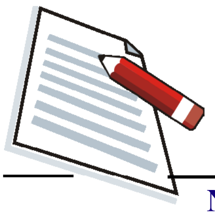


Figure 11.50



Step 4: Using set-square verify that l_2 and l_3 are parallel lines.

(c) By drawing interior angles having sum of 180°

Step 1: Draw a line l_1 and take two points A and B on it.

Step 2: Draw $\angle PAB$ at A.

Step 3: At B draw $\angle QBR = \angle PAB$ so that both the angles are on the same side of l_1.

Step 4: Extend AP and BQ and name them as l_2 and l_3 (figure 11.51).

Now observe that

$\angle PAB + \angle QBR = 180°$ (why)

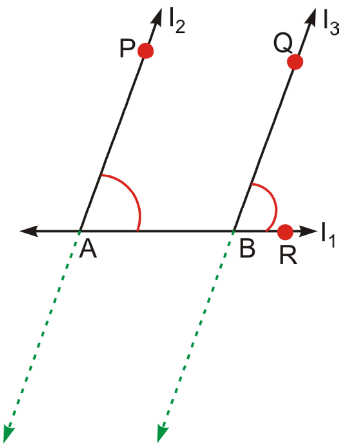


Figure 11.51

Step 5: Using set-square verify that l_2 and l_3 are parallel lines.

So we have learnt three properties of parallel lines. Using any one of these we can verify that two given lines are parallel or not. A transversal which intersects two parallel lines and any one of the angles formed with these is known, we can find the other angles.

Example 11.8: In the given figure identify the angles marked by:

(a) 7 and 5 (b) 5 and 3

(c) 5 and 8 (d) 2 and 3

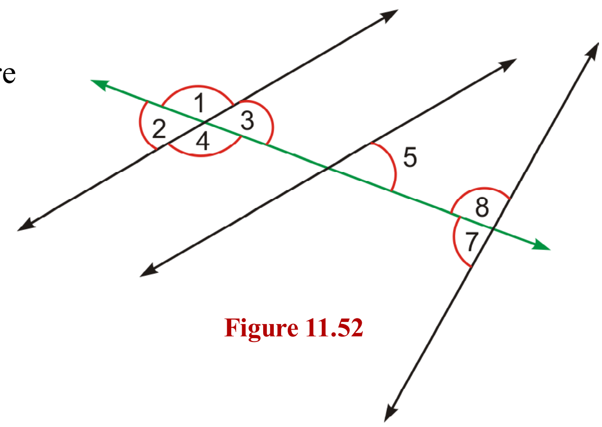


Figure 11.52

Solution:

(a) Alternate angles

(b) Corresponding angles

(c) Interior angles on the same side of the transversal

(d) Vertically opposite angles

Example 11.9: In figure 11.53, two parallel lines l_1 and l_2 are intersected by a transversal l_3. If $\angle u = 110°$ then find $\angle v$, $\angle x$, $\angle y$ and $\angle z$. Give reasons also.

Solution: $\angle v = \angle u = 110°$ (vertically opposite angles)

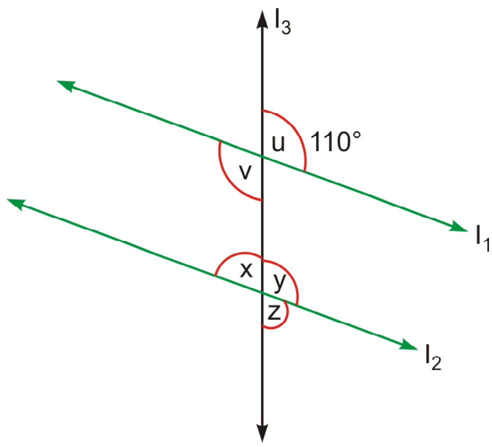


Figure 11.53

$\angle y = \angle u = 110°$ (corresponding angles)

$\angle x = 180° - \angle y = 180° - 110° = 70°$ (linear pair)

$\angle z = \angle x = 70°$ (vertically opposite angles)

Example 11.10: In each of the figure observe the angles and with reasons write whether lines l_1 and l_2 are parallel or not.

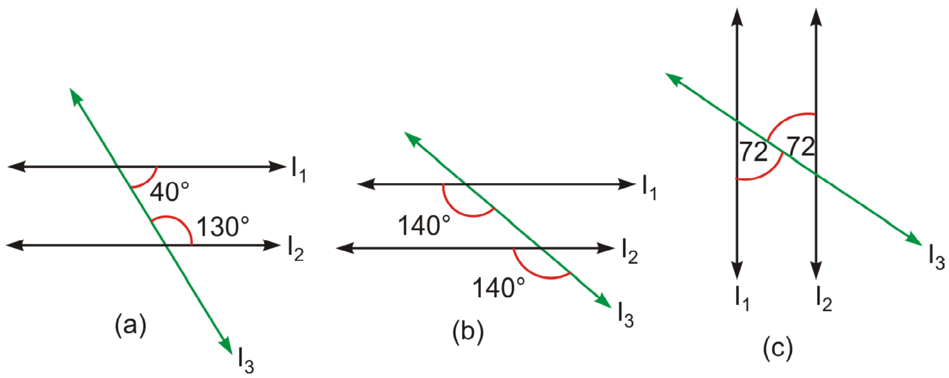


Figure 11.54

Solution:

(a) l_1 and l_2 are not parallel, because sum of interior angles on the same side of the transversal is 170° not 180°.

(b) l_1 and l_2 are parallel, because corresponding angles are equal.

(c) l_1 and l_2 are parallel, because alternate angles are equal.

Intext Questions 11.6

1. In figure 11.55 identify the following corresponding angles, alternate angles, interior angles on the same side of the transversal or vertically opposite angles:

    (a) $\angle 2$ and $\angle 6$

    (b) $\angle 3$ and $\angle 8$

    (c) $\angle 2$ and $\angle 7$

    (d) $\angle 5$ and $\angle 7$

    (e) $\angle 3$ and $\angle 4$

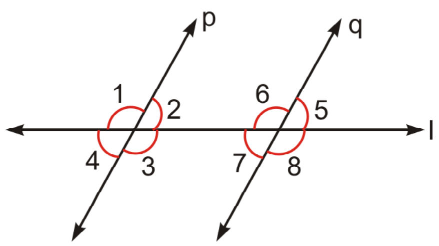


Figure 11.55

Note

2. In figure 11.56, l_1 and l_2 are two parallel lines and l_3 is a transversal. If $\angle 1 = 70°$ then find the measures of the following angles:

   (a) $\angle 4$

   (b) $\angle 5$

   (c) $\angle 6$

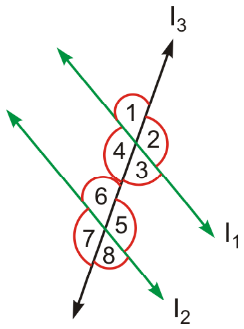


Figure 11.56

3. In figure 11.57, l_1 and l_2 are two parallel lines; l_3 and l_4 are two transversals. Find the measures of the following angles:

   (a) $\angle x$ (b) $\angle y$

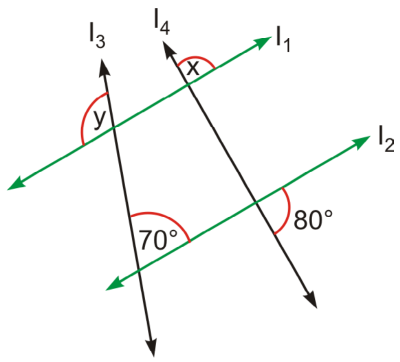


Figure 11.57

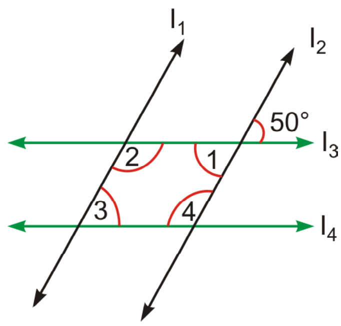


Figure 11.58

4. In figure 11.58, if lines l_1 and l_1 are parallel and lines l_3 and l_4 are parallel lines then find the measures of $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

5. In figure 11.59, if lines l_1 and l_2 are parallel lines, then find the measures of $\angle 1$, $\angle 2$ and $\angle 3$.

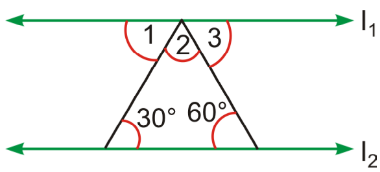


Figure 11.59

11.9 Drawing Parallel Lines

Now we will learn to draw a line parallel to a given line.

(a) With the help of a set-square

Example 11.11: Draw a line parallel to a given line l_1 which passes through a given point P.

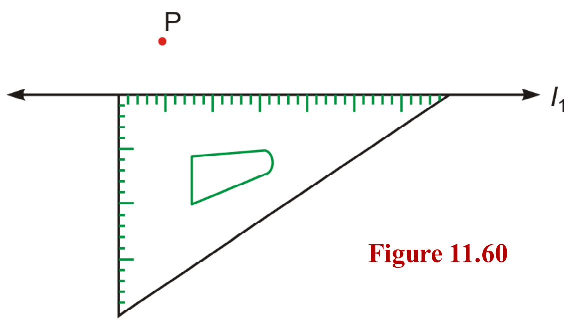


Figure 11.60

Solution:

Step 1: Place a set-square in such a way that its one edge coincides with l_1 (see figure 11.60).

Step 2: Now place a ruler along the other edge of the set-square without displacing it.

Step 3: Keeping the scale stationary, slide the set-square upwards along the edge of the ruler in such a way that its upper edge touches the point P (figure 11.61).

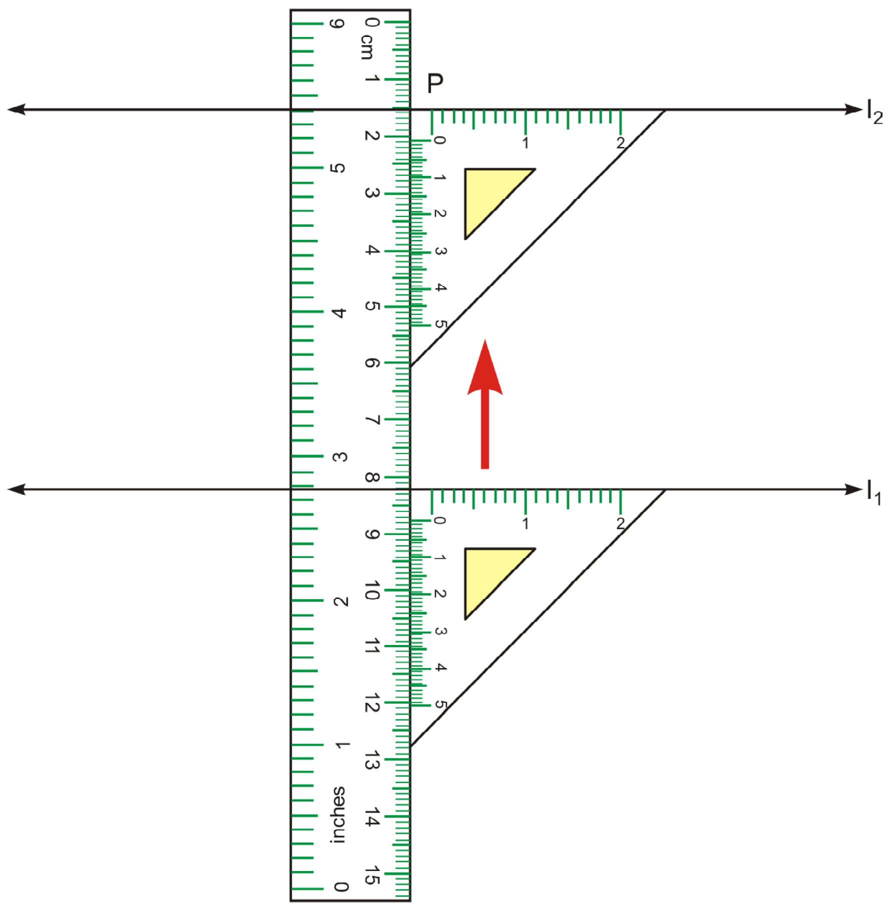


Figure 11.61

Step 4: Now keeping the set-square fixed at its place draw a line l_2 passing through P along the edge of the set-square. Line l_2 is our desired line which passes through P and is parallel to l_1.

(b) With the help of ruler and compass

(i) Using equality of alternate angles

Example 11.12: Draw a line parallel to a given line l_1 which passes through a given point P.

Solution:

Step 1: Take a point A on the line l_1 and join P with A.

Step 2: Taking a convenient radius and A as centre draw an arc. Mark the angle made at A by 1.

Step 3: Taking P as centre and the same radius draw an arc which intersects PA at point B.

Step 4: Cut the arc BC so that $\angle 2$ formed at P and $\angle 1$ formed at point A are equal.

Step 5: Join P with C and extend it on both sides (figure 11.62).

In this way we got line l_2 which is parallel to line l_1 and passes through point P.

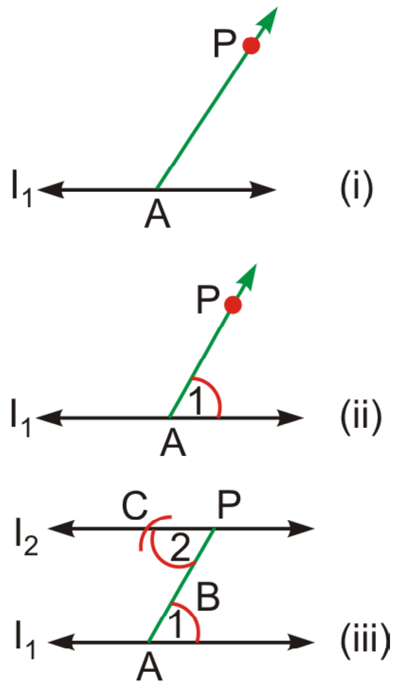


Figure 11.62

(ii) Using equality of corresponding angles

Step 1: Take a point A on the line l_1, join P with A and extend it upto point B.

Step 2: Taking A as centre and a convenient radius draw an angle and mark the angle by 1.

Step 3: Taking P as centre and the same radius cut PB at C as shown in figure 1_1.

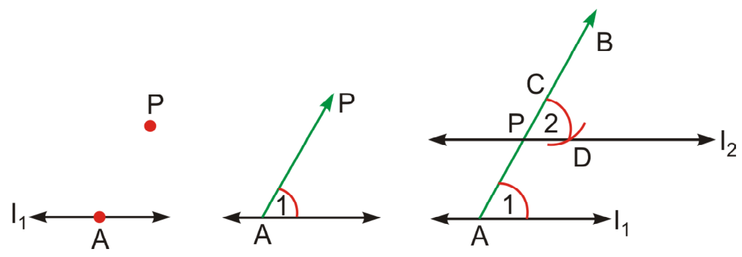


Figure 11.63

Step 4: At P draw $\angle 2$ such that $\angle 2 = \angle 1$.

Step 5: Draw PD and extend it on both sides. In this way we got line l_2 which is parallel toline l_1 and passes through point P.

Intext Questions 11.7

1. Take a line segment 6.8 cm long and a point below it. Draw a line passing through this point and parallel to a given line:

(i) using set-square

(ii) using compass

2. Take two lines l_1 and l_2 and a point P as shown in figure 11.64. From point P now draw lines parallel to both lines l_1 and l_2:

(i) with the help of set-square

(ii) with the help of compass

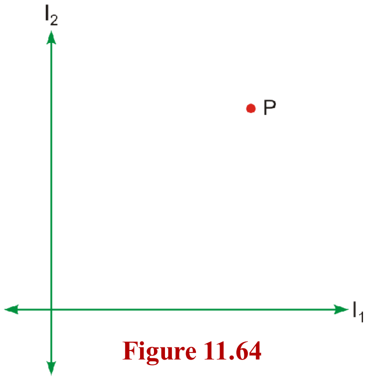


Figure 11.64

11.10 Perpendicular Lines

Observe the angles between the adjoining edges of a table or corners of a table.

These are examples of right angle. What can you say about angles at other corners? Angle at each corner is a right angle.

If there is a right angle between two lines (i.e. angle of 90°) then they are called perpendicular lines.
In figure 11.66, l_1 and l_2 are perpendicular lines. We say it like this also that l_1 is perpendicular to l_2 or l_2 is perpendicular to l_1. They are perpendicular to each other.

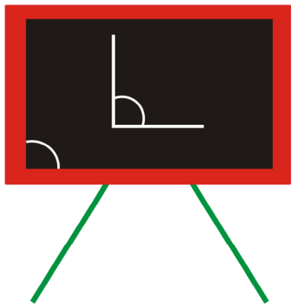


Figure 11.65

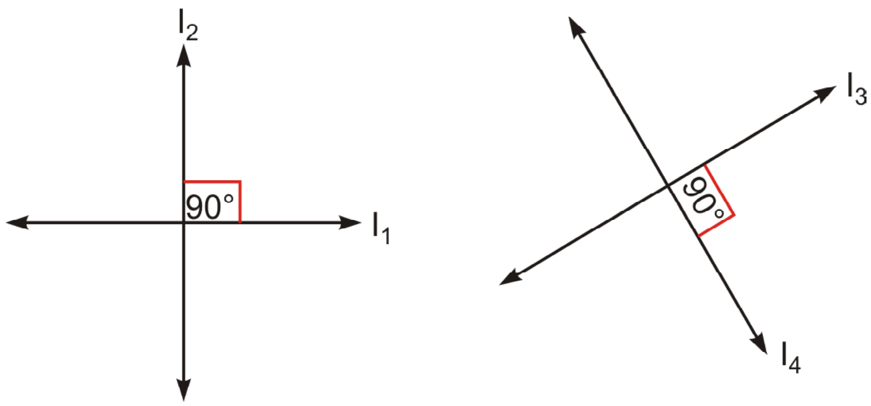
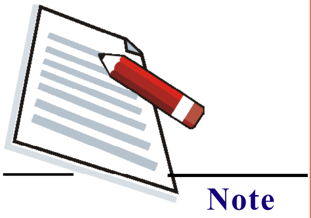


Figure 11.66



Drawing a line perpendicular to a given line

From a point given on a line we can draw a line perpendicular to it using protractor, set-square or compass.

(a) Using a protractor

Step 1: Draw a line l and take a point A on it.

Step 2: Place a base line of protractor on line l in such a way that centre of the protractor lie on point P.

Step 3: Keeping protractor stationary, mark a point at the place marked by 90° and name it as B.

Step 4: Join P with B, and get a desired perpendicular line by extending it.

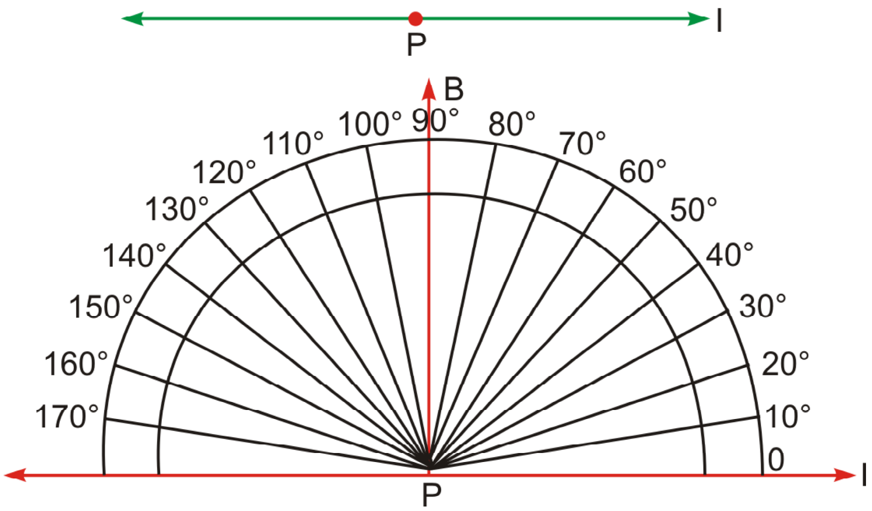


Figure 11.67

(b) Using set-square

Step 1: Draw a line *l* and take a point A on it.

Step 2: Place a set-square on line *l* in such a way that its corner with right angle is on point P and its one edge is on *l*.

Step 3: Keeping set-square stationary, draw a line AB along its other side. Line AB is the desired line which passes through A and is perpendicular to *l*.

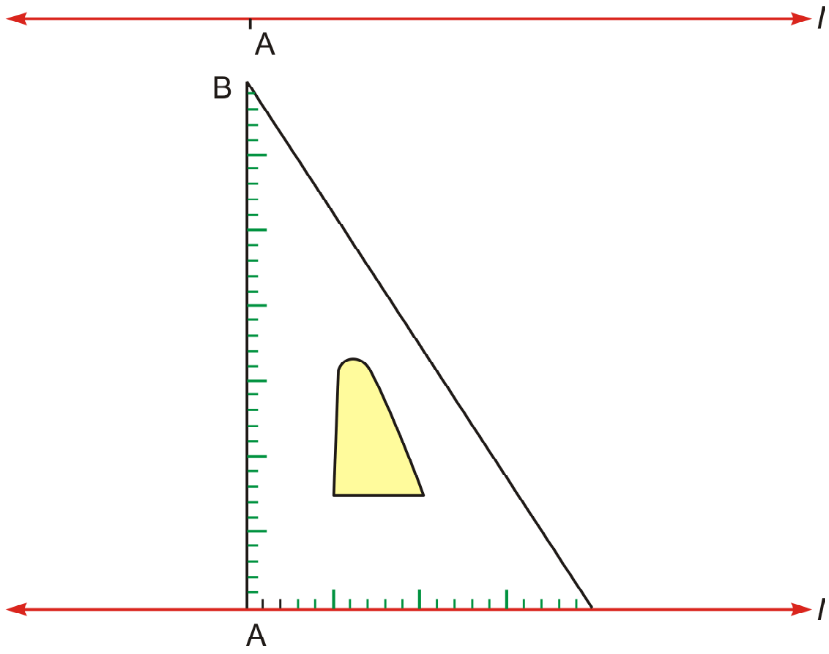


Figure 11.68

Let us Revise

- Angle of 90° is called a right angle.
- Angle smaller than 90° and greater than 0° is called an acute angle.
- Angle greater than 90° and smaller than 180°is called an obtuse angle.
- Angle having degree measure of 180° is called a straight angle.
- Two angles are known as Adjacent angles if they have a common vertex and one common arm, and the other arms are on the opposite sides of the common arm.
- Two angles form a pair of complementary angles if their sum is 90°.
- A pair of supplementary adjacent angles is called a linear pair.
- Vertically opposite angles are equal.
- A line which intersects two or more than two lines at different points is called a transversal line.
- When a transversal line intersects two lines in two points then eight angles are formed.

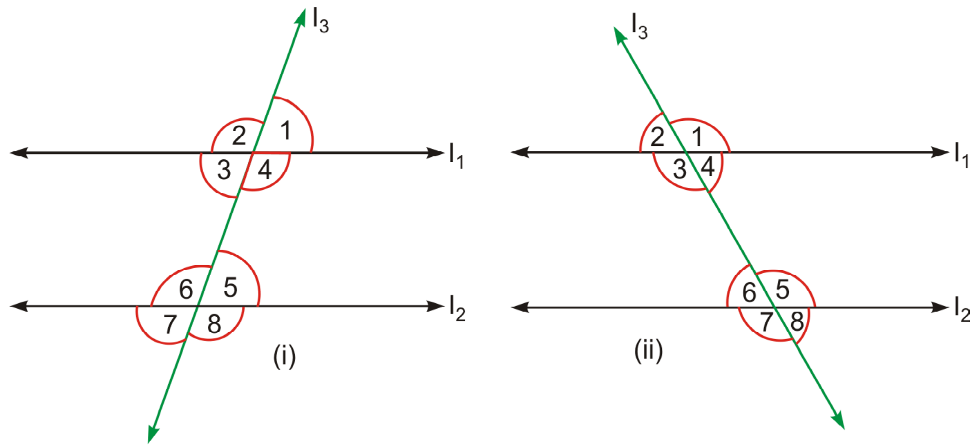


Figure 11.69

- $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are pairs of corresponding angles.
- $\angle 3$ and $\angle 5$, and $\angle 4$ and $\angle 6$ are pairs of alternate angles.
- $\angle 4$ and $\angle 5$, and $\angle 3$ and $\angle 6$ are pairs of interior angles on the same side of the transversal.
- If a transversal intersects two parallel lines then
  - a) pairs of corresponding angles are equal
  - b) pairs of alternate angles are equal
  - c) sum of interior angles on the same side of the transversal is 180°.
- Converse of all the three properties are also true.
- Using these properties parallel and perpendicular lines can be drawn.

Exercise

1. Using compass draw angles mentioned below:
   (a) Angle of 90°
   (b) Angle of 45°
   (c) $\angle PQR = 135°$
   (d) $\angle ABC = 75°$

2. In figure 11.70, write
   (a) Pairs of adjacent angles
   (b) Pairs of supplementary angles
   (c) Pairs of vertically opposite angles
   (d) Linear pairs of angles

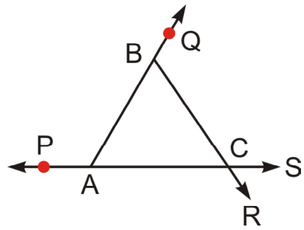


Figure 11.70

3. In the figure 11.71 given below $\angle BOD$ and $\angle AOC$ are right angles. Name the pairs of complementary angles.

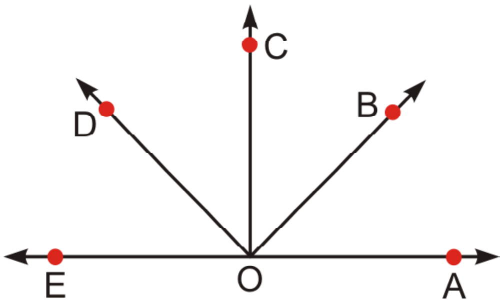


Figure 11.71

4. (a) Is it possible that both the angles of a pair of supplementary angles are acute angles?
   (b) Is it possible that one angle of a pair of complementary angles is
   (i) an acute angle?
   (ii) an obtuse angle?
   (c) Is it possible that both the angles of a pair of supplementary angles are right angles?

5. With the help of a ruler and a compass, draw the following angles and bisect them also. Verify by measuring with the help of a protractor.
   (a) Angle of 60°
   (b) Angle of 90°
   (c) Angle of 135°
   (d) Angle of 150°

6. In figure 11.72, l_1 is parallel to l_2 and l_3 is parallel to l_4. Find the measures of $\angle x$, $\angle y$ and $\angle z$.

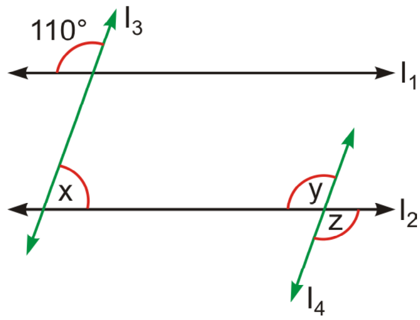


Figure 11.72

7. In figure 11.73, AB is parallel to DE. Find the measures of $\angle A$, $\angle B$ and $\angle ACB$.

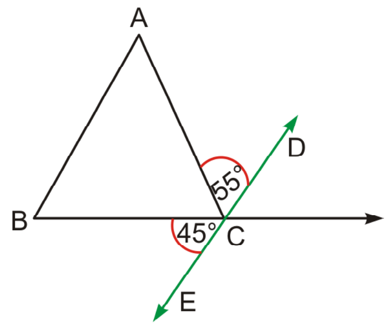


Figure 11.73

8. In figure 11.74, AB is parallel to DE. Find the measures of $\angle x$ and $\angle y$.

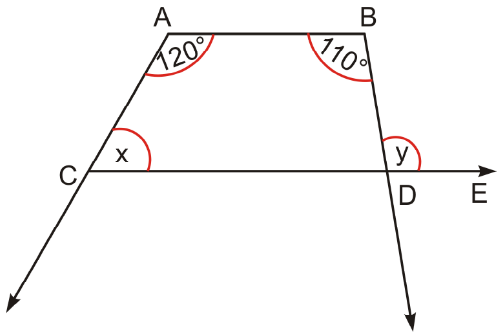


Figure 11.74

9. Looking at figure 11.75, write the parallel lines. Is AD parallel to BC?

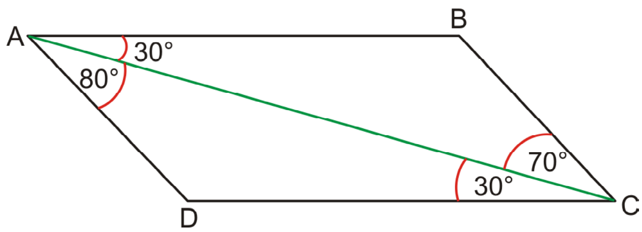


Figure 11.75

10. Draw a line segment AB=6.8 cm. Take a point P on it so that AP=4.2 cm. Draw a perpendicular line PQ=5.3 cm. Draw line QR parallel to AB which passes through Q. Is QR per perpendicular to PQ?

Answers

Intext Questions 11.1

1. (a) half (b) complete
   (c) one-fourth (d) half (e) one-fourth

2 (a) $\angle AOB$ (b) $\angle CAB$ (c) $\angle MOL$ (d) $\angle RPQ$
   (e) $\angle ROS$

3 $\angle EAC, \angle EAB, \angle DAB, \angle DAC$

4.

Vertex	Arms
O	OP and OQ
Q	QP and QR
M	ML and MN
S	SR and ST

Intext Questions 11.3

1. (a) obtuse angle (b) straight angle
   (c) right angle (d) acute angle
   (e) reflex angle (f) zero angle
   (g) complete angle

2 $\angle DAB$ is an obtuse angle.

   $\angle ABC$ is an acute angle.

   $\angle ADC$ is an obtuse angle.

   $\angle DCB$ is an acute angle.

   $\angle BPA$ is a straight angle.

Intext Questions 11.4

1. (a) in figure (ii), $\angle DBA$ and $\angle ABC$ form a pair of supplementary angles.

in figure (iii), $\angle DOA$ and $\angle COA$, $\angle AOC$, $\angle AOC$ and $\angle COB$,

$\angle COB$ and $\angle BOD$, and $\angle BOD$ and $\angle AOD$ are pairs of supplementary angles.

(b) in figure (i), $\angle ABD$ and $\angle DBC$ form a pair of complementary angles

(c) in figure (iii), $\angle AOC$ and $\angle BOD$; $\angle AOD$ and $\angle BOC$ are pairs of vertically opposite angles.

(d) in figure (ii), $\angle DBA$ and $\angle CBA$ form a linear pair of angles.

in figure (iii) $\angle DOA$ and $\angle AOC$, $\angle AOC$ and $\angle COB$,

$\angle COB$ and $\angle BOD$, $\angle BOD$ and $\angle DOA$ form linear pairs.

(e) in figure (i), $\angle DBC$ and $\angle DBA$ are adjacent angles

in figure (ii), $\angle ABD$ and $\angle ABC$ are adjacent angles'\.

in figure (iii), $\angle DOA$ and $\angle AOC$, $\angle AOC$ and $\angle COB$,

$\angle COB$ and $\angle BOD$, $\angle BOD$ and $\angle DOA$ are adjacent angles.

Intext Questions 11.5

1. (a) intersect (b) same (c) transversal
   (d) opposite, between (e) same, between (f) same, between

Intext Questions 11.6

1. (a) interior angles on the same side of the transversal
   (b) corresponding angles
   (c) alternate angles
   (d) vertically opposite angles
   (e) linear pair

Exercise

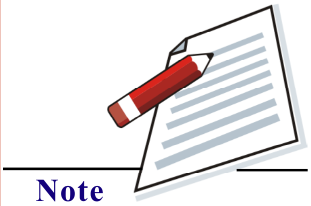
2. (a) adjacent angles:

$\angle ACB$, $\angle ACR$, $\angle ACB$, $\angle BCS$

$\angle ABC$, $\angle CBQ$

$\angle BAC$, $\angle BAP$

$\angle RCA$, $\angle RCS$



(b) as in (a)

(c) $\angle BCA$, $\angle RCS$; $\angle RAC$, $\angle BCS$

(d) as in (a)

3. $\angle AOB$, $\angle BOC$; $\angle BOC$, $\angle COD$; $\angle COD$, $\angle EOD$

4. (a) No

   (b) (i) yes (ii) No

   (c) yes

6. $\angle x = 70°$, $\angle y = 110°$, $\angle z = 110°$

7. $\angle A = 55°$, $\angle B = 45°$ and $\angle ACB = 80°$

8. $\angle x = 60°$, $\angle y = 110°$

9. AB and CD; No

10. yes

12

TRIANGLES AND ITS TYPES

We have learnt about lines and angles. Now we shall learn about a figure which is made of more than two line segments. Out of such figures Triangle is the easiest figure.

As it is clear from the name, a triangle is a three sided figure, which is represented by the symbol Δ.

This figure is very important in our daily life. We see many figures around us, some of which are triangular figures, such as two set squares of Geometry Box, triangular tiles used for constructing floor and traffic signals displayed on roundabouts of roads are also displayed in triangles, as shown below in figure 12.1 (i):

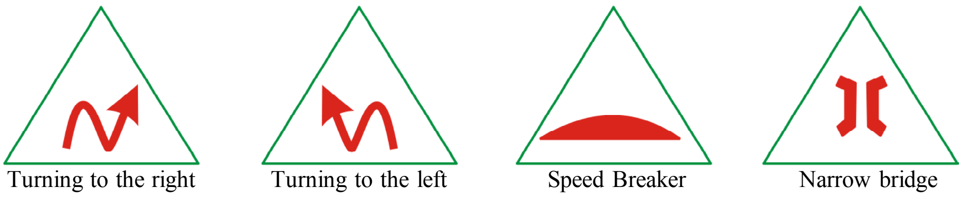


Figure 12.1 (i)

If A, B and C be any three non-collinear points, then figure formed by the line segments AB, BC and CA is called a Triangle with vertices A, B and C. This triangle is represented as 'Δ ABC'. This triangle has six components or parts as can be seen in figure 12.1(i). They are: three sides AB, BC and CA and three angles $\angle ABC$, $\angle ACB$ and $\angle BAC$.

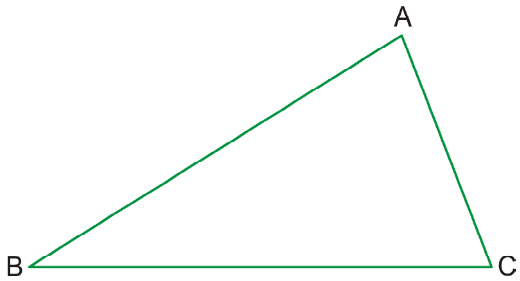


Figure 12.1 (ii)

From this lesson, you will learn:

- About vertices, sides and angles of a triangle
- Relation between angles of a Triangle

Note

- Categorization of Triangles
  - (a) On the bases of sides
  - (b) On the bases of angles
- Properties of special type of Triangles

12.1 Triangle

For understanding the figure of a triangle, we shall use thin straight wire as shown in figure 12.2 (i).

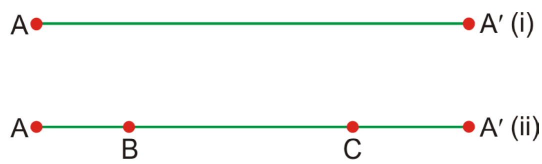


Figure 12.2

Marking two points B and C on this wire, tie a thread on these and place the wire as in figure 12.2 (ii).

Now turn the wire from the points B and C such that it looks like figure 12.3(i)

Join A and A' at one point and name it as A as shown in figure 12.3 (ii). This figure can be named as triangle.

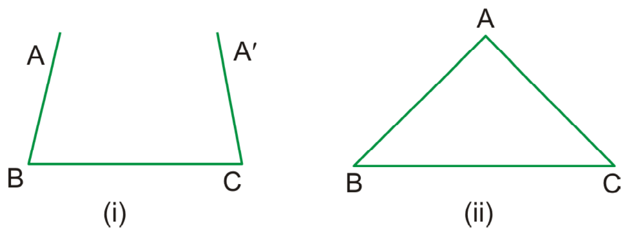


Figure 12.3

Thus we can say that a triangle is formed by joining three line segments lying in a plane such that a simple closed figure is formed. So,

A simple close figure formed by three line segments is called a Triangle.

Learn by doing:

(a) Take two match sticks. Join one of their ends with al-pin, as shown in figure 12.4. Can we get a close figure? No.

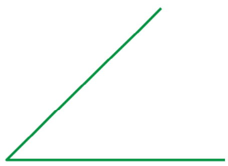


Figure 12.4

(b) Now with the help of third stick we can make following figure.

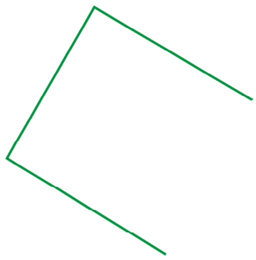


Figure 12.5

Above is not a simple closed figure.

Figure 12.6 (i) is a simple close figure, which is called a triangle.

As every simple closed figure, triangle also divides its plane in three parts. Look at figure 12.6 (ii), these are

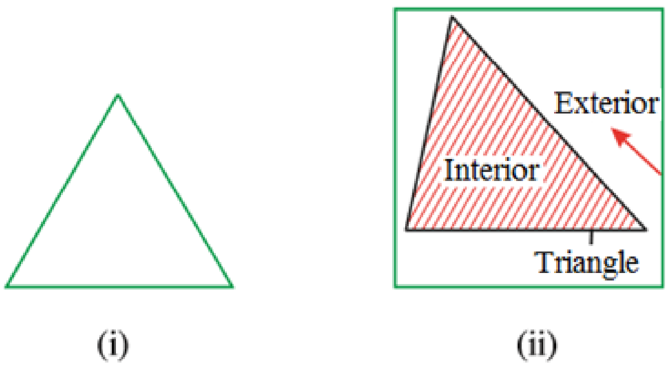


Figure 12.6

Interior of triangle (shaded)

External of triangle (unshaded)

Triangle itself

Triangle along with interior of the triangle is called Triangular Region.

12.2 Drawing a Triangle

Look carefully the three points A, B, C in figure 12.7 (i). Can a triangle be formed by joining these three points? No, because these three points are collinear.

Now look at the points P, Q, R in figure 12.7 (ii), triangle can be formed by joining them. So by joining three non-collinear points always one triangle is formed.

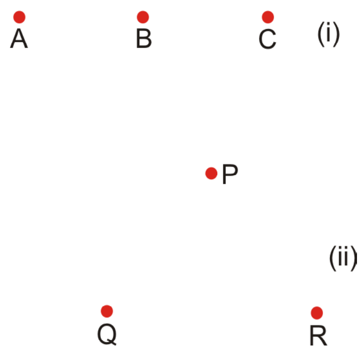


Figure 12.7

Note

12.3 Vertices, Sides, Angles, Exterior Angles, Vertically opposite Angles of Triangle

By now we have learnt that a closed figure formed by three non-collinear points is a triangle. These three points are called **Vertices** of the triangle.

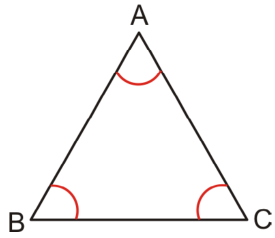


Figure 12.8

Three lines which make the triangle are called sides of the triangle. As it is clear from figure 12.8 that three **sides** of the triangle are AB, BC and CA and these sides are making three angles in the triangle.

These are represented by $\angle BAC$, $\angle ABC$ and $\angle BCA$ or $\angle A$, $\angle B$ and $\angle C$.

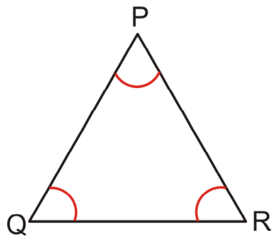


Figure 12.9

Thus three sides combined with three angles of a triangle are six components (parts) of a triangle.

Example 12.1: In triangular figure 12.9 shown on the right, name the sides, angles and vertices of the triangle.

Solution: In figure 12.9, sides of ΔPQR are PQ, QR and RP. Angles are $\angle P, \angle Q$ and $\angle R$. Vertices are P, Q and R.

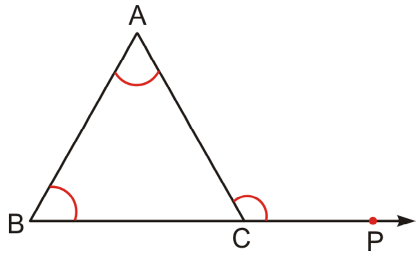


Figure 12.10

If we extend side BC of triangle ΔABC upto P then $\angle ACP$ is formed (as shown in figure 12.10). The $\angle ACP$ shall be called the exterior angle.

$\angle A$ and $\angle B$ are called interior opposite angles of exterior angle $\angle ACP$.

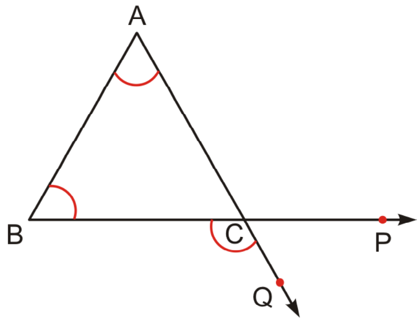


Figure 12.11

In figure 12.10, $\angle A$ and $\angle B$ are interior opposite angles of exterior angle $\angle ACP$. Similarly if AC is extended upto point Q then we get another exterior angle at the vertex of the triangle. Here we observe that exterior angles $\angle BCQ$ and $\angle ACP$ form a pair of vertically opposite angles. So $\angle ACP = \angle BCQ$.

Example 12.2: In figure 12.12 write the names of sides, exterior angle and interior opposite angles of the triangle.

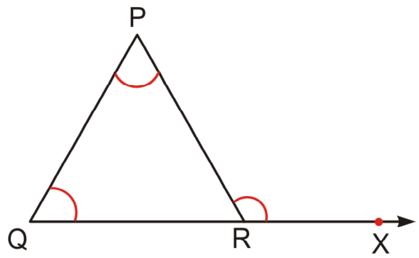


Figure 12.12

Solution: In figure 12.12, PQ, QR and PR are sides of ΔPQR. Exterior angle at the vertex R is $\angle PRX$. $\angle P$ and $\angle Q$ are the interior opposite angles of this exterior angle.

12.4 Altitudes and Medians of Triangle

Take a triangle ABC and draw perpendicular AD from A to the opposite side BC. AD is called an **Altitude** of a triangle (figure 12.13 (i)). Triangle has three Altitudes (figure 12.13 (ii)). The three altitudes are concurrent.

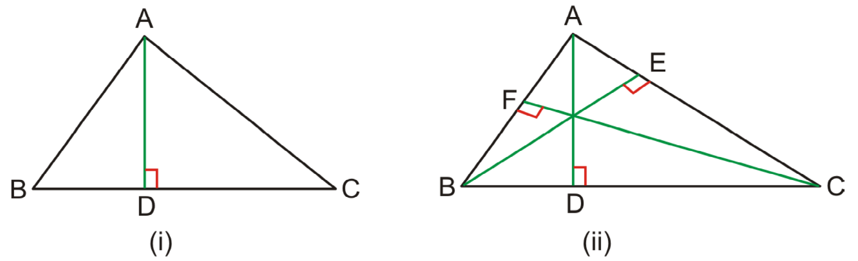


Figure 12.13

Now take any triangle PQR and join the middle point M of QR with the opposite vertex P (figure 12.14 (i)). Line segment PM is called a **Median** of triangle PQR. Triangle has **three medians** (figure 12.14(ii)), which are concurrent.

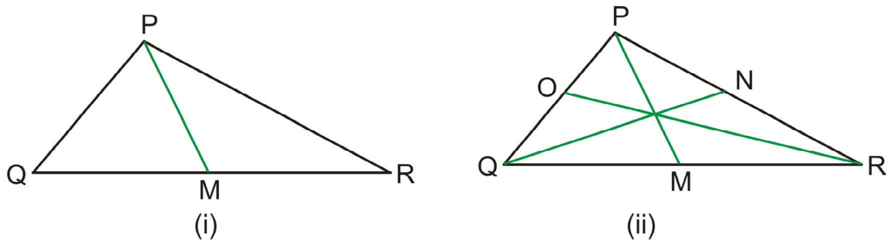


Figure 12.14

Intext Questions 12.1

1. Fill a word in the blanks so that statements are true:

   (a) A triangle has____________ vertices.

   (b) A triangle has ____________ sides.

   (c) A triangle has ____________ angles.

   (d) A triangle has ____________ components (parts).

   (e) A triangle has ____________ altitudes.

   (f) A triangle has ____________ medians.

2. Take three non-collinear points P, Q, R on any page of your notebook. Draw PQ, QR and RP. Is the shape drawn is a triangle? If not, then why not?

3. Take three non-collinear points A, B, C on any page of your notebook. Draw AB, BC and CA. Write name of the shape.

4. In figure 12.15 write the exterior angle and its interior opposite angles.

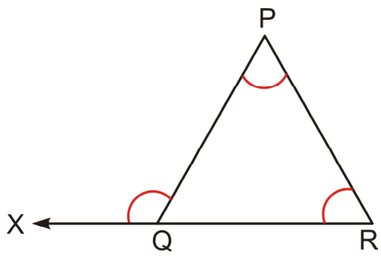


Figure 12.15

12.5 Sum of Angles of a Triangle

We have already studied about angles of a triangle. Now we will study about sum of angles of a triangle.

Learn by doing

Take a piece of paper and as in figure 12.16 make a ΔABC . Put marks on its three angles as in the figure and mark them by 1, 2, and 3. Cut this triangular region along its sides using scissors and cut it in three pieces.

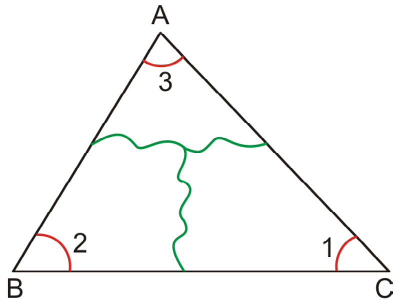


Figure 12.16

These three pieces represent different interior angles as is evident from figure 12.17.

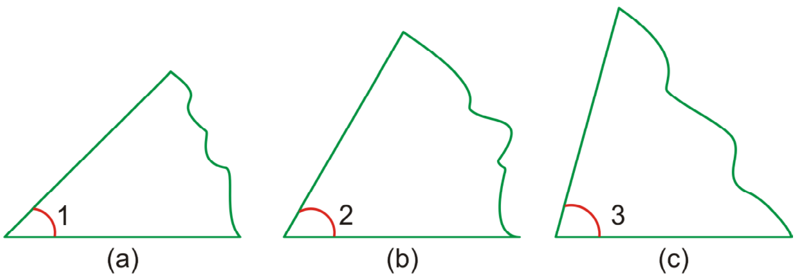


Figure 12.17

Now after drawing a line PQR place the three cut outs in such a way that the vertices of all the three angles fall at point O as in figure 12.18. In this way we observe that three cut-outs form a straight line. Sum of angles at a point is 180°. So in ΔABC

$\angle A + \angle B + \angle C = 180^\circ$

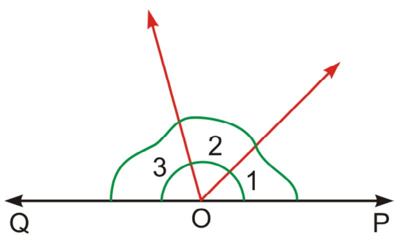


Figure 12.18

Learn by doing

Take three triangles as in figure 12.19 and represent them as (a), (b) and (c).

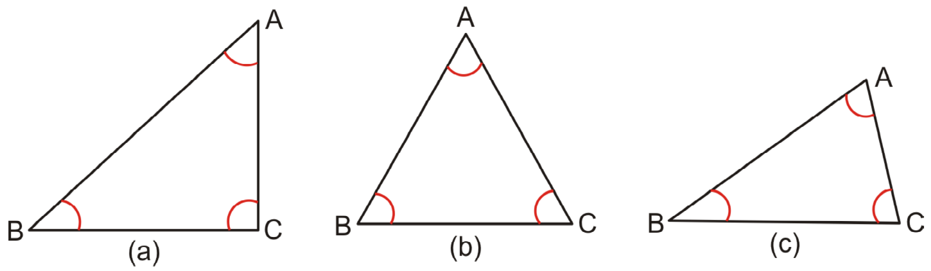


Figure 12.19

Now measure the three angles of every triangle and write in the following table and add them which has been represented by S.

Triangle	Measure of angle			Sum	180° – S	Remarks
	$\angle A$	$\angle B$	$\angle C$	$S = \angle A + \angle B + \angle C$		
(a)						
(b)						
(c)						

In this way we observe from the above table that difference 180°-S is zero or it is so small that it is negligible. This negligible difference can be because of inaccuracies in measuring angles.

In this way in any ΔABC, $\angle A + \angle B + \angle C = 180^\circ$.

Verification

After the above experiments we may reach at this conclusion in the following way also.

As in figure 12.20 draw a ΔABC and name the angle at A as 1, at B as 2 and at C as 3. Now draw a line passing from A and parallel to side BC of ΔABC . Name the other angles at A formed with line PQ as 4 and 5.

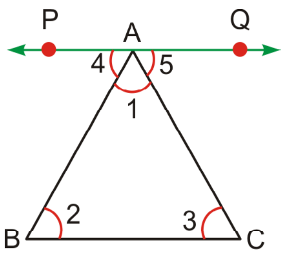


Figure 12.20

In this way, we get the following information from figure 12.20:

$\angle 2 = \angle 4$ .....(Alternate angles)

$\angle 3 = \angle 5$ .....(Alternate angles)

$\angle 1 = \angle 1$ ..... (Common to both)

Adding the two sides mentioned above separately, we get the following result.

$\angle 2 + \angle 1 + \angle 3 = \angle 4 + \angle 1 + \angle 5$

Or $\angle 1 + \angle 2 + \angle 3 = 180^o$

(Because $\angle 4 + \angle 1 + \angle 5$, are adjacent angles on a straight line whose sum is 180°)

So, $\angle A + \angle B + \angle C = 180°$

So sum of the three angles of any triangle is 180°.

Example 12.3: If in ΔPQR, $\angle P = 30°$ and $\angle Q = 45°$ then find the measure of $\angle R$.

Solution: We draw a rough figure of ΔPQR and write measures of given angles $(\angle P$ and $\angle Q)$.

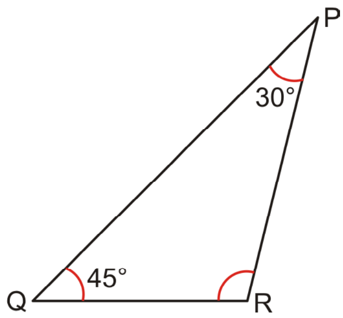


Figure 12.21

So in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$

$30° + 45° + \angle R = 180°$ (by putting the measures of $\angle P$ and $\angle Q$)

Or $75° + \angle R = 180°$

$\therefore \angle R = 180° - 75°$

or $\angle R = 105°$

Therefore in ΔPQR measure of the third angle $\angle R$ is 105°.

Example 12.4: Ratio of the angles of a triangle is 1: 2: 3. Find the measures of the three angles.

Solution: It is given that ratio of the angles of a triangle is 1: 2: 3.

Assume that measures of angles of the triangle are x, 2x, 3x.

As per property of triangles, sum of the angles of a triangle is 180°,

$x + 2x + 3x = 180°$

Or $6x = 180°$

Or $x = \frac{180°}{6} = 30°$

Thus, measure of first angle of the triangle = x = 30°

Measure of second angle of the triangle = 2x = 2 x30° = 60°

Measure of third angle of the triangle = 3x = 3 x30° = 90°

Thus angles of the triangle will be 30°, 60°, and 90°.

Intext Questions 12.2

1. Measures of two angles of a triangle are 75° and 55°. Find the measure of third angle.
2. All the three angles of a triangle are equal. Find the measure of each of the angle.
3. Two angles of a triangle are equal and the third angle is of 80°. Find the measure of equal angles.

12.6 Relation between the exterior angle and interior opposite angles

We have already studied about exterior angle and its interior opposite angles in triangle. We observe in figure 12.22 that $\angle ACP$ is the exterior angle of $\angle ABC$ and, $\angle BAC$ and $\angle ABC$ are its interior opposite angles.

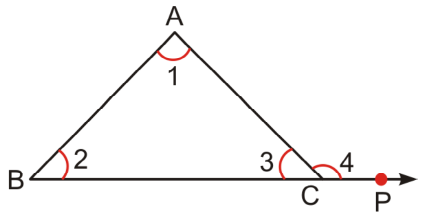


Figure 12.22

In ΔABC,

$\angle 1 + \angle 2 + \angle 3 = 180°$ ….. (1) (Sum of the three angles of a triangle)

$\angle 3 + \angle 4 = 180°$ ….. (2) (Linear pair)

Here we observe that right hand side of equation (1) and equation (2) are equal. Therefore left hand side will also be equal.

$\therefore \angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

Or $\angle 1 + \angle 2 = \angle 4$ ….. (Subtracting $\angle 3$ from both the sides)

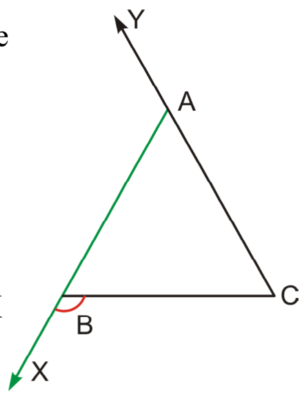
From this we conclude that

Exterior angle of a triangle is equal to sum of interior opposite angles.

Example 12.5: In figure 12.23, $\angle CBX$ is the exterior angle at vertex B of ΔABC. Write the names of its

(i) adjacent interior angle

(ii) interior opposite angles



Solution:

(i) In ΔABC adjacent interior angle of exterior angle $\angle CBX$ is $\angle ABC$.

(ii) Interior opposite angles of $\angle CBX$ are $\angle BAC$ and $\angle ACB$.

Example 12.6: In figure 12.24, find the measure of exterior angle $\angle ACD$.

Solution: In the figure it is given that $\angle A = 70°$ and $\angle B = 80°$, which are the interior opposite angles of exterior $\angle ACD$ of ΔABC.

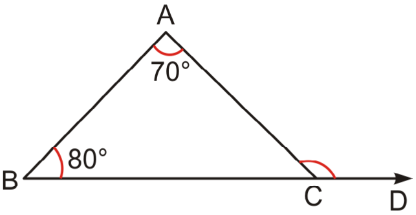


Figure 12.24

We know that in any triangle every exterior angle is equal to sum of the interior opposite angles.

$\therefore \angle ACD = \angle BAC + \angle ABC$

$= 70° + 80°$ (By putting the measures of angles)

$= 150°$

$\therefore$ in the figure, exterior angle $\angle ACD = 150°$

Intext Questions 12.3

1. In a triangle an exterior angle is 110° and one interior opposite angle is 30°. Find the other angles of the triangle.

2. In figure 12.25 find the measure of $\angle PRX$.

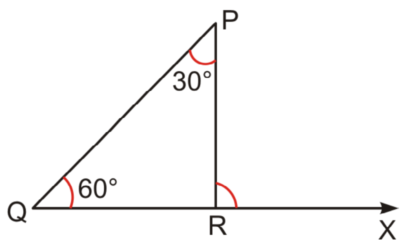


Figure 12.25

3. In a triangle measure of an exterior angle is 100° and both the interior opposite angles are equal. Find the measures of these angles and find the measure of the third angle also.

12.7 Sum of any two sides of a Triangle

Take any triangle ABC (figure 12.26).

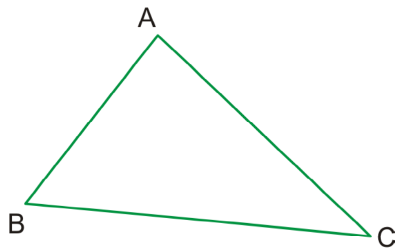


Figure 12.26

Measure its sides AB, BC and CA.

Is AB + BC > CA?

Is BC + CA > AB?

Is CA + AB > BC?

You will observe that

Sum of any two sides of a triangle is greater than its third side.

Intext Questions 12.4

1. Can 3.5 cm, 2.5 cm and 6 cm be the measure of sides of a triangle?

2. Can 7.2 cm, 3.8 cm and 4.3 cm be the measure of sides of a triangle?

3. Can 2.9 cm, 3.4 cm and 6.1 cm be the measure of sides of a triangle?

12.8 Categorisation of Triangles

(a) On the bases of sides

12.8.1 Scalene Triangle

A triangle in which no two sides are equal is called a Scalene Triangle. ΔABC is a Scalene Triangle because measures of all the sides of this triangle are different (figure 12.27)

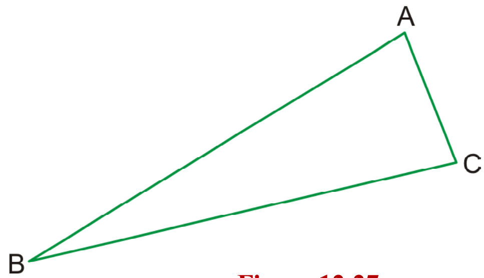


Figure 12.27

12.8.2 Isosceles Triangle

A triangle in which measures of two sides are equal is called an Isosceles Triangle. In ΔPQR sides PQ and PR are equal (see figure 12.28). Therefore, it is an Isosceles Triangle.

To show equality of sides we put the same sign on the sides.

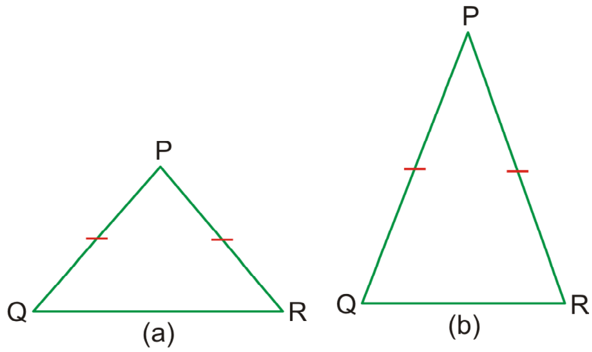


Figure 12.28

12.8.3 Equilateral Triangle

A triangle in which measures of all the three sides are equal is called an Equilateral Triangle. In ΔLMN side LM, side MN and side LN have the same lengths. Therefore it is an Equilateral Triangle (see figure 12.29).

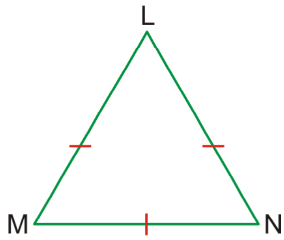


Figure 12.29

(b) On the bases of angles

Angles of a triangle can be acute angle, right angle or obtuse angle. On the bases of these we categorise the triangles.

12.8.4 Acute Angled Triangle

A triangle in which all the angles are acute angles is called an Acute angled Triangle. In figure 12.30 ΔABC is an Acute angled Triangle because it's all the angles $\angle ABC$, $\angle ACB$ and $\angle BAC$ are Acute angles.

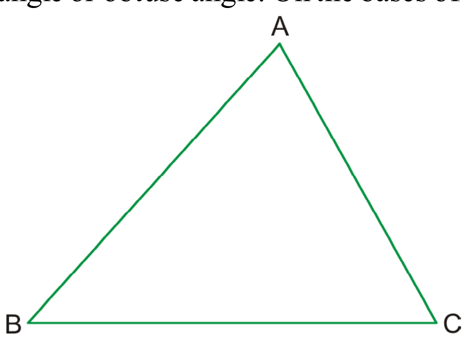


Figure 12.30

12.8.5 Right Triangle

A triangle in which one angle is Right angle is called a Right Triangle. In figure 12.31 ΔPQR is a Right Triangle because its one angle $\angle PRQ$ is a Right angle.

Observe carefully the sign used to show the Right angle.

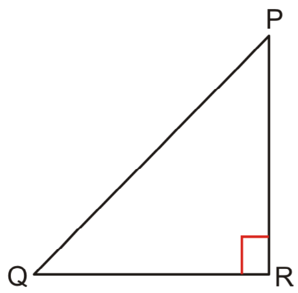


Figure 12.31

12.8.6 Obtuse Angled Triangle

A triangle in which one angle is an Obtuse angle is called an Obtuse Angled Triangle. In figure 12.32 ΔLMN is obtuse angled triangle since one angle $\angle MNL$ is an Obtuse angle.

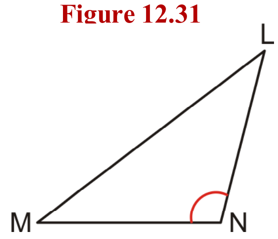


Figure 12.32

Intext Questions 12.5

1. On the bases of measures of sides categorise triangles given in figure 12.33:

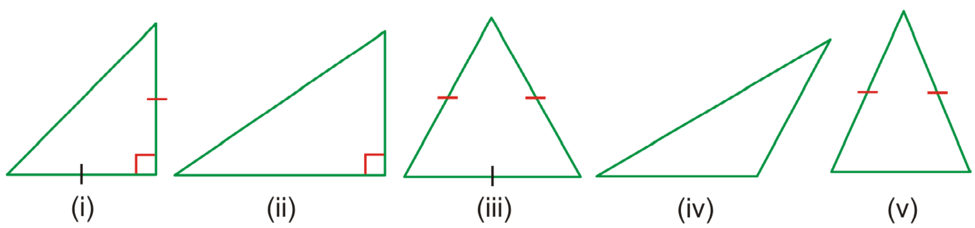


Figure 12.33

2. On the bases of measures of angles categorise the triangles drawn above.

3. How many triangles are there in the figure given below? Write their names and categorise them on the bases of angles.

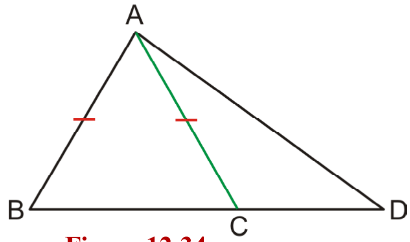


Figure 12.34

4. In figure 12.35 there are five triangles, measures of whose sides in centimetres have been written. On the bases of measures of the sides categorise the triangles as Scalene, Isosceles, and Equilateral triangle.

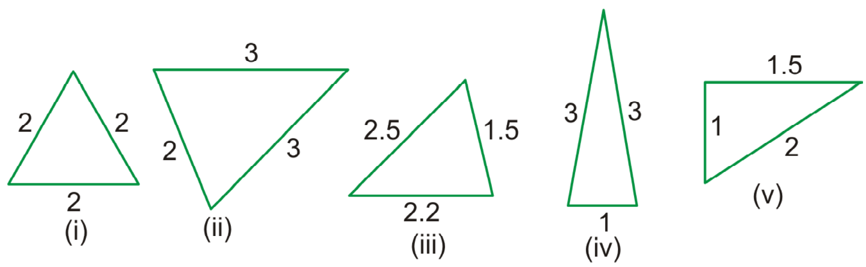


Figure 12.35

5. In figure 12.36 there are five triangles. Measures of some of the angles have been mentioned. Categorise the triangles in Acute angled triangle, Right triangle or Obtuse angled triangle also categorize as Scalene, Isosceles, and Equilateral triangle.

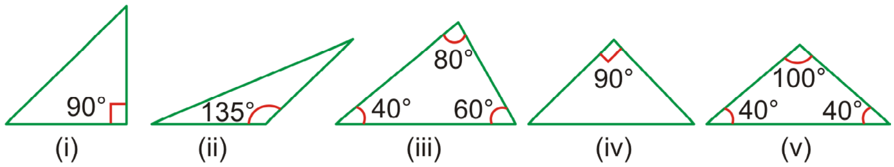


Figure 12.36

12.9 Properties of Isosceles Triangles

There are two very interesting properties of Isosceles Triangles:

(i) Angles opposite to equal sides are equal.

(ii) Sides opposite to equal angles are equal.

We will examine the truth of the statements in two ways-through experiment (by measuring) and by activity of paper folding.

When we look at any isosceles triangle, it seems that angles opposite to equal sides are equal. Actually the situation is like this. We will examine the truth of this property by doing the following experiment:

Experiment:

Draw a triangle ABC in which AB = AC = 7 cm and BC = 4 cm. Measure $\angle ABC$ and $\angle ACB$. Repeat the activity with other two isosceles triangles. Every time name the triangle as ΔABC and take AB = AC. Write your observations in the form of following table:

Serial No. of Triangle	$\angle$ABC	$\angle$ACB
1		
2		
3		

You will observe that in each situation $\angle ABC = \angle ACB$.

Conclusion: If any triangle two sides are equal then angles opposite to these sides will also be equal.

Note

Method of Paper Folding

Experiment: Draw a triangle ABC in which AB = AC = 7 cm and BC = 4 cm. Cut this triangle from the paper. Fold it in such a way that side AB falls on Side AC. When AB covers AC perfectly, then press the paper to get a crease. Open the paper and draw a line AD on the crease. Now fold the paper again along AD. Upon doing so you will observe that $\angle C$ has covered perfectly $\angle B$. Which means $\angle ABD = \angle ACD$ and $\angle ABC = \angle ACB$.

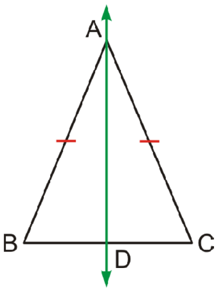


Figure 12.37

In Isosceles triangle sides opposite to equal angles are equal.

12.10 Property of Sides and Angles of Isosceles Triangles

Experiment: Draw a triangle ABC in which BC=6 cm and $\angle ABC = \angle ACB = 50°$. Measure sides AB and CD. Repeat the activity with other two triangles in which $\angle ABC = \angle ACB$. Write your observations in the form of following table:

Serial No. of Triangle	AB	AC
1		
2		
3		

You will observe that in each situation AB = AC.

Conclusion: If any triangle two angles are equal then sides opposite to these angles will also be equal.

Remark: You observed that the statements of two properties of Isosceles triangle are related to each other.

(i) In triangle ABC if AB = AC then $\angle ABC = \angle ACB$.

(ii) In triangle ABC if $\angle ABC = \angle ACB$ then AB = AC.

These statements involve 'if' and 'then'. Each statement has two parts. In statement (i) if we interchange the two parts then we get statement (ii). Similarly in statement (ii) if we interchange the two parts then we get statement (i). Such statements are called converse of each other. So statement (ii) is converse of statement (i) and statement (i) is converse of stamen (ii).

Method of Paper Folding

Draw a triangle ABC in which BC= 6 cm and $\angle ABC = \angle ACB = 50°$. Cut this triangle from the paper. Fold it in such a way that side $\angle C$ covers $\angle B$ and two parts of BC cover each other perfectly.

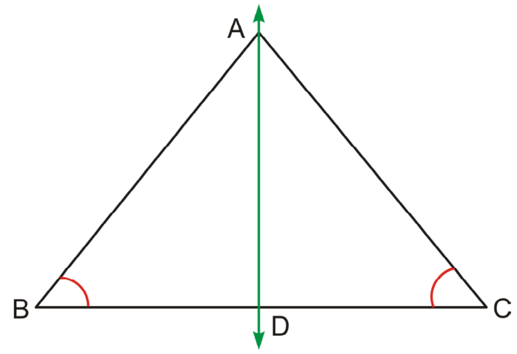


Figure 12.39

You will observe that CA and AB are covering each other perfectly. It shows that AB = AC.

If in any triangle two angles are equal then sides opposite to these angles are also equal.

Example 12.7: If in an Isosceles triangle ABC, AB = AC and $\angle BAC = 40°$. Bisectors of $\angle ABC$ and $\angle ACB$ meet at point O. Find the measure of the following:

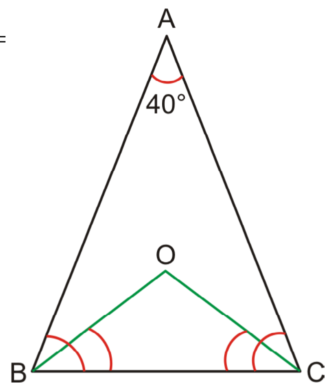


Figure 12.39

(i) $\angle ABC$

(ii) $\angle OBC$

(iii) $\angle BOC$

(iv) Is BO = CO? If yes, then why?

Solution:

(i) In ΔABC, AB = AC

$\therefore \angle ABC = \angle ACB$

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\therefore 2\angle ABC + 40° = 180°$

Or $2\angle ABC = 140°$

Or $\angle ABC = 70°$

Therefore $\angle ABC = 70°$

(ii) $\angle OBC = \frac{1}{2} \angle ABC$

$= \frac{1}{2} \times (70°) = 35°$

(iii) Similarly, $\angle OCB = 35°$.

Therefore $\angle BOC = 180° - (\angle OBC + \angle OCB)$

$= 180° - 70° = 110°$

(iv) In ΔOCB, $\angle OBC = \angle OCB$

$\therefore$ BO = OC

Intext Questions 12.6

1. In triangle ABC, AB = AC and $\angle BAC = 80°$. Find the measures of $\angle ABC$ and $\angle ACB$.
2. Triangle ABC is an Isosceles triangle in which AB = AC. If $\angle ABC = 50°$ then find the measure of $\angle BAC$.
3. In triangle PQR $\angle PQR = \angle PRQ = 50°$. Name the equal sides.
4. In triangle ABC if AB = BC then name the equal angles.

12.11 Property of Right Triangle (Pythagoras Theorem)

Draw a Right triangle PQR, in which $\angle Q = 90°$, hypotenuse PR=5 cm and side PQ = 3 cm. Measure the third side QR. Is its length 4 cm? Yes, it is like that.

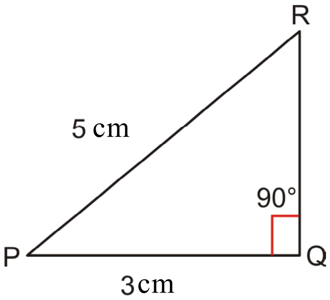


Figure 12.40

Now find $PQ^2 + QR^2$.

Here 32 + 42 = 25 = PR^2.

It means in this Right triangle,

square of the hypotenuse

= sum of the squares of the other two sides.

Repeat the activity by drawing Right triangle with different measures of sides. You will find that **in every Right Triangle, square of the hypotenuse is equal to the sum of the squares of the other two sides**. The result is generally called a Pythagoras Theorem. This result is called Bhaudhayan Pramey also.


Note

Intext Questions 12.7

1. Is ΔABC a Right Triangle if AB=13 cm, BC=5 cm and CA=12 cm? If yes, then which of its angle is a Right angle?

2. Which of the following can be sides of a Right Triangle?

   (a) 7 cm, 24 cm, 25 cm

   (b) 5 cm, 6 cm, and 8 cm

Let us Revise

- A figure which is formed by joining two points each out of three non-collinear points is called a Triangle.
- A triangle has three sides and three angles.
- Every triangle has three vertices and two exterior angles at each vertex.
- Every triangle has three altitudes and three medians.
- Sum of angles of triangle is 180°.
- In every triagle its ever exterior angle is equal to sum of it's interior opposite angles.
- Sum of any two sides of a triangle is greater than the third side.
- On the bases of measures of sides triangles are of following type (i) Scalene triangle, in which no two sides are equal.

  (ii) Isosceles triangle, in which any two sides are equal.

  (iii) Equilateral triangle, in which all three sides are equal.
- On the bases of measures of angles triangles are of following type

  (i) Acute angled triangle, in which all three angles are acute angles.

  (ii) Right triangle, in which one angle is Right angle.

  (iii) Obtuse angled triangle, in which one angle is an obtuse angle.
- Construction of Triangle (SSS, ASA, SAS and RHS constructions)
- Two properties of Isosceles tringle and their verification by experimentation
- In Isosceles triangle angles opposite to equal sides are equal.
- In Isosceles triangle sides opposite to equal angles are equal.
- In Right triangle square of the hypotenuse is equal to sum of the squares of other two sides. This result is called Pythagoras Theorem or Baudhayan Pramey.

- Solving questions using these properties.

Exercise

1. Fill in the blanks in the following to make the statements true:

   (a) Sum of angles of a triangle is ....

   (b) In every triangle an Exterior angle is .... sum of it's interior opposite angles.

   (c) In every triangle the three altitudes are ....

   (d) In every triangle sum of any two sides is .... the third side.

2. In a triangle two angles are 100° and 45°. Find the third angle.

3. In a triangle ratio between the angles is 2 : 3 : 4. Find the measure of the three angles.

4. In a triangle an exterior angle is 120° and it's interior opposite angles are equal. Find the measure of each of it's equal angles.

5. In figure 12.141 two angles have been shown, find the measure of $\angle ACX$.

6. In a triangle an exterior angle is 80° and ratio between it's interior opposite angles is 2:3. Find the measure of three angles of the triangle.

7. Draw one triangle each of the types Scalene triangle, Isosceles triangle and Equilateral triangle in your notebook.

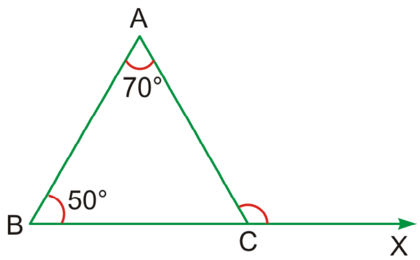


Figure 12.41

8. Can an equilateral triangle be taken as an Isosceles triangle?

9. In figure 12.42, how many triangles can you locate? Write the names of all of these. Write their type of triangle also viz Acute angled triangle, Right triangle or Obtuse angled triangle.

10. Write the definition of the following:

    (i) Scalene triangle

    (ii) Isosceles triangle

    (iii) Equilateral triangle

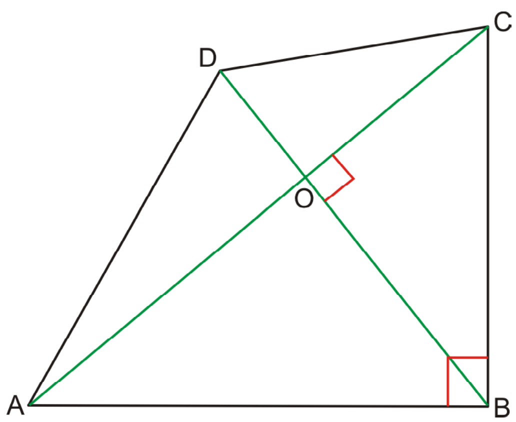


Figure12.42

11. Fill in the blanks:

    (i) In every triangle sides opposite to equal angles are …………..

    (ii) In a Right triangle square of the hypotenuse is ……….. to ………….. of the other two sides.

    (iii) In every triangle angles opposite to equal sides are …………

Answers

Intext Questions 12.1

1. (a) three (b) three (c) three
   (d) six (e) three (f) three
2. No, because triangle cannot be formed with three collinear points
3. Triangle
4. Exterior angle $\angle PQX$ and interior opposite angles $\angle P$, $\angle R$

Intext Questions 12.2

1. 50° 2. 60° 3. 50°, 50°

Intext Questions 12.3

1. 80°, 70° 2.90° 3. 50°, 50°, 80°

Intext Questions 12.4

1. No 2. Yes 3. No

Intext Questions 12.5

1. Scalene Triangle: ii, iv
   Equilateral Triangle: iii
   Isosceles Triangle: i, v
2. Acute angled Triangle: iii, v
   Right Triangle: i, ii
   Obtuse angled Triangle: iv
3. $\Delta ABC, \Delta ACD, \Delta ABD$ are three triangles.
   Acute angled Triangle: ΔABC

Right Triangle: ΔABD

Obtuse angled Triangle: ΔACD

4. Scalene Triangle: iii, v
   Isosceles Triangle: ii, iv
   Equilateral Triangle: i

5. Acute angled Triangle: iii
   Right Triangle: i, iv
   Obtuse angled Triangle: ii, v

Intext Questions 12.6

1. $\angle ABC = \angle ACB = 50°$
2. $\angle BAC = 80°$
3. PQ = PR
4. $\angle BAC = \angle BCA$

Intext Questions 12.7

1, Yes, $\angle C = 90°$ 2. (a) Yes (b) No

Exercise

1. (a) 180° (b) sum of (c) collinear (d) greater than
2. 35°
3. 40°, 60°, 80°
4. every angle 60°
5. 120°
6. 32°, 48°, 100°
8. Yes, we can say. Converse is not true.
9. ΔAOB, ΔBOC, ΔAOD, ΔCOD, ΔADB, ΔBCD, ΔACD and ΔABC
   Acute angled Triangle: ADB, BCD
   Right Triangle: BOC, AOD, COD, ABC, AOB
   Obtuse angled Triangle: ACD
11. (i) equal (ii) squares, equal (iii) equal

13

QUADRILATERALS AND ITS TYPES

Recall that in last lesson you have studied about the figure which is called Triangle. Triangle is a plane closed figure made up with three sides. It is worth noting that if you have only one or two line segments then closed figure can not be formed. Closed figures can be formed with three or more than three line segments only. In this lesson, we will discuss about a plane closed figure which is formed with four line segments or which has four sides.

Look at the four pegs fixed at four points on the floor. These points are such that no three of them are in a straight line.

Take a rope and move it around these four pegs in such a way that rope is properly stretched. In this way a simple closed figure is formed by the rope (figure 13.1). A simple closed figure by joining the four pegs with rope and all other such figures are called Quadrilaterals. This closed figure has four sides. If four points of the pegs are represented by A, B, C and D respectively then figure formed by joining these with stretched rope is called Quadrilateral ABCD.

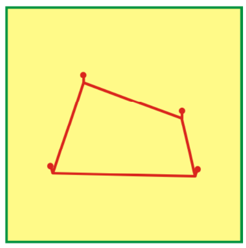


Figure 13.1

In our daily life at times we see such objects which are in the form of quadrilaterals. For example floor of the room, blackboard, surfaces of a dice, kite, fields, all are examples of quadrilaterals.

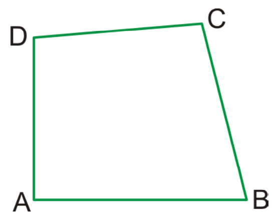


Figure 13.2

From this lesson, you will learn:

- different parts of quadrilateral
- sum of angles of a quadrilateral is 360°
- Special type of quadrilaterals, like Trapezium, Parallelogram, Rectangle, Square, Rhombus and Kite

13.1 Quadrilateral and its different parts

You observed that four sided plane closed figure is called a Quadrilateral.

In figure 13.3 look at the quadrilateral ABCD. it has four corners: A, B, C and D. These four points are called the vertices of the quadrilateral.

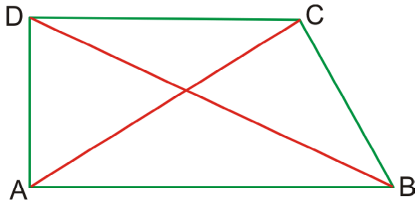


Figure 13.3

Quadrilateral is made of four line segments AB, BC, CD and DA. These are called four sides of the quadrilateral.

Quadrilateral ABCD has four angles. These angles are at $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle CDA$.

Line segment joining two opposite vertices is called a diagonal of the quadrilateral.

In figure 13.3 A and C, B and D are opposite vertices of the quadrilateral ABCD. So its two diagonals are AC and BD.

Quadrilateral ABCD has four sides AB, BC, CD and DA.

Look carefully at the sides AB and CD. Point B is common to both the sides. These two sides are known as adjacent sides. Which other sides of the quadrilateral will be adjacent?

These sides are BC, CD; CD, DA; DA, AB

Sides of the quadrilateral which are not adjacent are called opposite sides. In quadrilateral ABCD, AB and CD are opposite sides. Similarly BC and AD also are opposite sides.

Look at the figure 13.3 again. Here $\angle DAB$ and $\angle BCD$ are such angles that they do not have any arm common. Such angles are called opposite angles of the quadrilateral. You can say it like this also that

In quadrilateral angles formed at the opposite vertices are called opposite angles.

In quadrilateral ABCD which other angles are opposite angles? By looking carefully we come to know that in this quadrilateral $\angle DAB$ and $\angle ABC$ are opposite angles.

Now look at $\angle DAB$ and $\angle ABC$ of the quadrilateral. Both the angles are formed on side AB. These are called adjacent angles of the quadrilateral. Similarly, in quadrilateral ABCD, $\angle ADC$ and $\angle BCD$ are its adjacent angles. It has two more pairs of adjacent angles. Write these also.

Like triangle, a quadrilateral also divides its surface in three parts.

Note

(i) its interior, (ii) its exterior and (iii) the quadrilateral itself.

Intext Questions 13.1

1. In figure 13.4, in quadrilateral ABCD write

   (a) all the sides

   (b) all the vertices

   (c) all the angles

   (d) all the diagonals

   (e) all the pairs of adjacent sides

   (f) all the pairs of opposite sides

   (g) all the pairs of opposite angles

   (h) all the pairs of adjacent angles

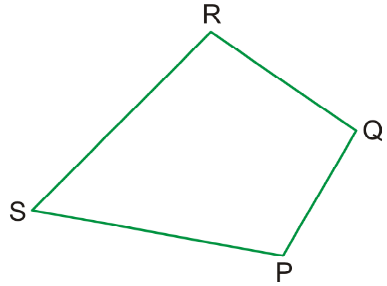


Figure 13.4

2. In any quadrilateral how many sides and how many angles are there? How many vertices and how many diagonals it has? How many pairs of adjacent sides and how many pairs of opposite sides it has? How many pairs of opposite angles are there and how many pairs of adjacent angles are there?

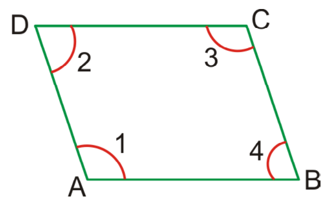


Figure 13.5

13.2 Sum of angles of a Quadrilateral

Look at the figure 13.5 carefully. It has four angles. On the same pattern draw a quadrilateral on your piece of paper. Cut it's all the four angles and place them adjacent to each other as shown in figure 13.6.

What do you get? Sum of the four angles is 360°, because these four angles complete one revolution. In this way we see that

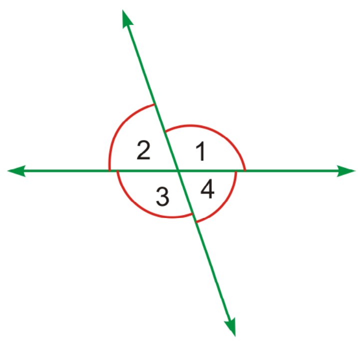


Figure 13.6

Sum of the angles of a quadrilateral is 360°.

You can understand the concept in this way also:

Draw a diagonal AC of quadrilateral ABCD. You get two triangles ABC and ACD.

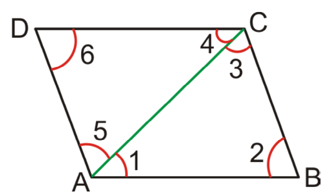


Figure 13.7

In the last lesson you have learnt that

$\angle 1 = \angle 2 = \angle 3 = 180°$ ...(1)

Similarly $\angle 4 = \angle 5 = \angle 6 = 180°$ ...(2)

Now $\angle 1 = \angle 5 = \angle DAB$

And $\angle 3 = \angle 4 = \angle BCD$

By adding (1) and (2) we get

$(\angle 1 = \angle 5) + \angle 2 + (\angle 3 + \angle 4) + \angle 6 = 180° + 180°$

Or $\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$

Therefore Sum of angles of a quadrilateral is 360°.

Intext Questions 13.2

1. Draw diagonal BD in quadrilateral ABCD (figure 13.8) and verify that

   $\angle A + \angle B + \angle C + \angle D = 360°$.

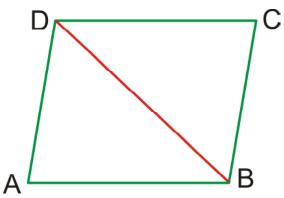


Figure 13.8

13.3 Trapezium, Parallelogram, Rectangle, Square, Rhombus, Kite

In this section you will be introduced with some special quadrilaterals.

13.3.1 Trapezium

Look carefully at the quadrilateral ABCD in figure 13.9. Its two opposite sides AB and CD are parallel. Perpendicular distance between these always remains same. Such quadrilateral is called Trapezium 'quadrilateral with equal altitudes'. Note that the other two opposite sides in a Trapezium may or may not be parallel.

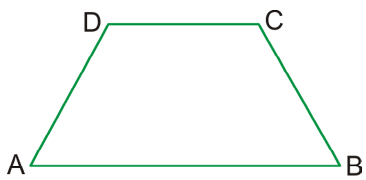


Figure 13.9

3.3.2 Parallelogram

In figure 13.10, ABCD is a quadrilateral in which all the pairs of opposite sides are parallel. It is called a Parallelogram.

Quadrilaterals in which all the opposite sides are parallel are Parallelograms. In Parallelogram opposite angles are also equal.

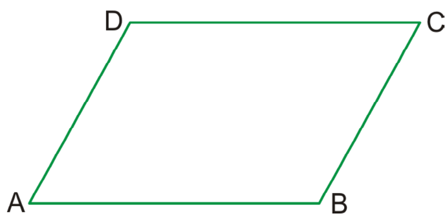


Figure 13.10

Note that not all trapeziums are parallelograms, but all parallelograms are trapeziums.

13.3.3 Rectangle

In figure 13.11, ABCD is such a quadrilateral in which all the angles are right angles. If you look carefully, you will

Figure 13.11

observe that sides AB, CD and AD, BC are parallel and equal. This quadrilateral is called a Rectangle.

A quadrilateral in which all the angles are right angles is called a Rectangle.

It is worth noting that in rectangle opposite sides are parallel and equal, and its all angles are equal.

Diagonals of a rectangle are equal and bisect each other.

13.3.4 Square

In figure 13.12 look at the quadrilateral PQRS. It is a rectangle, why? It has one more speciality. Its all sides are also equal.

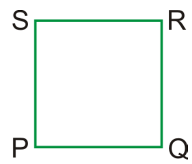


Figure 13.12

A quadrilateral in which all the sides are equal and all the angles are right angles is called a Square.

Note that a square is always a rectangle. But not all rectangles are squares.

Diagonals of a square are equal and bisect each other at right angle.

13.3.5 Rhombus

In figure 13.13 quadrilateral PQRS is such that it's all sides are equal but it's angles are not right angles. These can be right angles also. Such quadrilateral is called a Rhombus.

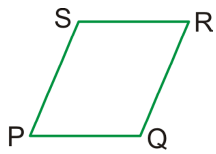


Figure 13.13

A quadrilateral with all sides equal is called a Rhombus.

Remember, every square is a rhombus, but a rhombus may or may not be a square. In this way a square is a rectangle as well as a rhombus.

Remark: Opposite sides of a rhombus are parallel.

Diagonals of a rhombus bisect each other at right angles.

13.3.6 Kite

In figure 13.14 quadrilateral PQRS is such that its adjacent sides PQ and PS are equal, and RS and RQ are equal.

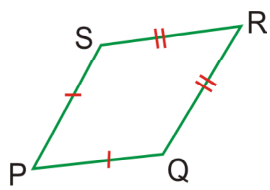


Figure 13.14

It is called a Kite. Note that every rhombus is a kite but kite may not be a rhombus.

Intext Questions 13.3

1. (a) Is every rhombus a parallelogram?
   (b) Is every rectangle a square?
   (c) Is every square a rhombus?

(d) Is every parallelogram a square?

(e) Draw a trapezium which is not a parallelogram.

(f) Draw a parallelogram which is not a rectangle.

(g) Draw a rhombus which is not a square.

(h) Draw a kite which is not a rhombus.

Let us Revise

- Four sided closed figure is called a quadrilateral.
- Quadrilateral has four angles, four vertices and two diagonals.
- Sum of all the four angles of a quadrilateral is 360°.
- Square, rectangle, parallelogram, rhombus, kite, trapezium are quadrilaterals.
- Diagonals of a rectangle are equal and bisect each other.
- Diagonals of a square are equal and bisect each other at right angle.
- In parallelogram opposite sides are equal and opposite angles are equal.
- In rhombus diagonals bisect each other at right angle.

Exercise

1. Answer the following questions with reasons:

   (a) Is square a rectangle?

   (b) Is square a parallelogram?

   (c) Is every rhombus a square?

   (d) Is every parallelogram a square?

   (e) Under what conditions trapezium will become a parallelogram?

   (f) Under what condition a parallelogram will become a rectangle?

2. What is the sum of all the four angles of a quadrilateral?

3. Draw a square whose area is 9 sq. cm.

4. Fill in the blanks:

   (a) In a parallelogram opposite sides are ....

   (b) In a parallelogram opposite angles are ....

   (c) If one diagonal of a rectangle is 12 cm then its other diagonal will be ....

Answers

Intext Questions 13.1

1. (a) PQ, QR, RS, SP
   (b) P, Q, R, S
   (c) $\angle SPQ, \angle PQR, \angle QRS, \angle RSP$
   (d) PR, SQ
   (e) SP, PQ; PQ, QR; QR, RS; RS, SP
   (f) SP, QR; PQ, RS
   (g) $\angle SPQ, \angle QRS, \angle RSP, \angle PQR$
   (h) $\angle RSP, \angle SPQ, \angle SPQ, \angle PQR, \angle PQR, \angle QRS, \angle QRS, \angle RSP$
2. 4 sides, 4 angles, 4 vertices, 2 diagonals, 4 pairs of adjacent sides,
   2 pairs of opposite sides, 2 pairs of opposite angles and 4 pairs of adjacent angles.

Intext Questions 13.3

1. (a) Yes
   (b) No
   (c) Yes
   (d) No

Exercise

1. (a) Yes, because its every angle is 90°.
   (b) Yes, because it's all the sides are equal.
   (c) No, it's angle may be different from 90°.
   (d) No, it's angle may be different from 90°.
   (e) When it's other pair of opposite sides also is parallel.
   (f) When it's one angle is 90°.
2. 360°
4. (a) equal
   (b) equal
   (c) 12

14

CIRCLE

You might have seen people of your village taking their animals in the morning outside the village for grazing. They tie their animals with rope and tie the rope with the peg fixed on the ground.

You must be getting surprised that even though they are tied yet they are able to graze. How much area the animals are able to graze?

Can you say something about the shape of the region which they are grazing?

If you look carefully you will observe that rope allows these animals to move in a circular region (figure 14.1). You will observe that boundary of that region will be available only when they move with the stretched rope.

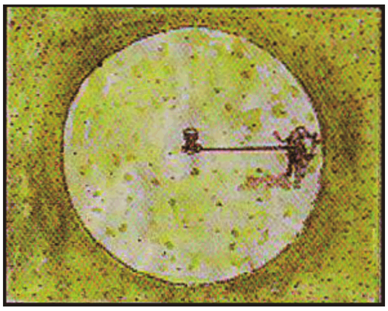


Figure 14.1

So you can say that the region which the animal can graze to the maximum will be circular.

In day-to-day life you come across many objects which are circular. For example, sun, full moon, plates, edge of the upper portion of a cup, different coins (like 5, 1, 50 paise, 25 paise coin), wheels of car and wheels of cycle.

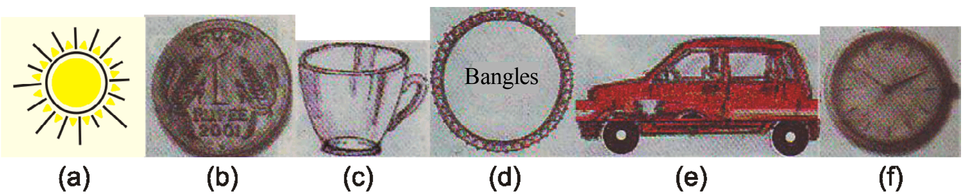


Figure 14.2

From this lesson, you will learn:

- Different parts of circle and its elements
- Drawing a circle of given radius
- Angle formed in a semicircle is 90°

14.1 Parts and elements of circle

If you look at the figure 14.3 carefully, you will find that distance of every point on the boundary from the nail is same. So we can say that Circle is a collection of all those points in a plane which are equidistant from a fixed point (here it is the peg).

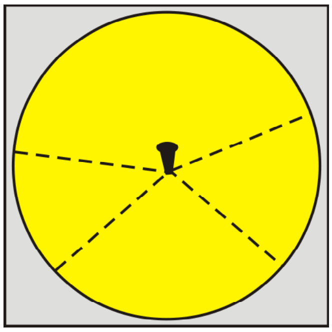


Figure 14.3

Let us represent the fixed point (where peg is fixed) by O and take four points A, B, C and D on the boundary. Then OA = OB = OC = OD.

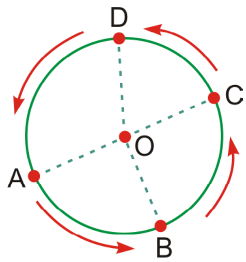


Figure 14.4

Measure of the boundary is called circumference of the circle and constant distance is called the radius. So radius 'r = OA = OB = OC = OD'. In other words, perimeter of circle is called its circumference. Generally it is represented by C, if you start moving from point A (figure 14.4) and reaches at point B by moving along the boundary, then at point C, then at point D and in the end at point A again, then distance covered is same as circumference of the circle.

Distance between the centre of the circle and any point on the circle is called the radius of the circle.

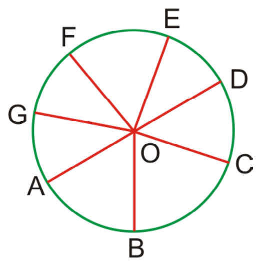


Figure 14.5

Generally radius of the circle is represented by 'r'. In figure 14.4, O is the centre of the circle and OA, OB, OC and OD are the radii of the circle. A circle can have any number of radii, but all radii are of same length. In figure 14.5 you can verify by measuring that all the radii have same length.

Now, take two points A and B on the circle (figure 14.6). If you join these points then you will get a line segment AB.

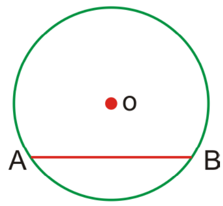


Figure 14.6

Line segment joining two points on a circle is called a Chord of the circle.

In figure 14.6 AB is a chord of the circle. As there are infinite points on the circle, so there can be infinite chords of the circle. In figure 14.7, AB, CD, EF, GH and PQ are chords of the circle. But chords EF and GH are of special type. Can you explain in what manner these are different from other chords? Chords GH and EF pass through O the centre of the circle. You can verify that chords EF and GH are longest chords. All other chords which do not pass through O are of shorter length.

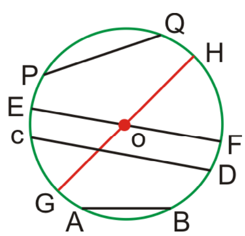


Figure 14.7

Chord passing through centre of the circle is called a Diameter.

Therefore diameter is the longest chord of the circle. You can verify that length of a diameter is double of the radius. Therefore **diameter = 2 x radius**. In figure 14.7, EF and GH are two diameters of the circle. Here EF = GH = 2 GO = 2r. As shown in figure 14.8, a circle can have infinite diameters, like AB, CD, EF, GH and PQ are diameters of the circle with centre O.

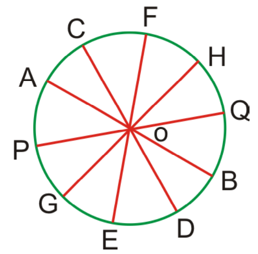


Figure 14.8

Some part of the circle is called its Arc.

In fig 14.9 part of the circle shown in dark line ABC is an arc of the circle. Similarly CDEA also is an arc of the circle. Smaller arcs CD, DE, EA and DEA and those shown in dark AB and BC lines are arcs of the circle. An arc is generally shown by $\frown$. Therefore, Arc ABC and Arc CDEA are written

as $\overparen{ABC}$ and $\overparen{CDEA}$.

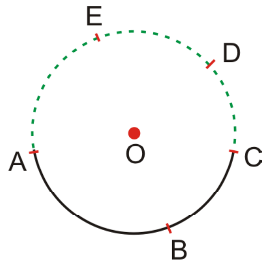


Figure 14.9

Now look at arcs ACB and ADB in figure 14.10. What do you observe? Are these of some special type? If you join their end points A and B then it passes through O, the centre of the circle. We have already said that chord passing through the centre of the circle is called a diameter. So AB is the diameter of the circle (see figure 14.10).

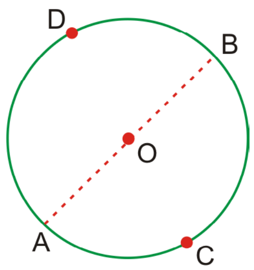


Figure 14.10

An arc whose end points are the end points of a diameter is called a semicircle. So in figure 14.10, ACB and ADB are semicircles. Like each closed figure in a plane (triangle, quadrilateral) a circle also divides its plane in three pars (see figure 14.11).

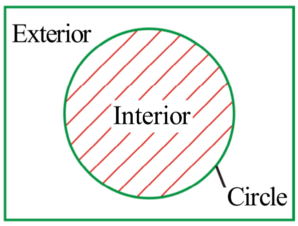


Figure 14.11

(i) its interior,

(ii) its exterior and

(iii) circle itself.

The region by joining interior and circle is called **Circular region**.

If you look at the diameter AB in figure 14.10 then you will see that it divides the circular region in two equal parts. Each part is called semi-circular region.

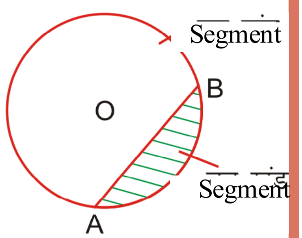


Figure14.12

In other words, you may say that a semi-circular region is enclosed by a diameter and an arc whose end points are end points of the diameter. In figure 14.10, ACBOA and AOBDA are two semi-circular regions.

Take a circle and draw a chord AB (figure 14.12). This chord divides the circular region in two parts. Each part is called a **Segment of the circle**. In the figure one part is shaded and the other is unshaded.

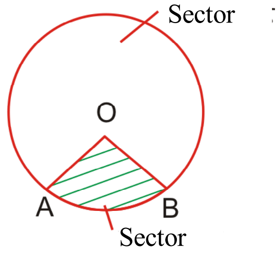


Figure 14.13

Now take a circle and draw its two radii OA and OB (figure 14.13). Radii OA and OB also divide the circular region in two parts. Each part is called a **Sector**.

Example 14.1: In the figures given below write the name of centre, radius, diameter, chord, arc, semi-circle and semi-circular region:

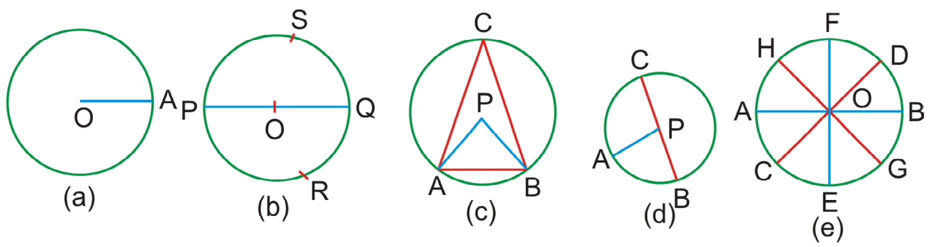


Figure 14.14

Solution:

(a) Centre is O and radius is OA.

(b) Centre is O and OP and OQ are the radii and, PQ is the diameter. PS, PR, QS, SR, RPS and RQS are the arcs. Two semi-circles are PRQ and PSQ. PRQOP and POQSP are two semi-circular regions.

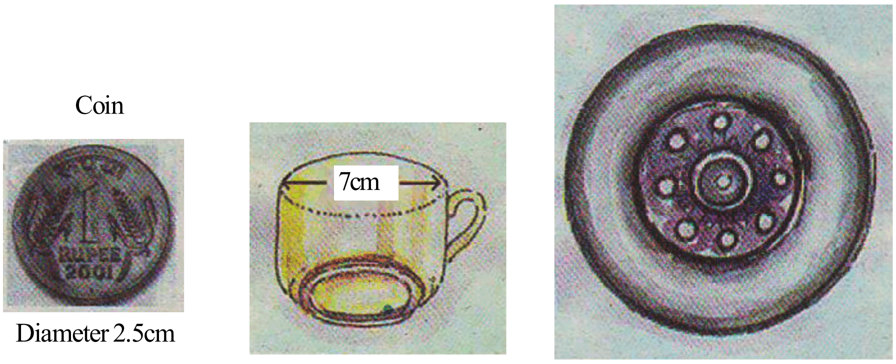
(c) P is the centre and PA and PB are the radii. AB, BC and AC are the chords. $\overset{\frown}{AC}$, $\overset{\frown}{AB}$, $\overset{\frown}{CA}$, $\overset{\frown}{CAB}$, $\overset{\frown}{ABC}$ and $\overset{\frown}{BCA}$ are the arcs.

(d) P is the centre, AP, BP and CP are the radii and BC is the diameter. $\overset{\frown}{AC}$, $\overset{\frown}{AB}$, $\overset{\frown}{ACB}$ are the arcs and BAC is a semi-circle. BACPB is a semi-circular region.

(e) O is the centre. AB, CD, EF and GH are the diameters.

Example 14.2: Calculate the radius in the following figures:

Note



Coin — Diameter 2.5cm (a) | (b) | Wheel — Diameter 1.4cm (c)

Figure 14.15

Solution:

(a) Diameter = 2 × radius

$\therefore$ Radius = diameter/2

Diameter of the coin is 2.5 cm

$\therefore$ Radius = 2.5/2 cm = 1.25 cm

(b) We know that diameter = 2 × radius

$\therefore$ Radius = diameter/2

Therefore radius of the upper portion = 7/2 cm = 3.5 cm

(c) Diameter of the wheel = 1.4 m

Therefore radius of the wheel = 1.4/2m = 0.7 m

14.2 Drawing a Circle

A Geometrical instrument which is used to draw a circle is called a pair of compasses.

By keeping the pointed side of compasses at the point O on a paper and revolving its pencil end, we get a circle as shown in figure 14.16

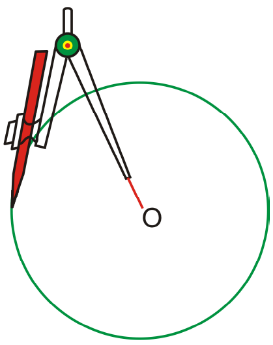


Figure 14.16

14.2.1 Drawing a circle of given radius

Assume that you have to draw a circle of radius 5 cm. For that follow the steps given below:

Step 1: With the help of ruler draw 5 cm long line segment (figure 14.17 (a)).

Step 2: Mark a point O on the paper.

Step 3: Open the compasses keeping the distance between its pointed end and pencil end is 5 cm (figure 14.17 (b)).

Step 4: Place the pointed end of the compasses at O.

Step 5: Rotate the pencil end around O. In figure 14.17 (c) a circle with radius 5 cm has been drawn.

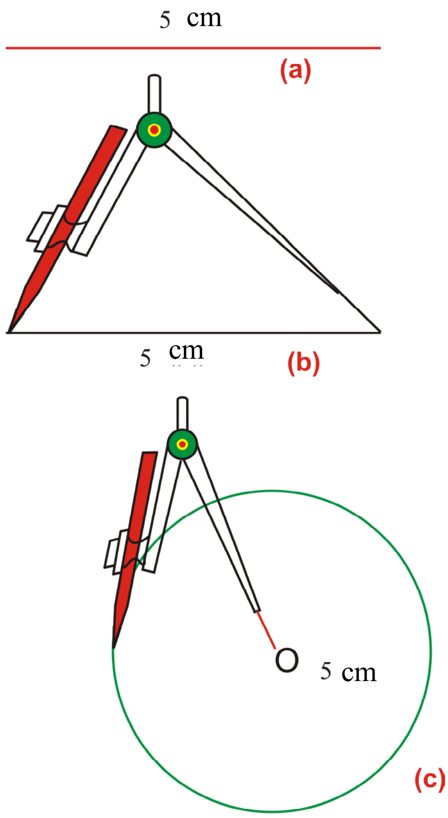


Figure 14.17

Intext Questions 14.1

1. In figure 14.18 find the centre, radius, diameter,
   chord, arc, semi-circle and semi-circular region:

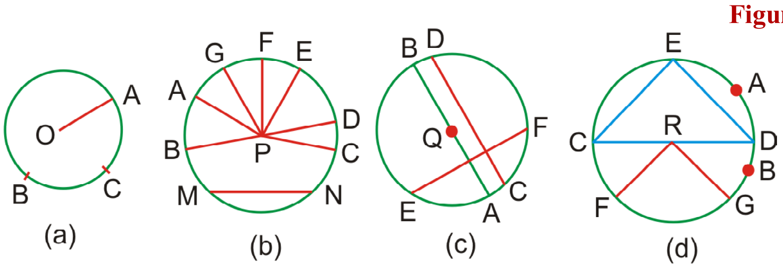
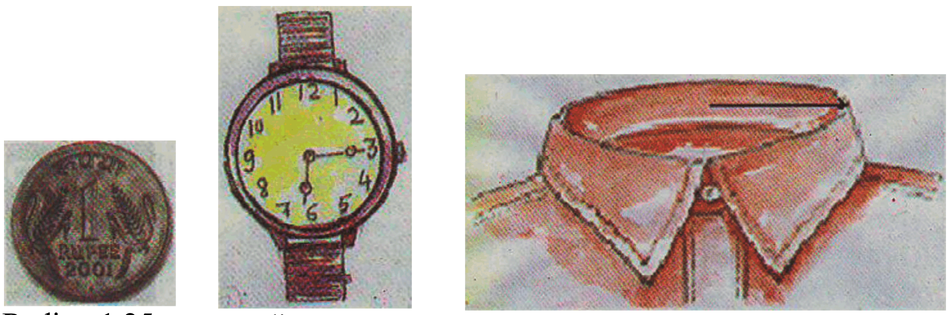


Figure 14.18

2. In the following circles find the length of the diameter:



Radius :1.25cm (a) Radius : 4 cm (b) Radius:7.2 cm (c)

Figure 14.19

Note

14.3 Measure of an angle in a semi-circle

Draw a circle with centre O and AB is its diameter as shown in figure 14.20. Take two points C and D on a semicircle. Draw DA, DB and CA, CB to get $\angle ADB$ and $\angle ACB$ respectively.

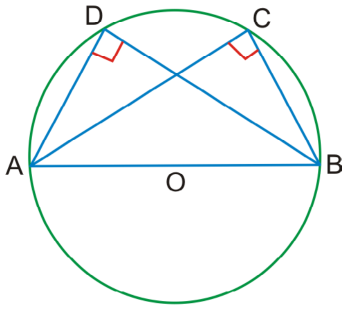


Figure 14.20

Now measure these angles with the help of protector (You have learnt to measure an angle with the help of protector in chapter 11). What did you get? You will observe that measure of both the angles is 90° which means each angle is a right angle.

So $\angle ADB = 90° = \angle ACB$.

If you take more angles in a semicircle as shown in figure 14.21, then you will find that measure of each angle is 90°.

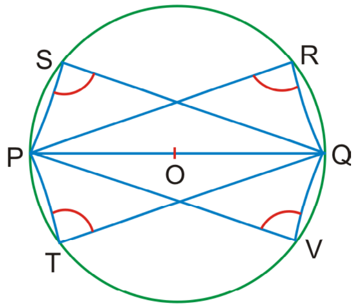


Figure 14.21

$\therefore \angle PRQ = \angle PSQ = \angle PTQ = \angle PVQ = 90°$

So we conclude that every angle in a semicircle is a right angle.

Angle in a semicircle is a right angle.

14.4 Distance of chords from the centre

Draw a circle with centre O and draw any two chords AB and CD of the circle (figure 14.22).

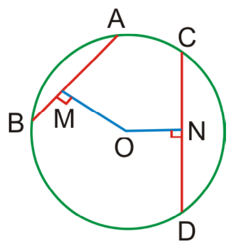


Figure 14.22

Draw perpendiculars OM and ON from O on AB and CD.

Measure AB, CD, OM and ON.

What did you observe?

You found that AB < CD and OM > ON. Which means **In a circle longer chord is nearer to the centre.**

Let us Revise

- Circle is a collection of points in a plane which are equidistant from a fixed point.
- Perimeter of a circle is called its circumference.
- Distance between the centre of a circle and any point on the circle is called the radius of the circle.
- Circle can have infinite radii.

- Line segment joining two points on the circle is called a chord of the circle.
- Chord passing through the centre of a circle is called a diameter.
- Every diameter is the longest chord of the circle.
- Circle can have infinite diameters.
- Diameter = 2 x radius
- Part of a circle is called an arc.
- End points of a diameter divide the circle in two equal parts. Each part is called a semicircle.
- Angle in a semicircle is 90°.
- Longer chord is nearer to the centre.

Exercise

1. Fill in the following blanks making the statements true:

   (a) Circle has ………….. centre.

   (b) Diameter is the ……………. chord of a circle.

   (c) Diameter of a circle is …………… of radius of the circle.

   (d) Line segment joining two points on a circle is called …………

   (e) Chord passing through the centre of a circle is called ………..

   (f) Every angle in a semicircle is ………..

2. Diameters of different types of Hats are as under:

   (a) 18 cm

   (b) 21 cm

   (c) 24 cm

   Find the radius of each Hat.

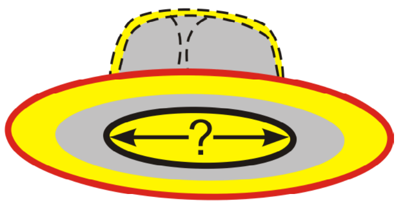


Figure 14.23

3. Draw circles with the following radii.

   (i) 4 cm

   (ii) 6 cm

4. DE and PQ are two chords of a circle, DE = 8 cm and PQ = 6 cm. Which chord is at more distance from the centre?

Answers

Intext Questions 14.1

1. (a) Centre: O ; radius: OA ; arc: $\overset{\frown}{BC}$, $\overset{\frown}{CA}$, $\overset{\frown}{BAC}$, $\overset{\frown}{ACB}$

   (b) Centre: P; radius: PA, PB, PC, PD, PE, PF and PG; chord: MN; arc: $\overset{\frown}{BAG}$, $\overset{\frown}{NCDE}$ etc.

   (c) Centre: Q; diameter: AB; chord: CD, EF; radius: QA, QB; arc: $\overset{\frown}{EAC}$, $\overset{\frown}{ACF}$ etc,

   (d) Centre: R; radius: RF, RG, RD, RC; diameter: CD; chord: CE, DE; arc: $\overset{\frown}{BDA}$ etc. semicircle: CFGBD, DAEC; semi-circular region: CFGBDRC, CEADRC.

2. (a) 2.5 cm

   (b) 8 cm

   (c) 14.4 cm

Exercise

1. (a) only one

   (b) longest

   (c) double

   (d) chord

   (e) diameter

   (f) right angle

2. (a) 9 cm

   (b) 10.5 cm

   (c) 12 cm

4. PQ

15

CONGRUENT AND SYMMETRIC FIGURES

In our day-to-day life, we come across with many things, whose shape and size are same. For example Blades of a same company, Biscuits of the same brand etc. Things of this type are called Congruent. We come across with figures as given in figure 15.1 also.

(i) Picture of TajMahal

(ii) Picture of a Butterfly

Figure 15.1

We can fold these along a line, in such a manner that one part of the figure covers completely the other part. Such figures are called **Symmetric figures.**

From this lesson, you will learn:

- About Congruent figures
- About conditions of Congruency of triangles like SSS, SAS, ASA and RHS
- Symmetric figures, especially figures with linear symmetry
- About axis of symmetry of symmetric figures

15.1 Congruency

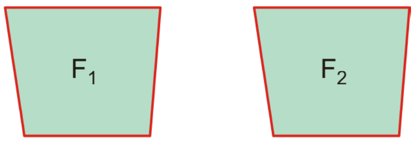


Figure 15.2

Look at the shapes F_1 and F_2 given in figure 15.2, from these cut F_1 and try to place it on F_2. You will observe that F_1 covers F_2 completely. It means shape and size of the two shapes are same. In other words, F_1 and F_2 are congruent.

This method of examining congruency is called method of **Super-position.** By Super-position you can verify that shapes given in figure 15.3 (i) are not congruent, but shapes given in figure 15.3 (ii) are congruent. To represent the congruency we use the symbol $(\cong)$. Here $F_1 \cong F_2$ and $B_1 \cong B_2$.

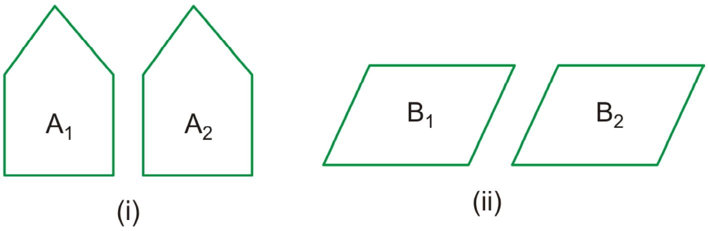


Figure 15.3

15.2 Congruency of Triangles

Think about the two triangles ABC and DEF given in figure 15.4. Cut and remove the triangle DEF and try to place it over the triangle ABC. You will observe that triangle DEF has covered triangle ABC completely, when vertex D lies on vertex A, vertex E lies on vertex B and vertex F lies on vertex C (figure 15.5).

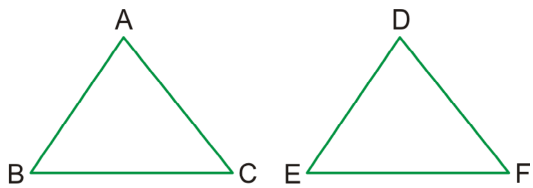


Figure 15.4

We say that with the correspondence $\Delta ABC \leftrightarrow \Delta DEF$, triangle ABC is congruent to triangle DEF. Symbolically, we write it as $\Delta ABC \cong \Delta DEF$. Writing it as $\Delta ABC \cong \Delta EDF$ or $\Delta ABC \cong \Delta FDE$ will not be appropriate. In figure 15.5 you can see that AB = DE, BC = EF, CA = FD, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

In correspondence $ABC \leftrightarrow DEF$, AB and DE are the corresponding sides of the two triangles. Similarly BC and EF are corresponding sides, $\angle B$ and $\angle E$ are corresponding angles etc. it means all the six corresponding elements (parts) of triangles ABC and DEF are mutually equal.

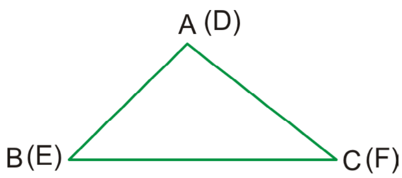


Figure 15.5

15.3 Conditions of Congruency of Triangles

For verifying congruency of two triangles it is not always necessary to verify equality of its six elements (3 sides and 3 angles) or parts. In reference to congruency of two triangles, we can verify their congruency by the following rules for which their only three corresponding elements are required:

Note

(i) Side-Side-Side (SSS) congruency Rule: Draw two triangles ABC and DEF in such a way that AB=DE=7 cm, BC=EF=5 cm and CA=FD=4 cm (figure 15.6). In the two triangles three sides of one triangle are equal to the corresponding sides of the other triangle. Now cut off ΔDEF and try to place it over ΔABC. You will find that with the correspondence ABC$\leftrightarrow$DEF triangle DEF covers the triangle ABC completely, so $\Delta ABC \cong$ DEF. If we draw such more pairs of triangles even then we will get the same result. Therefore, **if in any two triangles, three sides of one triangle are equal to three corresponding sides of the other triangle then the two triangles are congruent.** It is called **Side-Side-Side (SSS) congruency Rule.**

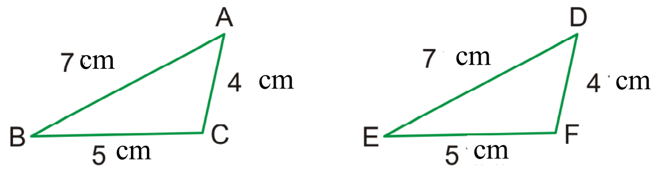


Figure 15.6

(ii) Side-Angle-Side (SAS) congruency Rule: Draw two triangles ABC and DEF in such a way that AB=PQ=6 cm, $\angle A = \angle P$ =50°and AC=PR=5cm (figure 15.7). Here two sides and an included angle of one triangle are equal to two corresponding sides and their included angle respectively. Now cut off ΔPQR and try to place it over ΔABC. You will find that with the correspondence $\Delta ABC \leftrightarrow \Delta PQR$ triangle ΔPQR covers the triangle ABC completely, so $\Delta ABC \cong \Delta PQR$. If we draw such more pairs of triangles even then we will get the same result. Therefore, **if in any two triangles, two sides and their included angle of one triangle are equal to two corresponding sides and their included angle of the other triangle respectively then the two triangles are congruent.It is calledSide-Angle-Side (SAS) congruency Rule.**

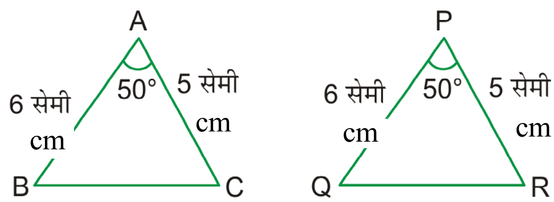


Figure 15.7

(iii) Angle-Side-Angle (ASA) congruency Rule: Draw two triangles PQR and DEF in such a way that QR = EF = 5 cm, $\angle Q = \angle E$ = 50° and $\angle R = \angle F = 60°$ (figure 15.8). Here two angles and the included side of one triangle are equal to two corresponding angles and their included sides respectively. Now cut off ΔDEF and try to place it over ΔPQR. You will find that with the correspondence $PQR \leftrightarrow DEF$ triangle DEF covers the triangle PQR completely, so $\Delta PQR \cong \Delta DEF$. If we draw such more pairs of triangles even then we will get the same result. Therefore, **if in any two triangles, two angles and their included side of one triangle are equal to two corresponding angles and their included sideof the other triangle respectively, then the two triangles are congruent.It is called Angle-Side-Angle (ASA) congruency Rule.**

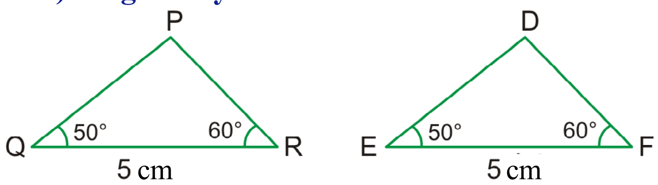


Figure 15.8

(iv) **Right angle-Hypotenuse-Side (RHS) congruency Rule:** Draw two right angled triangles PQR and XYZ in such a way that $\angle Q = \angle Y = 90°$, hypotenuse PR = hypotenuse XZ=6 cm and side QR= side YZ = 4 cm (figure 15.9). Here hypotenuse and one side of a right angled triangle are equal to the hypotenuse and one side of the otherrespectively. Now cut off ΔXYZ and try to place it over ΔPQR. You will find that with the correspondence $\Delta PQR \leftrightarrow \Delta XYZ$ triangle, XYZ covers the triangle PQR completely, so $\Delta PQR \cong \Delta XYZ$. If we draw such more pairs of right angled triangles even then we will get the same result. Therefore, **if in two right angled triangles, hypotenuse and one side of one triangle are equal to hypotenuse and one side of the other trianglerespectively then the two triangles are congruent. It is called Right angle-Hypotenuse-Side (RHS) congruency Rule.**

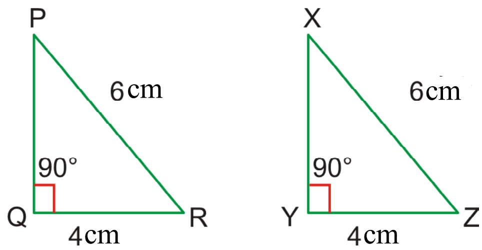


Figure 15.9

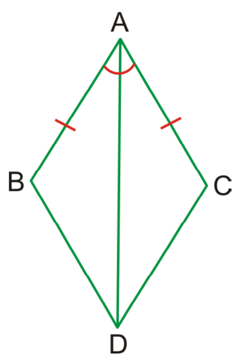


Figure 15.10

Example 15.1: In figure 15.10, AB = AC and $\angle BAD = \angle CAD$. Is $\Delta ABD \cong \Delta ACD$? If yes, under which rule and why?

Solution: Yes, because in ΔABC and ΔACD, AB=AC (given), $\angle BAD = \angle CAD$ (given) and AD = AD (common side)

Therefore by SAS rule, $\Delta ABD \cong \Delta ACD$.

Example 15.2: Look at figure 15.11, where pairs of equal parts of triangles have been shown by same signs. Find out by which rule they are congruent. Also write it symbolically.

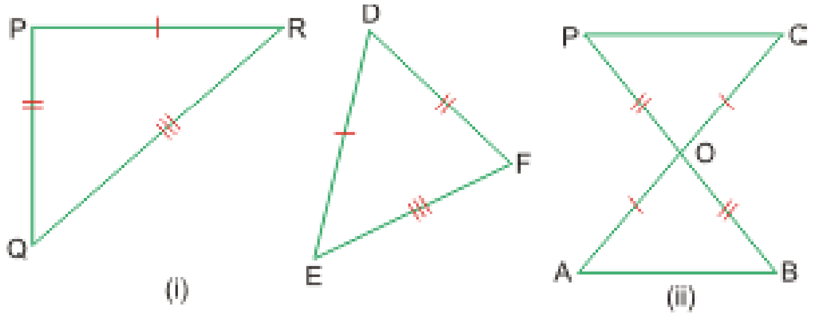


Figure 15.11

Solution: In (i) PR= DE, PQ = DF and QR = EF. Therefore correspondence is $PQR \leftrightarrow DFE$. Therefore by applying SSS Congruency rule $\Delta PQR \cong \Delta DFE$.

In (ii) AO = QO, BO = PO and $\angle AOB = \angle QOP$ (vertically opposite angles). Therefore, by applying SAS Congruency rule? $\Delta AOB \cong \Delta QOP$.

Intext Questions 15.1

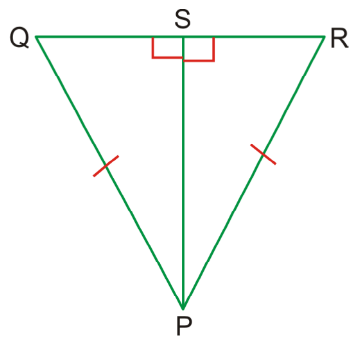


Figure 15.12

1. Fill in the blanks:

   If $\Delta ABC \cong \Delta QPR$

   (i) AB = ... (ii) BC = ...

   (iii) $\angle C$ = ...

2. In figure 15.12,

   $PQ = PR$ and $PS \perp QR$.

   Is $\Delta PSQ \cong \Delta PSR$?

If yes, under which rule?

15.4 Symmetry

Look at the shapes given in figure 15.13.

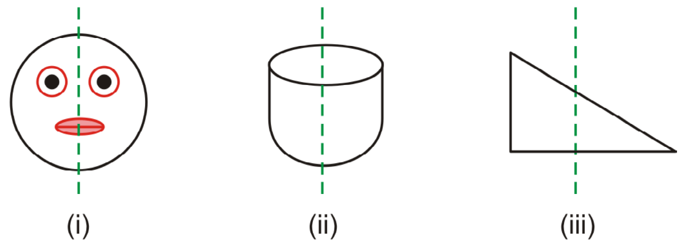


Figure 15.13

If you fold shape (i) along the dotted line then one part of the shape covers completely the other part. Therefore shape (i) is symmetrical with respect to the dotted line.

Dotted line is its axis of symmetry or line of symmetry. Similarly shape (ii) also is symmetrical with respect to the dotted line, but shape (iii) is not symmetrical because we are not able to find a line along which if we fold the shape and one part covers completely the other part. Such shapes are called **asymmetrical shapes.**

15.5 Number of Axis or Lines of symmetry

In different shapes, number of axis of symmetry or lines of symmetry may be different (figure 15.14).

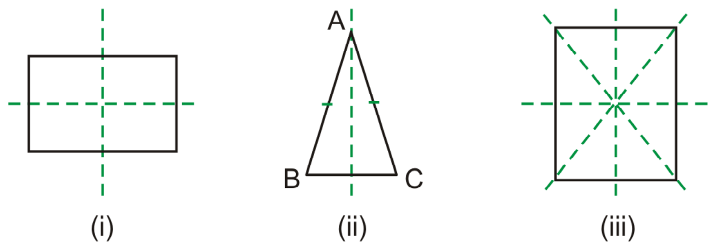


Figure 15.14

Shape (i) is a rectangle. It has two axis of symmetry. Shape (ii) is an isosceles triangle. It has only one axis of symmetry. Shape (iii) is a square. It has four axis of symmetry.

Example 15.3: Look at figure 15.15,

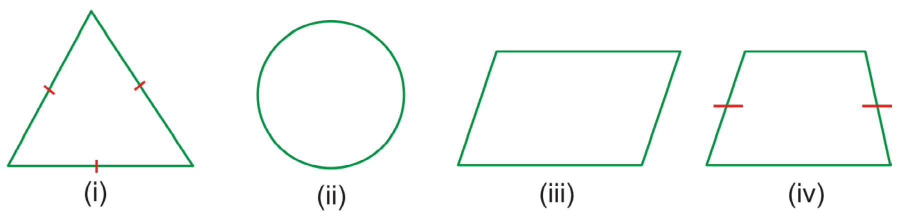


Figure 15.15

(a) In this figure which shapes are symmetrical and which are not?

(b) In symmetrical shapes, what is the number of axis of symmetry?

Solution: Symmetrical: (i), (ii) and (iv) Asymmetrical: (iii)

(i) It is an equilateral triangle; it has three axis of symmetry.

(ii) It is a circle. In it every diameter is an axis of symmetry. So number of axis of symmetry is infinity.

(iv) It is an isosceles trapezium. It has only one axis of symmetry.

Intext Questions 15.2

1. In figure 15.16 separate symmetrical and asymmetrical shapes. Write the number of axis of symmetry for symmetrical shapes.

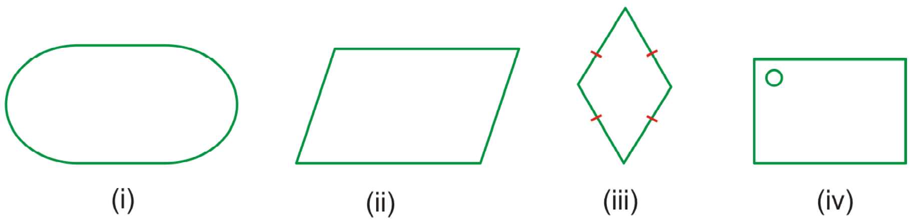


Figure 15.16

Let us Revise

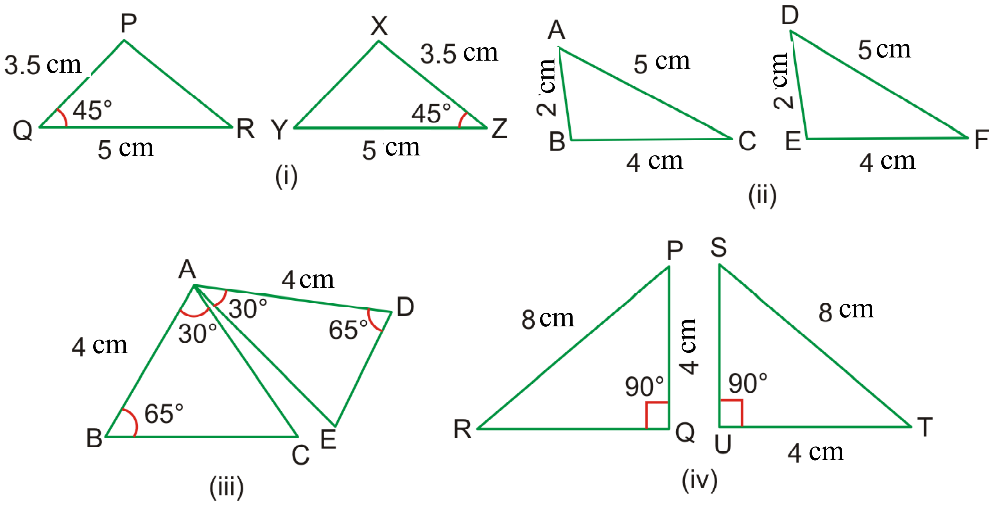
- Shapes with same size and same shape are called congruent shapes. For congruency '$\cong$' sign is used.
- Verification of congruency of two triangles can be done by the following rules:

  (i) SSS Congruency rule (ii) SAS Congruency rule

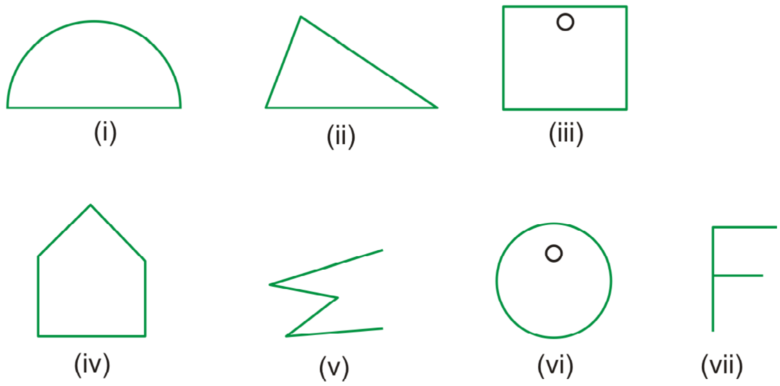
  (iii) ASA Congruency rule (iv) RHS Congruency rule
- If we find a line for a shape such that upon folding the shape along the line one part of the shape covers completely the other part then the shape is called symmetrical shape, otherwise asymmetrical shape. That line is called axis of symmetry or line of symmetry of the shape.
- Different shapes have different number of axis of symmetry.

Exercise

1. In the figure, identify congruent shapes and quoting the congruency rule of write them symbolically:



2. In the figure given below, separate symmetrical and asymmetrical shapes. Write the number of axis of symmetry for symmetrical shapes.



Intext Questions 15.1

1. (i) QP (ii) PR (iii) $\angle R$

2. Yes, by RHS

Intext Questions 15.2

1. Symmetric: (i), (ii), (iii)

   Asymmetric: (iv)

   Number of axis of symmetry: (i) two, (ii) two,(iii) one

Exercise

1. (i) Congruent, $\Delta PQR \cong \Delta XYZ$ (SAS)

   (ii) No

   (iii) Congruent, $\Delta ABC \cong \Delta ADE$ (ASA)

   (iv) Congruent, $\Delta PQR \cong \Delta TUS$ (RHS)

2. Symmetric: (i), (iii), (iv), (v) and (vi)

   Asymmetric: (ii) and (vii)

   Number of axis of symmetry:

   (i) one

   (iii) one (iv) five (v) one

   (vi) one

Module - V

Mensuration and Statistics

Mensuration

You are acquainted with closed plane shapes, triangles, rectangles, square, quaddrilateral, circle etc.

You are also aware of solids cuboid and cube etc. Finding perimeter, area of plane figures including surface areas and volume of solids is studied under mensuration

The knowledge and methods to find the area etc is importent for surveyor, architects, engineers and other common people.

In our daily life, we come across different situations, where we need to find the following:

1. The area of four walls of rooms for getting white washed
2. The area of ceiling for getting it plastered
3. The length of barbed wire for fencing
4. No of tiles to be used on a floor
5. The capacity of different shapes utensiles

   There are certain situations where the application of mensuration is useful. This module will help the learners in developing skills to solve such situations.

From this lesson, you will learn:

- To find the perimeter and area of plane figures like, triangles, rectangles, squares, parallelograms etc.
- To know the faces, vertices and edges of solids like cuboid and cube.
- To calculate the surface area and volume of solids like cuboid, cube etc.

Statistics

In the modern society there is great role of statistics. You wan to know the literacy level of a country, the need for rains for a slate or Per capita income of a particular group, we need to make use of the knowledge of "Statistics". The relation of this branch of mathematics, when the Govt. machinery was required to control the society. Govt needs to keep records of all those meterial, which will be used for preparing the plan of development.

In this module, you will be introduced to the basic elements of statistics as - data-primary and secondary, demonstration and drawing inference from the data.

You will learn to collect data, keep in tabular form, represent the data in barchart/ bargraph and preparing frequency table.

16

AREA OF PLANE FIGURE

In our daily life we need to take the following type of questions

(i) To find the area of residential plots.

(ii) To find the area of four walls of a room.

(iii) The find the length of the boundary of a rectangular or circular park.

(iv) To find the area and perimeter of triangular, rectangular, parallelgram type of objects

All these are inter related that how to find the perimeter area of these figure

From this lesson, you will learn:

- About conceptual understanding of perimeter & area of plane regious
- Satandard units of measuring area

  To find the ways of calculating perimeter and areas of plane figure
  - Triangle
  - Rectangle
  - Square
  - Parallelogram
- To know the methods of calculation of perimeter & areas of circle.

  The application of all the above in daily life, will also be dealt in this unit.

16.1 Area

See the figure in 16.1

Out of these, which figure is larger?

We can say just looking at figure A & B, that figure B' is larger than 'A' as the region of the plane covered by figure B is larger than the region covered by A.

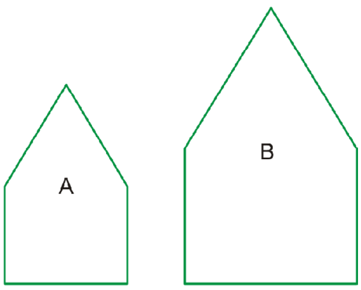


Figure 16.1

Similarly in fig 16.2, out of figure 'C' & 'D' we can say that figure 'D' is larger then figure 'C' because figure 'D' covers more region on the paper then figure 'C'. Look at the figure E & F in 16.3. It is difficult to say which figure is larger than other. The answer to this question is from the following question.

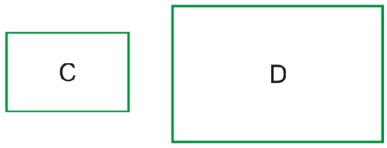


Figure 16.2

Out of these two figures E & F, which one covers the more region on the plane. Can we now say that the measurement of region covered on a plane by a closed figure is it's Area.

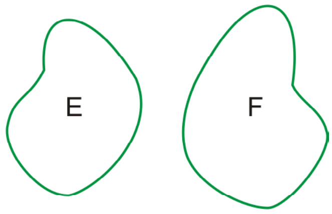


Figure 16.3

16.2 Area of a plane region

Recall that plane figure like triangle, rectangle, square etc are called rectilinear figure because these are formed by line segments. A linear figure is called simple as it's two sides, except the common point, never meet each other. For example in figure 16.4. (i) is a simple linear figure and (ii) is not a simple linear figure.

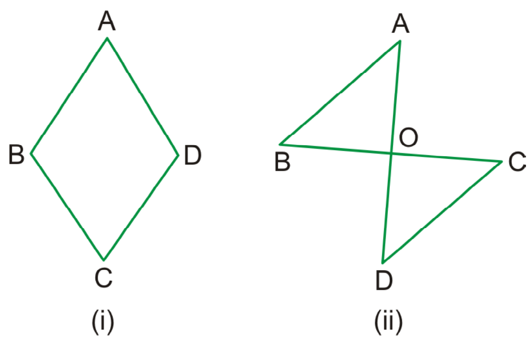


Figure 16.4

If we start moving from a point on the boundary of a figure and moving around this we reach at the starting point, then this is called a closed figure.

In figure 16.5, (iii) is a closed figure where as (iv) is not a closed figure.

Draw a rectangle ABCD on a paper and shade the region covered by this. The shaded region is called rectangular area see fig 16.6 similarly, a triangle ABC drawn on a pieace of paper and region covered is shaded.

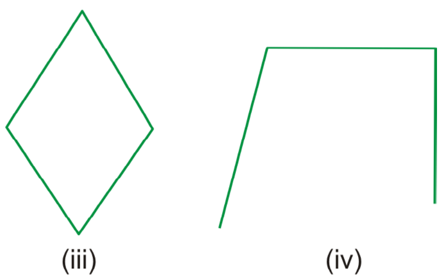


Figure 16.5

Note

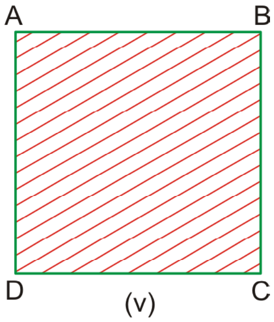


Figure 16.6

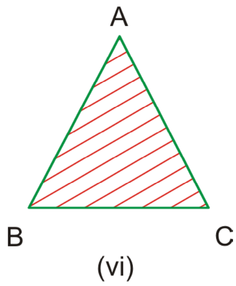


Figure 16.7

This shaded region as in fig 16.7 (vi) is called triangular region. Now we can say that the region bounded by a closed figure of a plane is called "region" of that figure and it's measure is called "area" of that closed figure.

16.3 Standard units of measure of area

Recall that the measure of a line segment in linear units is measured in meter, centimeter, milimeter etc., similarly the area of a plane region is measured in square units.

This unit is a square with 1m side or 1cm side or 1mm side

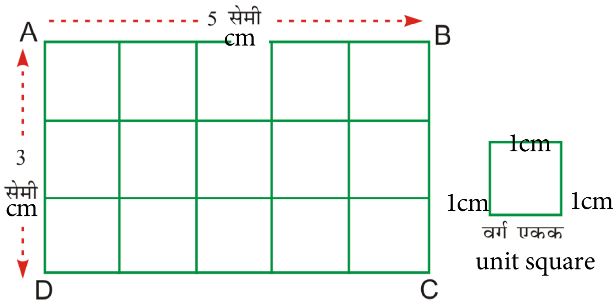


Figure 16.8

Let us take a rectangle with length 5cm and breadth 3cm. Divide AB into 5 equal parts (each part is 1cm) and AD into 3 eqaul parts. We get 15 sqaures of unit cm length after joing these points (see fig 16.8). Hence, this figure covers 15 times region in comparison of a unit square of 1 unit. Hence, the area of rectangular figure ABCD is $15cm^2$. Similarly the area of a square PQRS in figure (16.9) is $9cm^2$. Hence, if the side of a unit square is 1km or 1m then it will cover an area of $1km^2$ or $1m^2$ on a plane.

Note

16.4 Perimeter of rectilinear figures

The distance covered around a closed figure on a plane is called it's perimeter. In this way in figure 16.10, the perimeter of triangle ABC is (AB+BC+CA) and that of rectangle ABCD is (AB+BC+CD+DA) or 2 (AB+BC)

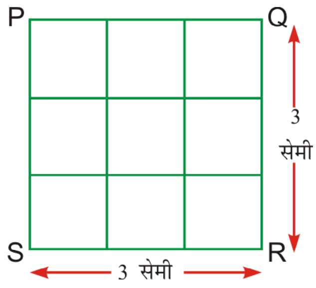


Figure 16.9

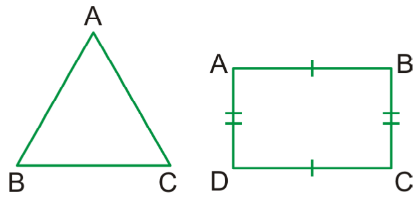


Figure 16.10

As the perimeter is a linear measure, hence the unit of it's measure is in cm or km.

Example 16.1: Find the perimeter of a triangle ABC, Where AB = 5cm, BC = 7cm and CA = 3cm.

Sol. Perimeter of triangle ABC = (AB+BC+CA) = (5+7+3)cm = 15cm

Example 16.5 : An equilateral triangle the side is 5cm, find it's perimeter

Sol. All the sides of an equilateral triangle are equal.

Hence, the perimeter of an equilateral triangle = (Side+Side+Side) or 3×side(length)

= (3×5) cm = 15cm

Example 16.3 : The base BC of an isocess triangle is 6cm and one side of the two equal side is 5cm. Find it's perimeter.

Sol. Perimeter of isoceless triangle = (BC+AB+AC)

= (BC+2×AB) [$\because$ AB=AC]

= (6+2×5) = (6+10)cm = 16cm

Intext Questions 16.1

1. In figure 16.11 find the area of all the figure where each small square is of 1 cm^2 area.

Note

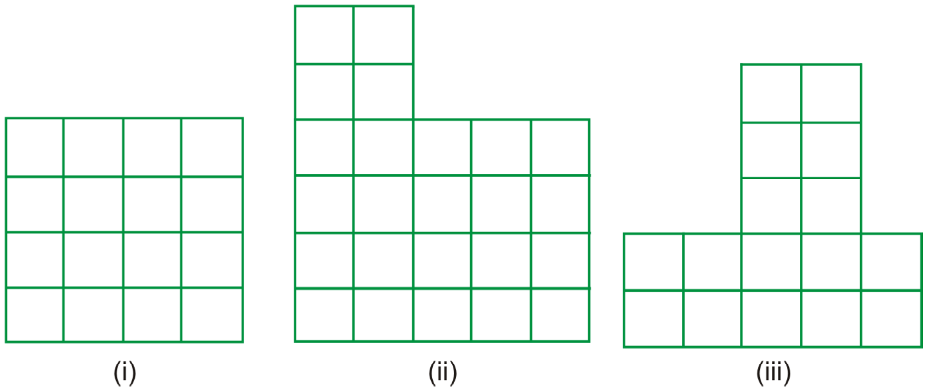


Figure 16.11

2. Fill in the blanks

   (i) The part of a plane which is covered by a simple closed figure is called it's ___________.

   (ii) The measure of that region on a plane whicch is covered by a closed figure is called the ____________ of it's area.

   (iii) The standared unit of area is ____________

   (iv) The measure of a covered along a closed figure on a plane is called it's ______________ .

   (v) The perimeter of an equilateral triangle is ___________

3. Find the perimeter of a triangle with side 3cm, 4cm and 5cm.

4. Find the perimeter of an equilateral triangle whose sides are 8cm

16.5 Area of a triangle with the help of Graph Paper

In figure 16.12, a triangle is drawn on a graph paper with 1cm^2 squares. Let us count the complete and in complete squares on the graph paper

The number of complete squares (shaded) = 12

In complete squares = 8

Each in complete square is half

$\therefore$ Area of triangle ABC = $(12+\frac{1}{2}\times 8)\text{cm}^2$

$= 16\text{cm}^2$

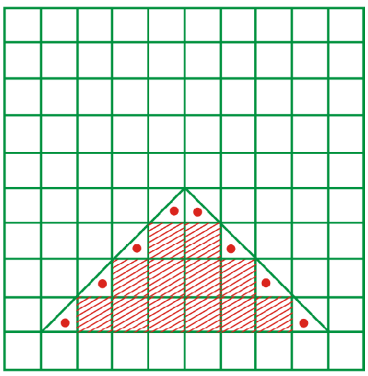
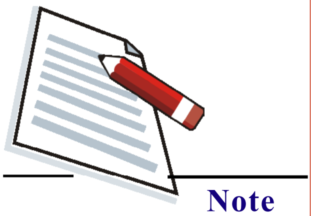


Figure 16.12



In fig 16.13

No. of complete square = 14

In complete squares which are half= 5

No of squares more them half= 3, No of square less them half= 3 we follow the following principle

(a) No. of squares more than half will be treated as complete squares

(b) No. of squares less than half will be left without counting

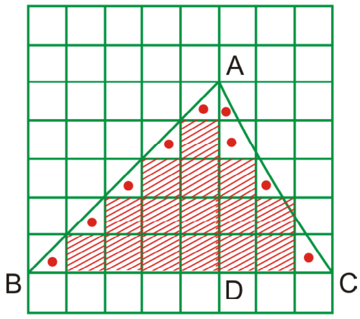


Figure 16.13

$$\therefore \text{ Area of } \Delta ABC = \left(14+\frac{1}{2}\times 5+3\times 1+3\times 0\right)cm^2$$

$$= (14+2.5+3)cm^2 = 19.5cm^2$$

Finding area using graph paper, will be approximate area. The exact area will be when there are all complete or half squares as in the above figure 16.12.

16.6 Area of triangle using formula

Observe fig 16.13 once again, we see that

(i) The length of the base of the triangle is 8cm (This is equal to 8 units of a unit square)

(ii) Height of the triangle is equal 4 units or 4cm = AD)

(iii) Mutliplication of numbers in (i) & (vi) is $8\times4 = 32cm^2$

This is double the area calculated using graph in fig 16.12.

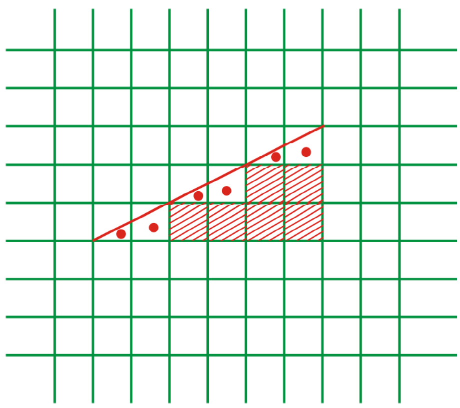


Figure 16.14

$$\therefore 2\times \text{ area of } \Delta ABC = 32cm^2$$

$$\therefore \text{ Area of triangle} = 16cm^2 = \frac{1}{2}\times(BC\times AD)$$

$$= \frac{1}{2}\times \text{Base} \times \text{corresponding height}$$

Now observe figure in 16.13

(i) The length of base BC = 8cm

(ii) Height AD = 5cm

$\therefore$ area of $\Delta ABC = (8\times5)cm^2$

Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

Can we say from the above two examples that

Area of the triangle $\Delta = \frac{1}{2} \times \text{Base} \times \text{Corresponding height}$

Area of triangle using Heron's formulas

If the sides of a triangle are a, b, c then area of Δ

$= \sqrt{s(s-a)(s-b)s-c)}$ where $2s = a+b+c$

$$LC = \frac{a+b+c}{2}$$

This formula is known as Heron's formula, The name of Greek mathematicean (Heron of alexendria). This formula was also derived by Indian mathematicians Brahmgupt & Arya Bhatt.

Let us now calculate the area of a Δ, using this formula when the sides of triangle are 25cm, 60cm and 65cm.

Let us suppose a = 25cm, b=60, c=65cm

$$\therefore\ s = \frac{25+60+65}{2} = \frac{150}{2} = 75$$

$$\therefore \text{Area of triangle} = \sqrt{75(75-25)(75-60)(75-65)}$$

$$= \sqrt{75\times50\times15\times10}$$

$$= \sqrt{3\times5\times5\times2\times5\times5\times3\times5\times2\times5}$$

$$= \sqrt{2\times2\times3\times3\times5\times5\times5\times5\times5\times5}$$

$$= 2\times3\times5\times5\times5$$

$$= 750\ cm^2$$

Example 16.4 In figure 16.14, find the area of triangle shown in the figure.

Sol. In the figure, no. of complete square = 6

(a) In complete squares more them half = 3

(b) Incomplete square less then half = 3

$\therefore$ Area of triangle $= (6+3\times1+3\times0)cm^2 = 9cm^2$

Example 16.5 Find the area of a Δ, whose base is 9cm and height 6cm.

Sol. Area of triangle $= \frac{1}{2} \times$ base $\times$ corresponding height

$$= \left(\frac{1}{2} \times 9 \times 6\right) = 27\text{cm}^2$$

Example 16.6. Find the length of the base of a Δ PQR, when it's area is 30cm^2 and height is 6cm.

Sol. Let the base of $\Delta PQR = x$cm

So, $\frac{1}{2} \times x \times 6 = 30$ or $x = 10$

$\therefore$ The length of the base is 10cm.

Example 16.7 Find height AD of a triangle, whose area is 112cm^2 and the base is 32cm.

Sol. Let the height of the Δ is $x$cm.

So, $\frac{1}{\not{2}} \times \not{32}^{16} \times x = 112$

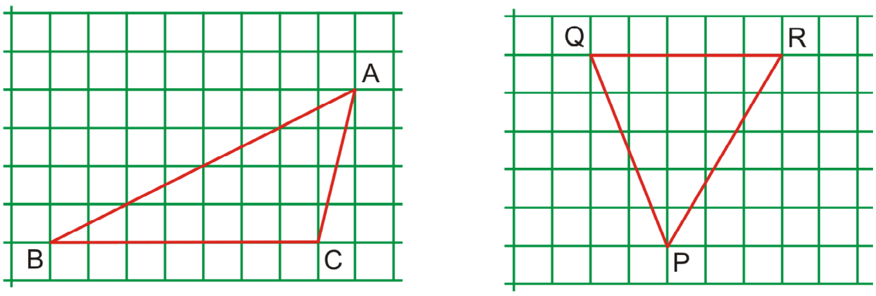
$\Rightarrow 16x = 112$

$\Rightarrow x = 7$

$\therefore$ Height of the triangle is 7cm.

Intext Questions 16.2

1. The figures 16.15 (a) & (b) showing two triangles. Find their area.



(a) (b)

Figure 16.15

2. Using the given data, find the area of Δ ABC.

	Base	Height
(i)	8cm	4cm

(ii)	16cm	2cm
(iii)	9cm	7cm

3. For a triangle, fill in the blanks.

	Area	Base	Height
(i)	$30cm^2$	10cm	________
(ii)	$120cm^2$	________	16cm
(iii)	$50cm^2$	10cm	________
(iv)	$90cm^2$	________	18cm

4. Using heron's formulas, find the area of a triangle with sides 51m, 52m & 53m.

16.7 Perimeter of a rectangle

We have discussed the perimeter of rectangle ABCD as 2 (AB+BC).

If the perimeter is denoted by P, length as 'ℓ' and width as 'w'

Then, $P = 2\ (\ell + w)$

Example 16.8 Find the perimeter of a rectangle whose length is 20cm and width as 8cm.

Sol. Perimeter of rectangle $= 2\ (\ell + w)$

$\therefore$ P = 2 (20+8)cm

Example 16.9 The perimeter of a rectangle is 40cm, length as 15cm, find the breadth of the rectangle.

Sol. $P = 2\ (\ell + w)$ given P = 46cm, ℓ =15cm, w=?

46 = 2 (15+w

$\therefore$ 23 = 15+w or w = 8cm

$\therefore$ Width of the rectangle is 8cm.

Example 16.10 If the perimeter of a rectangle 2m84cm and breadth is 30cm. Find the length.

Sol. We know the formula for perimeter as-

$P = 2(\ell + w)$, here P = 2m 84cm = 284cm

W = 30cm, ℓ =?

$\therefore$ 284 = 2 $(\ell + 30)$

Note

or $142 = \ell + 30 \Rightarrow \ell = 112$

$\therefore$ Length of the rectangle of is 112cm or 1m 12cm.

Example 16.11 If the length of a rectangle is 10cm more then it's width and perimeter is 100cm. Find the dimension of the rectangle.

Sol. Here P = 100cm, If the width is wcm, then length = (w+10) cm

$\therefore$ $P = 2\ (\ell + w)$

$= 2\ (w+10+w)$

$50 = 2w + 10 \Rightarrow 2w = 40 \Rightarrow w = 20cm$

$\therefore$ length = 20 + 10 = 30cm

width = 20cm

Intext Questions 16.3

1. Fill in the following blanks for a reactangle

	Perimeter	Length	Width
(i)	120cm	__	20cm
(ii)	60cm	__	10cm
(iii)	100cm	30cm	__
(iv)	80cm	30cm	__

2. If the total length of the fence of a field is 30m, the longer side is 8m find the length of the shorter side.

3. The length of a rectangular is 100m and the perimeter is 216, then find the width of the ________.

16.8 Perimeter of a square

We know that square is a special rectangle, in which length & width are equal.

$\therefore$ Perimeter of square $= 2 \times (\ell + \ell) = 4\ell$ or 4times the length of the side of the square.

Example 16.12 A square Carromboard is of side 90cm. Find it's perimeter.

Sol. Perimeter of Carromboard = (4×90) cm = 360cm

Example 16.13 A sqare park of side 10m has inner road around it of width 1m as showing fig 16.16.

Find the length of the barbed wire for fencing ABCD and PQRS

Sol. Perimeter of PQRS = 4×10=40m

Side of internal square = (10-2)m = 8cm

∴ Perimeter of ABCD = 4×8=32cm

∴ Total length of the wire = 40+32=72m

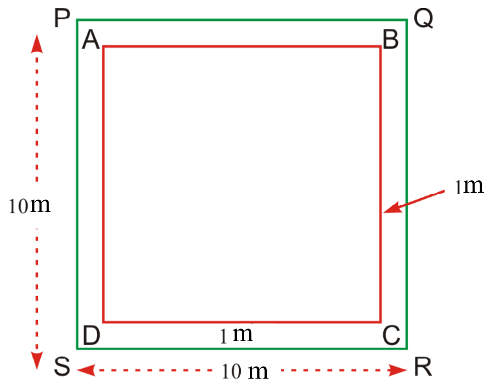


Figure 16.16

16.9 Area of a rectangle and square

Let us see fig 16.8 again. We calculated the area of rectangle ABCD using graph paper which was 15cm^2.

Let us see the length of the rectangle's, This is 5cm and width is 3cm. The multiplication of 5 & 3 is 5×3=15

∴ We can say that the area of reactangle = (length × width)

Area of square = length × length = (length)^2 = (side)^2

Knowing any two out of length, width & area, third can be calculated using above formula.

Example 16.14: Find the area of a rectangle whose length is 2 meter and width 50cm.

Sol. Side = 50cm, area or A=?

We know that area of a square = (side)^2 = 50×50=2500cm^2

∴ A = 2500cm^2

Example 16.16 Find the length of a rectangle whose area is 400cm^2 and the width is 16cm.

Sol. Here area = 400 cm^2, ℓ = 3, w = 16 cm

∴ A = ℓ×16

∴ 400 = ℓ×16

So $\ell = \frac{400}{16} = 25$

or ℓ = 25 cm

Note

Example 16.17 The area of a square is 784m^2. Find it's side.

Sol. We know that area of square = side × side = (side)^2

$\therefore 784 = (\text{side})^2$

or $\text{side} = \sqrt{784}$

$\therefore$ side of square = 28cm

Example 16.8 Find the length of the side of the square whose area is 2.25m^2.

$\therefore$ Area = (side)^2

$$\therefore \text{Side} = \sqrt{2.25} = \sqrt{\frac{2.25}{100}} = \frac{15}{10} = 1.5$$

$\therefore$ Length of the side of square = 1.5m

Intext Questions 16.4

1. Fill in the blanks for a sqaure

	Perimeter	Side
(i)	_____	20cm
(ii)	_____	2m
(iii)	200cm	_____
(iv)	1200cm	_____

2. Fill in the blanks for rectangle/square

	Area	Length	width
(i)	_____	40cm	15cm
(ii)	600cm^2	30cm	_____
(iii)	2500cm^2	1m	_____
(iv)	600cm^2	______	15cm
(v)	_____	40cm	40cm

16.10 Area of a parallelogram from by graph paper

Draw a parallelogram ABCD on a centimeter paper graph as in fig 16.17. Draw $AP \perp DC$ and $CQ \perp AB$.

$$\text{Area of } \Delta\ APD = \left(\frac{1}{2} \times 2 \times 4\right) \text{cm}^2$$

Note

$= 4cm^2$

Similarly area of Δ CQB = $4cm^2$

Combining Δ ADP & Δ QBC and the middle area we get parallelogram ABCD

Area of parallelogram = Area of Δ ADP + Area of rectangle ΔPCQ + area of Δ BQC

$= (4+24+4)\ cm^2 = 32cm^2$

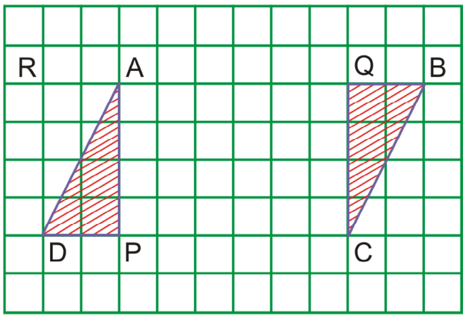


Figure 16.17

Now we also see DC = 8cm and AP = 4cm

If we multiply 8 and 4, we also get 32.

$\therefore$ Area of parallelogram = Base (DC) $\times$ height (AP) = $32cm^2$

$\therefore$ Area of a parallelogram = Base $\times$ Corresponding height

Also area of rectangle DC QR

$= DC \times DR$

$= 8cm \times 4cm$

$= 32cm^2$

Hence the area of a parallelogram and rectangle is equal when these are on the same base and between the two parallel line

16.11 Area of parallelogram

In figure 16.18, area of parallelogram

ABCD = Area of ΔADC + area of ΔACB

$$= \frac{1}{2} DC \times h + \frac{1}{2} AB \times h$$

$$= \frac{1}{2} h\,(DC+AB)$$

$$= \frac{1}{2} h\,(DC+DC) \quad [\because AB = DC$$

opposite sides of a parallelogram

$$= \frac{1}{2} h \times 2DC = h.DC = AP \times DC\ [\because h=AP]$$

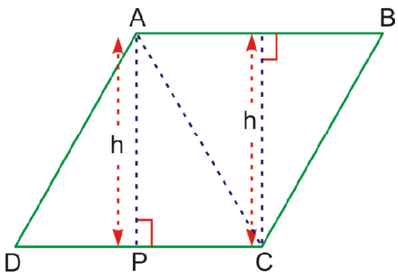


Figure 16.18

$\therefore$ Area of parallelogram = Base $\times$ Corresponding height

Note

16.11.1 Area of a trapezium

In fig 16.18.1 ABCD is trapezium where AB||CD.

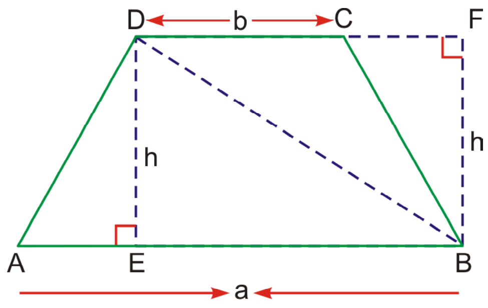


Figure 16.18.1

Area of trapezium = Area of ΔABD + Area of ΔBCD

$$= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h$$

$$= \frac{1}{2}(a+b) \times h$$

Hence, area of a trapezium = (half the sum of parallelogram) × height

Let us find the area of a trapezium whose parallel side are 20cm & 12cm and the distance between parallel line is 3cm.

We know that the area of a trapezium is $= \left(\frac{1}{2}\text{sum of parallel side}\right) \times \text{height}$

$$= \frac{1}{2}(20+12) \times 3\text{sq cm}$$

$$= \frac{1}{\cancel{2}} \times \cancel{32}^{16} \times 3 = 48\text{sq cm}$$

Example 16.19 A parallelogram with base 5cm and the corresponding height is 6cm. Find the area.

Sol. Area of a parallelogram = Base × Corresponding height

$$= (5 \times 6)\ \text{cm}^2 = 30\text{cm}^2$$

Example 16.20 Area of a parallelogram is 216 cm^2 and one side is 32cm

Find the corresponding height.

Sol. Area = Base×height

216 = 32 × height

$$\therefore \frac{216}{32} = \text{height}$$

= 6.75cm = height

Example 16.21 Find the length of the base of a parallelogram, whose area is 3000 sqm and the height beetween two long sides is 30m.

Note

Sol. Area of parallelogram = Base × height

Let the length of base = bm

$\therefore$ b×30=3000

or b = 100

Hence the base of parallelogram is 100m.

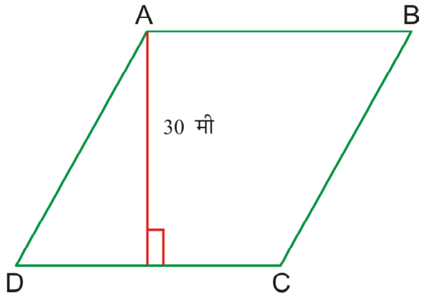


Figure :16.19

Example 16.22 The area of parallelogram of base 42 m is twice the area of the triangle whose height is 36 m and base is 63 m. Find the height of the parallelogram.

Sol. Let the height of parallelogram = hm

$\therefore$ Area of parallelogram $= 42 \times hm^2$

Area of triangle $= \frac{1}{2} \times$ base × height

$$= \left(\frac{1}{\not 2} \times 63 \times \not{36}^{18}\right)$$

$= 1134 \text{ m}^2$

It is given that the two areas are equal

$\therefore 1134 = 42 \times h$

$\Rightarrow h = \frac{1134}{42} = 27$

$\therefore$ Height of parallelgram = 27m

Intext Questions 16.5

1. Fill in the blanks for a parallelogram

	Base	Height	Area
(a)	32m	17m	____
(b)	____	14m	$11m^2$
(c)	1.2cm	____	$108cm^2$
(d)	13.5m	$1\frac{1}{7}$m	____

Note

2. Figures are drawn below in fig 16.20 find their area.

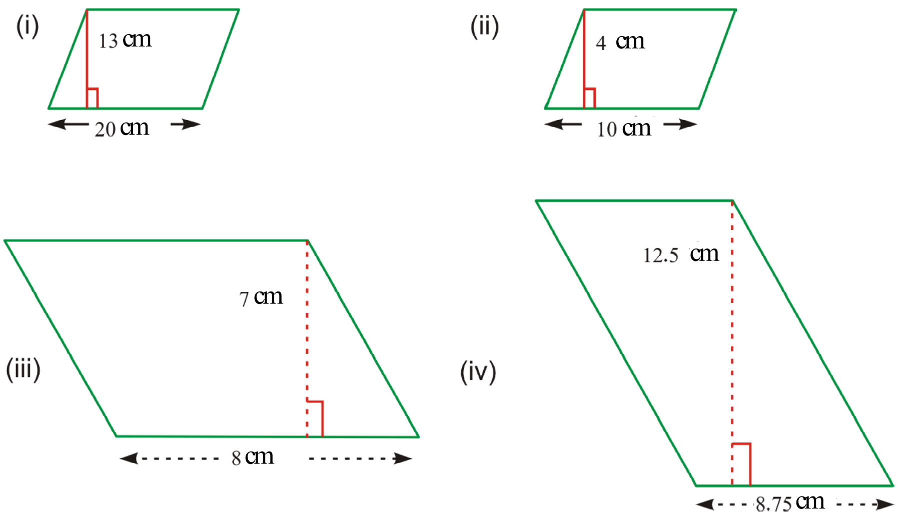


Figure 16.20

3. Find the area of the following trapazium.

Lengths of parallel sides	Distance between them
(i) 30m & 20m	15m
(ii) 17cm & 40cm	14.6cm

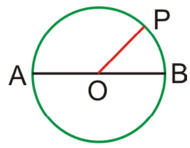


Figure 16.21

16.12 Circumference of a circle

Recall that a circle is the path of a point which is always at a constant distance from a fixed point.

In figure 16.21 O is a fixed point, which is the center of the circle, OP is the radius of circle and AB is the chord passing through center is called the diameter of the circle, you can see, diameter is twice of radius. A circle is not made of line segments, hence the perimeter cannot be joind like the earlier methods for linear figure like triangle, rectangle, square etc.

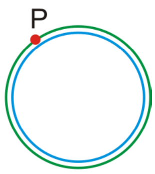


Figure 16.22

Starting from point P and reaching at 'P' after moving around the circle, the distance so covered is called circumference of O

Measuring the circumference of a circle

Wrap a thread around any circular article so that the thread may not be loose and overlap. Measure this by a measuring scale as the thread is linear. This is approximately the circumference of the circular object.

Another Method: Mark a point 'P' on the circle, move it on the line such that point 'P' again touches the line at 'p' (fig 16.23).

Then the measure of 'PP' is the circumference of the circle.

Note

Relation between circumference and the diameter of circle

Experiment: Draw three circles of 4cm, 6cm and 9.5cm radius. Calculate the circumference of all these circles by any method explained above and write the results in the table given below

S.No	Diameter 'D'	Circumference 'C'	C÷d
1	4 cm	12.6 cm	3.15
2	6 cm	19 cm	3.16
3	9.5 cm	30 cm	3.15

You can see the table, in each case the value of $\frac{c}{d}$ is approximater same and this is denoted by π

$$\therefore \frac{\text{Circumference}}{\text{Diameter}} = \pi$$

or circumference = $\pi \times$ diameter

Hence, the circumference a circle = $2 \times \pi \times$ radius

Note: Interesting and important information/knowledge about π is given here.

Babilonian has taken π as 3. Ancient greeks given $\pi = \frac{22}{7} = \pi$ or 3.14

Indian mathematician Aryabhatt (476A.D - 550AD) had given the value of π approximate at as 3.1416. Now a days with the help of computer we have know the value of π upto 5lacks palces of deciml. Value of π upto 20 places of decimal is 3.14159 265358979323846

You can observe that this number is neither recurring decimal nor terminating decimal. Hence π is an irrational number. For practical purposes we take the value of π as $\frac{22}{7}$ approximate or 3.14.

Example 16.33 Find the circumference of circle when

(i) Radius = 3.5cm (ii) Diameter = 1.75cm $\left(\text{Take } \pi = \frac{22}{7}\right)$

Sol. (i) We know that circumference = $2\pi r$

$$\therefore \text{Circumference} = 2\times\frac{22}{\cancel{7}}\times\frac{\cancel{7}}{2} = 22\text{cm}$$

Note

(ii) Circumference = $2\pi r$

$$= 2\times\frac{22}{7}\times\frac{1.75}{2}$$

$$= \cancel{2}\frac{22}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{8}4} = \frac{22}{4} = 5.5\text{cm}$$

16.13 Area of circle

Draw a circle of any radius say rcm and divide it into 16 equal parts. Arrange these parts as shown in fig 16.24. As half the parts of the circle are above and half below as shown in fig 16.25. Fig 16.25 represents approximates a parallelogram, whose opposite sides are $\frac{1}{2}$ of 2π r and the height is r cm apporox meters.

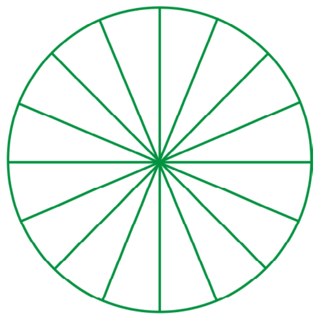


Figure 16.24

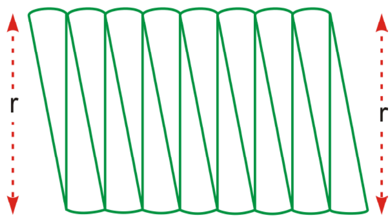


Figure 16.25

Hence the area of circle = area of the approximate parallelogram

$$= \pi r \times r$$

$\therefore$ Area of circle $= \pi \times (\text{radius})^2$

Example 16.24: The perimeter of a circular mat and rectangular mat is 132cm. Which of these two will cover more region?

Sol. (i) Perimeter of square mat = 132cm

$$\therefore \text{ Side of this mat } = \frac{132}{4} = 33\text{cm}$$

$\therefore$ Area of this mat = (33×33) cm^2 = 1089 cm^2

(ii) Perimeter/Circumferene of circular mat = 132cm

$$\therefore \text{ Radius of this mat} = \frac{132}{2\pi} = \frac{132}{2}\times\frac{7}{22}$$

$$= 21 \text{ cm}$$

Note

$\therefore$ Area of circular mat $= \frac{22}{7} \times 21 \times 21 = 66 \times 21 = 1386 cm^2$

Hence, circular mat will cover more region.

Intext Questions 16.6

1. For a circle, fill in the following blanks:

	Radius	Circumference	Area
(i)	13.5cm	____	____
(ii)	14cm	____	____
(iii)	____	8.8cm	____
(iv)	____	____	$2464 cm^2$

2. Find the radius and area of a circular plate whose circumference is 77cm.

3. The area of a metal circular plate is 256cm2. After melting it another square plate is made. Find the perimeter & side of the square plate.

4. A circular necklace is of 7cm radius some beads have been roped in it, each one of these covers 2cm length. If 4cm is the space between two beeds, then find the number of beads in the necklace.

5. The radius of a wooden circular sheet 15 14cm. Second sheet is rectangular, whose length is 25cm and width is 20cm compare the areas covened by these two wooden sheets.

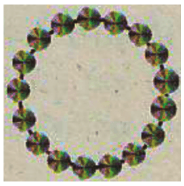


Figure 16.26

Let us Revise

- On a plane the space covered by any figure is called the region of that figure and it's measure is called it's area.
- One cm^2 or $1m^2$ is the unit of area.
- Distance covered along the sides/otherwise of a figure is called it's perimeter.
- The perimeter of a triangle is the sum total of the measure of all sides.
- The perimeter of an equilateral triangle is 3times the length of one side.
- Area of a triangle $= \frac{1}{2} \times$ base $\times$ corresponding height sq units
- Perimeter of rectangle = 2 (length + width)
- Area of rectangle = length x width sq units
- Perimeter of square = 4×length of side

Note

- Area of square = $(\text{side})^2$ sq units
- Area of parallelogram = base × corresponding height
- $\pi = \dfrac{\text{Circumference of Circle}}{\text{Diameter of circle}}$
- Value of π is approxmeter 3.14 or $\dfrac{22}{7}$
- Circumference of a circle = $2\pi r$, r is the radius of circle
- Area of a circle = πr^2, r is the readius of circle

Exercise

1. Find the perimeter of a triangle whose sides are given:

   (a) 13.5cm, 14.1cm & 16.2cm

   (b) 12m, 14m & 18m

2. Find the perimeter of equilateral triangles whose sides are:

   (a) 5.1cm (b) 7.2mm (c) 8.25m

3. Find the area of following triangles:

(i)	Base	Corresponding Height
(ii)	8.5cm	5.6cm
(iii)	8m	15m

4. Fill in the following blanks for a triangle:

	Base	Height	Area
(a)	18cm	____	36cm2
(b)	____	5m	36cm2
(c)	2.8cm	3.5cm	____

5. Fill in the following blanks for a ractangle:

	Length	Width	Perimets
(i)	8.3cm	1.7cm	____
(ii)	____	6cm	48m
(iii)	37cm	____	100cm

Note

6. Fill in the following blanks for a rectangle:

	length	width	area
(i)	length	width	area
(ii)	5.5m	4.5m	___
(iii)	___	15m	270m2

7. Fill in the following blanks for a square:

(a) Side — Perimeter

Side	Perimeter
(i) 5m	___
(ii) ___	72m

(b) Side — Area

Side	Area
(i) 4m	___
(ii) ___	144m2

8. Find the area of a parallelograms whose data is given:

	Base	Height
(i)	15cm	32cm
(ii)	8cm	22cm
(iii)	16m	12m

9. Find the radii and area of a circle whose circumference is given below:

(i) 4400m (ii) 110cm

10. Find the radii and circumference of the circle:

(i) 154 cm2 (ii) 66cm2

11. A cow is tied with a rope of 105m length in the corner of a field whose dimensions are 20m×15m. What area out side the field the cow can graze the grass?

12. Whose area is more and how much?

A square whose perimeter is 44cm or a circle with circumference 44cm?

Answer

Intext Questions 16.1

1. (i) 16cm2 (ii) 24cm2 (iii) 16cm2
2. (i) Area (ii) measure (iii) Unit Square
   (iv) Distance (v) 3×side of equilateral triangle
3. 12cm
4. 24cm

Intext Questions 16.2

1. (a) 14 unitsquare (b) $12\frac{1}{2}$ unit square
2. (i) 16cm^2 (ii) 16cm^2 (iii) $\frac{63}{2}$ cm^2
3. (i) 16cm (ii) 15cm (iii) 10cm
4. 1170m^2

Intext Questions 16.3

1. (i) 40cm (ii) 20cm (iii) 20cm
   (iv) 10cm
2. 7m
3. 8m

Intext Questions 16.4

1. (i) 80cm (ii) 8m
   (iii) 50cm (iv) 300cm
2. (i) 600cm2 (ii) 20cm (iii) 25cm
   (iv) 40cm (v) 1600cm2
3. Length 150cm width 100cm
4. 25m

Intext Questions 16.5

1. (a) 544m2 (b) 8m
   (c) 0.9cm (d) $15\frac{3}{7}$ m2

Note

2. (i) 260cm2 (ii) 40cm2

   (iii) 56cm2 (iv) 109.375cm2

3. (i) 375m2 (ii) 416.1cm2

Intext Questions 16.6

1. (i) 22cm $\frac{77}{2}$ cm^2 (ii) 88m, 616m^2

   (iii) 14cm, 616cm^2 (iv) 28cm, 176cm

2. Radius $\frac{49}{4}$ cm, Area = $\frac{3773}{8}$ cm^2

3. 16cm 64cm 4. 20

5.

Exercise

1. (a) 43.9cm (b) 44m

2. (a) 15.3cm (b) 21.6mm (c) 24.75m

3. (i) 23.8cm^2 (ii) 11.9cm^2 (iii) 60m^2

4. (a) 4cm (b) 4m (c) 4.9cm^2

5. (i) 20cm (ii) 18m (iii) 13m

6. (i) 24.75m^2 (ii) 7m (iii) 18m

7. (a) (i) 20m (ii) 18cm

   (b) (i) 16m^2 (ii) 12cm

8. (i) 48cm^2 (ii) 176cm^2 (iii) 192cm^2

9. (i) Radius = 700m, Area = 1540000m^2

   (ii) Radius = 17.5cm, Area = 962.5cm^2

10. Radius Circumference

   (i) 7cm 44cm

   (ii) 14cm 88cm

11. 259.875m^2

12. The area of circle park is 33cm^2 more than the area of square park.

13. 22m

14. 10164m^2

17

VOLUME OF SOLIDS

You have seen that majority of things in the market are sold in the boxes, Tins and other types of boxes. All these are mostly in cuboidal shapes. Also around us in our homes, steel almirah, refrigerator, boxes etc are cuboidal in shape. Hence it is useful for us to know about cuboids. Especially the number of faces, edges, surface area and volume are very useful for us.

From this lesson, you will learn

- About number of vertices, edges & faces of cuboid.
- Identifying cube as a special cuboid
- Formula for cubes and cuboids for finding surface areas.
- Understanding volume as the three dimensional space
- Formula of calculating volume of cube & cuboid
- Problems based on these concepts

17.1 Faces of a Cuboid

We see in our daily life objects/things like shoe box, Tea box, match box, brick etc. All these resemble with the figure given along side (171).

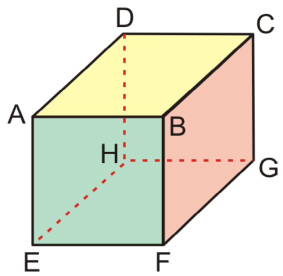


Figure 17.1

A cuboid has six faces all are rectangular.

Each oposite face is identical/ccongruent. Tap & botton faces are ABCD & EFGH in the figure 17.1 other four faces are

EHDA, FGCB, HGCD & EFBA

17.2 Edges and Vertices of a Cuboid

Two adjacent faces meet in a line. This line is called the edge of cuboid. In this way faces ABCD & BCGF meet as BC edge (See fig. 171) similarly faces BCGF and EFGH meet as edge FG (Common points in the two cuboids will from edge) There are in all 12 edges. Their names are-

AE, AB, AD, BF, BC, DH, DC, CG, EF, EH, FG, HG.

Two adjacent edges meet at a point. This is called vertex hence A , B, C, D, E, F, G, H are 8 Vertices

17.3 Cuboid's Special form

A cuboid, whose all edges are equal, is called a cube hence all the faces of a cube are squares, where as in cuboid at least two face are rectangular.

Intext Questions 17.1

1. Below drawn the figures of cube & cuboid. Write the names of all faces, edges & vertices

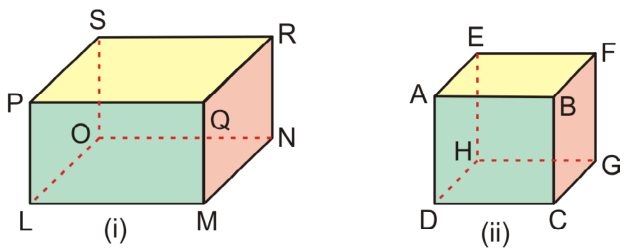


Figure 17.2

2. Answer the following question from the figure 17.3

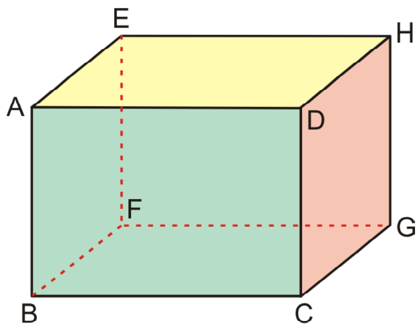


Figure 17.3

(i) Write the name of the face parallel to face ABCD

(ii) Write the faces which are adjacent to face OCGH.

(iii) Write the faces which meet as edge AD

(iv) Write three edges meet at point/vertex F

Note

3. Fill in the blanks
   (i) No of edges in a cube are ____________
   (ii) No of faces in a cube are _____________
   (iii) No of all vertices of a cube are __________

17.4 Surface area of a cube and cuboid

You may recall there are 6 rectangular faces. Hence the surface area of a cuboid is the sum total of areas of these six faces. (Fig. 17.4)

Area of two parallel faces ABCD & EFGH = $2\times\ell\times b$ sq.units

Similarly area of parallel faces ABFE & DCGH = $2\times b\times h$ sq.units

Area of parallel faces ADHE & BCGF = $2\times\ell\times h$ sq.units

$\therefore$ Surface area of cuboid = $2\ (\ell b + bh + h\ell)$

= (length × width + width × height + height × length)

you may recall that a cube is the the special case of a cuboid, in which length, width & height are equal and this the side of a cube. Hence there area of a cube = $6\times(\text{side})^2$ sq.units. Let us understand these formulae with the help of examples.

Example 17.1 The dimensions of a cuboid are 8cm, 9cm & 10cm. Calculate it's surface area.

Sol. Here length = 8cm, width = 9cm & height = 10cm

$\therefore$ Surface area of cuboid = $2\ (8\times9+9\times10+10\times8)$ sq.cm & 1cm

= $2\ (72+90+80)$ sq.cm

= 484cm^2 sq.cm

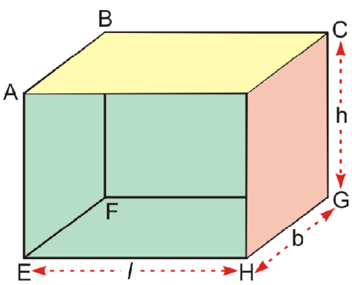


Figure 17.4

Example 17.2: The dimensions of a cuboidal box are 50cm, 40cm, & 30cm. Find the cost of sheet required to make this box at the rate of ₹125sq.meter.

Sol. The area of the sheet required = $2\ ((\ell b + bh + h\ell)$

= $2\ (50\times40+40\times30+30\times50)$ sq.cm

= $9400\ \text{cm}^2$

= $0.94\ \text{m}^2$

$\therefore$ The cost of the sheet = ₹0.94×125

= ₹ 117.50

Example 17.3: The side of a cube is 15cm. Find it's surfaces.

Sol. Surface area of cube = $6 \times (\text{side})^2$

$= 6 \times 15 \times 15$ sq.cm km^2

$= 1350$ cm^2

Example 17.3 : The side of a cube is 15cm. Find it's surface areas.

Sol. Surface area of cube = $6 \times (\text{Side})^2$

$= 6 \times 15 \times 15$ sq. cm/cm^2

$= 1350$ cm^2

Intext Questions 17.2

1. Find the surface area of the cubes with the given edge.

   (i) 11cm (ii) 25cm (iii) 5m (iv) 2m 15cm

2. The dimensions of an oil Tin are 25cm, 35cm & 45cm.

   What will be cost of colouring the Tin at the rate of 5 paise/cm^2?

3. Find the surface area of a cuboid whose dimensions are $30 \times 10 \times 125$cm

17.5 Volume

We face problems in our daily routine regarding finding the capacity of box, these are related to the volume. Here we shall discuss the volume of cube and cuboid only. Recall the volume of a solid is the measure of space occupied in a three dimensional space. We used the square unit for the area in the same way we shall also use a unit for the measurement of volume of solids/utensils/boxes etc.

For volume we shall use a cubic unit. This is for a cube of 1 unit a cube whose side is 1cm is called 1 cubic centimeter or 1cm^3 similarly a cube with side 1m will be called 1m^3 or 1 cubic meter

17.6 The volume of a cube and cuboid

In the figure 17.6 given below, it has two layers of cube each 18 unit squares, height 2cm. Total unit cubes are 36.

$\therefore$ Volume of cuboid = 36cm3

If we multiply 6, 3 & 2 we get 36cm^3

$\therefore$ Volume of cuboid = $\ell \times b \times h$ cubic units

For cube $\ell \times b \times h \quad (\ell = b = h)$

$\therefore$ Volume of cube $= (\ell)^3$

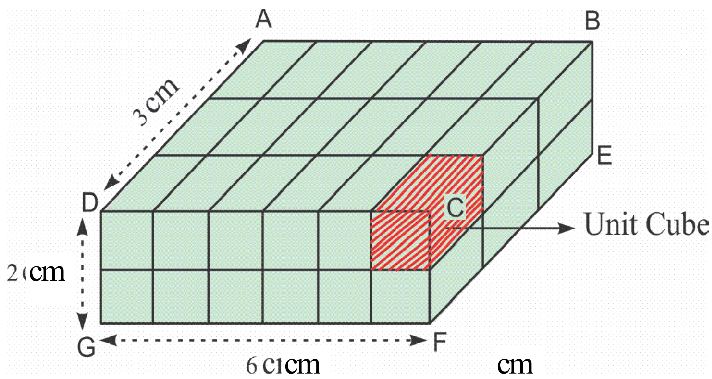


Figure 17.6

Example 17.4 The dimensions, of a wooden cuboid piece are 10cm, 8cm & 6cm. Find it's volume.

Sol. We know that volume of cuboid = $\ell \times b \times h$ cubic units

$\therefore$ The volume of wooden piece = $(10\times8\times6)$ cm^3

$= 480$ cm^3

Example 17.5 The dimensions, of a card board are 80cm, 40cm & 20cm. How many cubical box can be put into the width with 10cm side cube.

Sol. The volume of box = $(80\times40\times20)$ cm^3

Volume of a cubical box = $(10\times10\times10)$ cm^3

$$\therefore \text{ No of cubical boxes} = \frac{{}^{8}\cancel{80} \times {}^{4}\cancel{40} \times \cancel{20}^{2}}{\cancel{10} \times \cancel{10} \times \cancel{10}} = 64$$

Example : 17.6 The length width of a cuboidal box are 6cm & 3cm

If it's volume is 72cm^3, find it's height.

Sol. Volume = $\ell \times b \times h$, v = 72cm^3, $\ell = 6$cm, b=3cm

$\therefore$ 72 = 6×3×h

h = 4

$\therefore$ height of box = 4cm

Note

Intext Questions 17.3

1. Find the volume of a cuboid in wheels

   (i) ℓ =10cm, b = 8cm, h=4cm

   (ii) ℓ =8.5cm b = 6.5cm, h = 5.5cm

   (iii) ℓ =1.5cm, b = 25cm, h=15cm

2. The side of a cube is12cm. Find it's volume

3. Compare the volume of two cubes with edges 3cm and 6cm respectively.

Lets us Revise

- A cuboid is a figure with 6 faces, 12 edges & 8vertices
- The length, width (breadth) & height are called it's dimensions.
- A cuboid, with all sides equal, is called a cube.
- The three diemensional region occupied by a solid is it's measure of it's volume.
- A cube with 1cm edge whose volume is 1cm^3 is the unit of volume.
- The formulae in this chapter.

  (a) Surface area of a cuboid = 2 ($\ell \times b \times b \times h \times h \times \ell$) cubic units

  = 2 (length × breadth + breadth × height + height × length

  (b) Surface area of a cube = 6 ℓ^2 = 6×(side)2 cubic units

  (c) Volume of a cuboid = $\ell \times b \times h$

  = (length×breadth×height) cubic units

  (d) Volume of a cube = (Side)^3 cubic units

Exercise

1. The length, breadth depth of a swimming pool are 20m, 15m & 8m respectively. What will be the cost of plastering it's floor & walls at the rate of 250/sq meter? (Hint : Subtract the area of roof from the total surface are of a cuboid, as the pool is open at the top)

2. Find the volume of a box whose edge is 15cm

3. How much card board will be required for making a box 0.5m long 30cm wide & 20 cm height?

4. The dimensions of a chalk box are 16cm, 18cm & 6cm. Find it's surface area.

5. The dimensions of a soap are 10cm, 6cm & 5cm. Find it's volume.

6. What will be the volume of a cube?

   (i) If it's edge is doubled.

   (ii) If it's edge is halved.

7. The length and breadth of a cuboidal utensil are 10cm, 8cm. If it can accomodate 480 cm^3 liquid, what is the height of this utensils?

8. The dimensions of a match box are 4cm, 3cm, 2.5cm.

   Find the volume of a packet, in which 10 such match boxes can be placed.

9. The volume of a cuboid is 640cm3. If the length, height are 10cm & 8cm respective, find the width of cuboid.

10. A tea box is of 10cm×6cm×5cm dimensions. How many such tea boxes can be kept in a card board box of dimension 60cm×36cm×cm30?

Answers

Intext Questions 17.1

1.		Face	Vertex	Edge
	(i)	PQRS, LMNO	P, Q, R, S	LM, OQ, PQ, SR
		LMQP, ONRS	L, M, N, O	PS, LO, MN, QR
		PSOL, QRNM		LP, MQ, OS, NR
	(ii)	ABCD, EFGH	A, B, C, D	AB, DC, EF, HG
		ADHE, BCGF	E, F, G, H	AD, BC, EH, FG
		ABFE, DCGH		DH, CG, AE, BF

2. (i) DCGH (ii) ADHE, EHGF, BCGF, ABCE

   (iii) ADHF, ADCB (iv) BF, GF, EF

3. (i) 12 (ii) 6 (iii) Congruent (Equal)

Intext Questions 17.2

1. (i) 726 cm^2 (ii) 3 7.50 cm^2 (iii) 150 m^2 (iv) 27.735 cm^2

2. ₹ 357.5

3. 1600 cm^2

Intext Questions 17.3

1. (i) $320cm^2$ (ii) $303.875cm^3$ (iii) $\frac{9}{160}cm^3$

2. $1728cm^3$

3. (i) 1:8 (volume of small $27\ cm^3$

Volume of big cube = $216\ cm^3$

Revise

1.	215000	2.	$1350cm^2$
3.	$6200cm^2$	4.	$544cm^2$
5.	$300cm^3$	6.	(i) 8times (ii) $\frac{1}{8}$ times
7.	6cm	8.	$300cm^3$
9.	8cm	10.	216 boxes

18

INTRODUCTION TO STATISTICS

Are you aware that India is the second largest populated country in the world? In India there are 940 females per 1000 male. Literacy rate is 74.04%. These are some of the figures which you might have read in your social science book or have heard from friends or teachers. Have you ever thought about the largest village in your neighbourhood? How many females are there in your village in comparison to males. Are you aware that how many of your friends are not attending school and how many of them helping their parents in agricultural activities? It may be difficult to answer such questions. Statistics is the branch of Mathematics which keep record of such informations. Let us see how statistics can solve such problems. Let us take an example. Suppose you have taken Maths Examination. One day your friends are happy while coming out of the school. You asked them what happened? They answered that go to school and know your marks. Ah! I have obtained 65 marks out of 100; you also enjoyed with them.

When you reach home, your father asks about your marks and also some other questions. Maximum or minimum marks, no of students failed/passed. How many secure more than 60% marks? What will you do now? Through this chapters we shall learn to answer such questions:

From this lesson, you will learn

- What are data?
- How many types of data are there?
- How do we collect data? How are data presented?
- Reading bar chart and draw inference.
- Taking appropriate interval for drawing graph and drawing graph with the given data.
- Reading pie-chart and drawing pie-chart of given data.

18.1 Collection of Data

To answer the questions raised by your father on the previous page, you will do some work. First, you will know the marks obtained by all students in your class. Collecting there marks, there are two methods. First, you will ask each student to know his/her marks, secondly, you will collect this information from school records.

In the first method, you will collect information from each student and will record on a paper. Suppose, there are 20 students, then let the data be

20	25	30	30	65	72	49	57	25	45
30	57	57	72	49	57	45	38	38	65

In this example the source of data are the students, as you are collecting data directly from the students. These numbers are called data. There individual number (marks of an individual) is called observation as every time you have asked the student and write in the table.

The data, which is collected directly from source, are called Primary Data. Here the students are source of data and marks obtained are data

The data, which is original and is collected personally, are called "Primary Data" and the source, from where there data collected, is called the "Primary Source".

In the second case, the marks are collected from school records. Here the data, obtained from school records. Here the data, obtained from school register, are "secondary data" and school office/records is "Secondary Source".

Can you think now that you can answer the questions raised by your father? Directly it is difficult to answer the maximum & minimum numbers, how many students did get pass or fail! To know all this you need to do some more activities. But for all this the data is the basic material. These data are as recorded or ungrouped. Hence the basic data, you collect, are called ungrouped data. We need to group these data further, to answer the questions raised by your father.

Intext Questions 18.1

Fill in the blanks with the correct words:

(a) Data, collected directly from the source, are called __________ data.

(b) The source of primary data is called ________ source.

(c) When you use the data collected by others or from available source these data are called _____

Note

(d) The source of secondary data is called _______ source.

(e) Directly collected data, from primary/secondary source, are called _________ data.

18.2 Presentation of Data

After collecting the data, the next step is to present in order. The data can also be presented in tabular form. Data can also be presented through figure, graph and chart. Primary aim is to arrange data in such a way so that required information could be drawn from the presentation.

If you want to know the maximum or minimum then you have to arrange the data in ascending/descending order. Then find the maximum/minimum. This is only possible when the observations are less in numbers, when the data is large it may be difficult to know this information. You would like to present the data in a correct form. The simple way to present the data is ascending/descending order. The data from the previous page is arranged below:

Descending order

(i)	72	72	65	65	57	57	57	57	49	49
	45	45	38	38	30	30	30	25	25	20

Ascending order

(ii)	20	25	25	30	30	30	38	38	45	45
	49	49	57	57	57	65	65	72	72	

From (i) & (ii) above you can say that maximum number is 72 and minimum number is 20 and others are in between. Now you are in a position to answer some of the questions of your father, for others we need to do some more activities.

18.3 Use of frequency to present the data

Now you will know that which observation occur maximum number of times and which less number of times. How many students got failed and how many got passed, how many did get marks more than 60%, how many did get the same marks etc? To answer all these questions, the data will be written in the form of a table from the previous question.

Note

Marks	Tally marks	Frequency
20		1
25		2
30		3
38		2
45		2
49		2
57		4
65		2
72		2

Remarks: Instead of writing again and again, we use tally marks '/' mark represents a particular experiment/event represents the frequency of a particular event. We see from the above table, 20 comes 01 time, 25, 38, 45, 49, 65 & 72 come two times and 30 comes 3 times. The above table is called frequency distribution.

In the above example, 4 students have obtained 57 marks when only one student has obtained 20 marks. Maximum number of students obtained 57 marks, hence it's frequency is 4. Maximum marks obtained are 72, minimum marks obtained is 20.

The difference between the maximum & minimum is called the "Range" of data. Hence in the above example the "Range" is 72 - 20 = 52.

Intext Questions 18.2

1. Write the following data in ascending order and find "Range".

25	23	54	85	62
27	19	54	59	48
42	37	61	74	81

2. In your neighbourhood, the number of children in 24 families is:

4	3	5	2	4	1	0	2
3	3	5	1	2	4	3	4
2	1	6	2	3	2	2	3

Make a frequency distributer table using "Tally marks" also find the "Range".

18.4 Classification of data in different categories

Suppose you want to know that how many students have been declared failed or obtained less than 33 marks. Here you will divide the data in two categories. No. of students obtained marks less than 33 failed and the number obtained marks 33 (failed) and above 33 (pass).

Category/class	Marks	Tally marks	Frequency
Failed	20 25 25 30 30 30	𝍸	6
Passed	38 38 45 45 49 49 57 57 57 57 65 65 72 72	𝍸 𝍸 |||| 𝍸 𝍸 ||||	14

Remarks: '\' symbol in tally marks denotes 5^{th} observation. Hence '\' denotes 5 this makes counting easy by 5-5. In the above example 6 students are failed in Maths. From this method you may know the number of students obtained more than 50% or 60% marks and how many in between 50. After this you will be able to answer all the questions raised by your father. Can you recall the work you have done till now, data collect, divided into two categories.

All what you have learnt so far make you happy as all this comes under "Statistics".

Statistics is defined as a collections, presentation, analysis of numerical data and drawing inference in a scientific manner.

$\therefore$ There are three steps:

(i) Collection of data

(ii) Presentation of data

(iii) Drawing inference

Statistics deals with collection, presentation, analysis of numerical data related to persons or objects in systematic manner.

Now you have some knowledge of statistics. Now you can collect data in your neighbouring village and compare the density of population of two villages.

Intext Questions 18.3

Following are the marks obtained by 10 friends in social science:

63	45	54	72	55
48	59	66	68	42

Note

Your mother wants to give two chocolates to those who scored more than 60 marks and one for those obtained less than 60 marks. With the help of frequency distribution table, find how many will get 2-2 and how many will get 1-1 (chocolate) Tell your mother number of chocolates to be distributed.

18.5 Bar Graph

There are 20 students in your class. If you want to see daily attendance in a week draw a bargraph of these data which will look like figure 18.1. Height of each bar represents it's numerical value. This shows the clear form of data. Suppose your teacher asks you that.

- On which day all students were present?
- On which day least number of students attended school?
- On which days equal number of students were present?
- How many students were present on Wednesday? Etc.

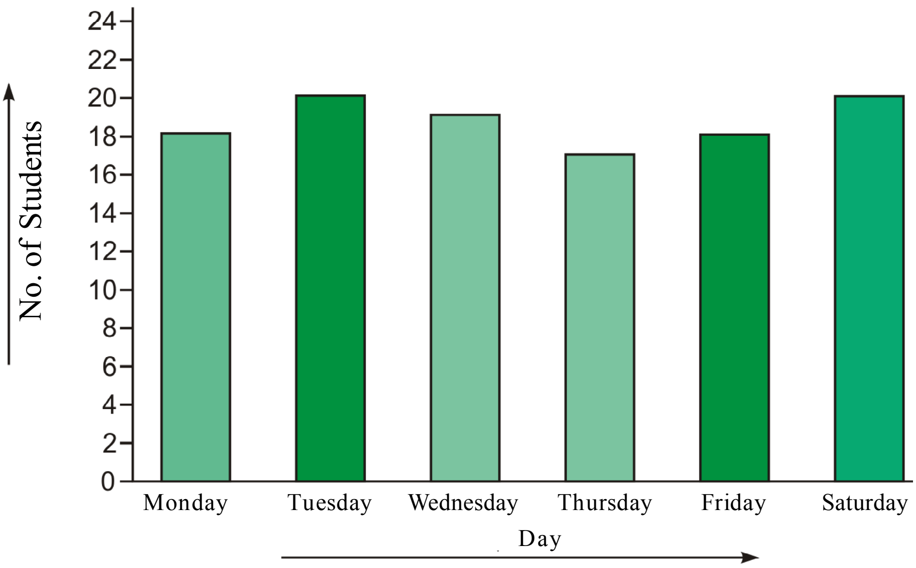


Figure 18.1

To answer these questions, you need to read the paragraph and answer the above questions.

18.6 Reading the Bar Graph

You will read the above graph in the following manner:

(i) This graph reflects the number of students present on a day of the week.

(ii) In the horizontal the names of days are written.

(iii) The column represents the number of students present on that day.

(iv) Each bar represents a particular day.

(v) There are six bars each for one day with the same width and same gap between them.

18.7 Explanation of Bar Graph

Let us see how will you answer the questions asked by your teacher

(i) On which days all the students were present? You know that there are 20 children in the class. Look at the days when the bar touches 20 marks then all students are present. So Tuesday and Saturday all students are present.

(ii) On which day minimum number of students are present? Look at the shortest bar, this is Thursday, when only 17 students are present.

(iii) When the same number of students came to the school. The days when the height of the bars is same, will tell that same number of students are present on Monday & Friday and number of students on Tuesday and Saturday are same.

(iv) How many students are present on Wednesday? Look at the height of the bar on wednesday. This is infront of the number 19 on wednesday, hence 19 students are present on Wednesday.

Intext Questions 18.4

1. Read the following bar chart and answer the following questions.

   (a) What informations does the bar give?

   (b) Name the planet, which has maximum number of ....

   (c) Name the planets, who do not have any ....

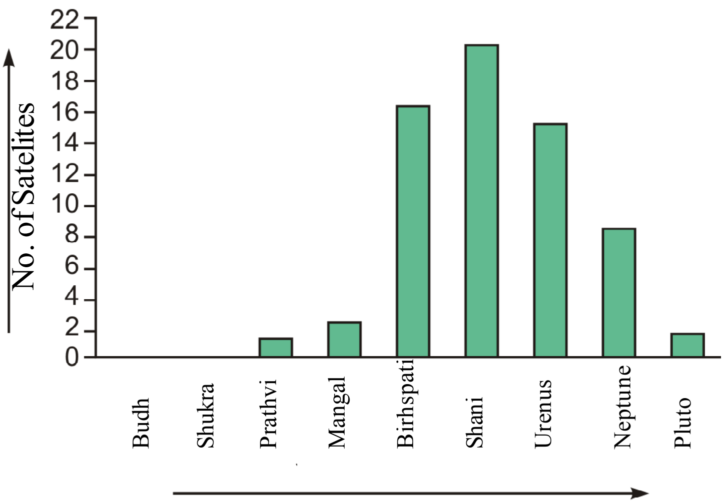


Figure 18.2

18.8 Drawing Bar Graph

Before drawing a Bar Graph, you need to remember the following:

(a) The width of all bars to be same.

(b) Same distance between two bars.

(c) The height of the bar will be in proportion to the number they represent.

Now you can read the Bar chart 18.1 and tell the number of students present on a particular day:

Monday	:	18
Tuesday	:	20
Wednesday	:	19
Thursday	:	17
Friday	:	18
Saturday	:	20

You can draw bar chart/graph on a paper/graph paper. First we will learn, how do we draw a graph on a paper.

Steps:

(i) Draw a horizontal line and a vertical line crossing it at a point.

(ii) On the horizontal line write the names of week and on the vertical line students number from fig. 18.1.

(iii) As you have six days, draw six bars of equal width and equal distance between them. The height will be as the number of students present on that day.

(iv) On the vertical line, with the help of a scale mark the number equal to the students on that day. Suppose 1 cm for one student e.g. for 18 students 18 cm etc.

(v) Each bar will represent a day and write below the bar, name of day and height according to the number of students present as per scale choosen.

(vi) To make the bars attracting you may fill them with colours and you may get the fig. as shown below 18.3.

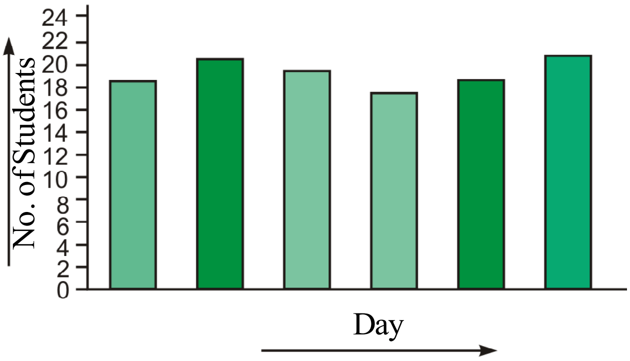


Figure 18.3

Intext Questions 18.5

1. The following information is related to the vehicles in your village.

Scooter	15
Motor Cycle	22
Car	12
Tractor	8
Truck	10

Represent this data by Bar Graph

18.9 Need of appropriate scale

In the previous example we have shown 1 cm as student on the vertical side. One cm height will represent one student on the bar. As 17 students are present on Thursday, hence 17 cm will represent 17 students on the vertical bar. Similarly the height for Friday is 18 cm for 18 students.

Here, total number of students were only 20. Hence the highest bar was representing 20 cm for Monday and Friday. This can be shown on paper easily. Please look at the situation when you have to represent the data of population of villages and to draw the bar graphs with figures involving 1000 and more numbers.

Let the population of five villages are shown below:

A	5000
B	3500
C	4500
D	2000
E	5500

How will you show these figures on paper? To solve this problem you will derive a method to take a scale so that the large figures are converted to small figures proportionality. You may choose a scale 1 cm for 500 people to reduce the height substantially. For 5000, the height will be 10 cm.

For village A the height of the bar = 10 cm (5000 $\div$ 500) = 10

For village B the height of the bar = 7 cm (3500 $\div$ 500) = 7

For village C 4500 $\div$ 500 = 9 cm

For D 2000 $\div$ 500 = 4 cm

For E 5500 $\div$ 500 = 11 cm

Now you can draw the bar graph with the help of this new scale lengths.'

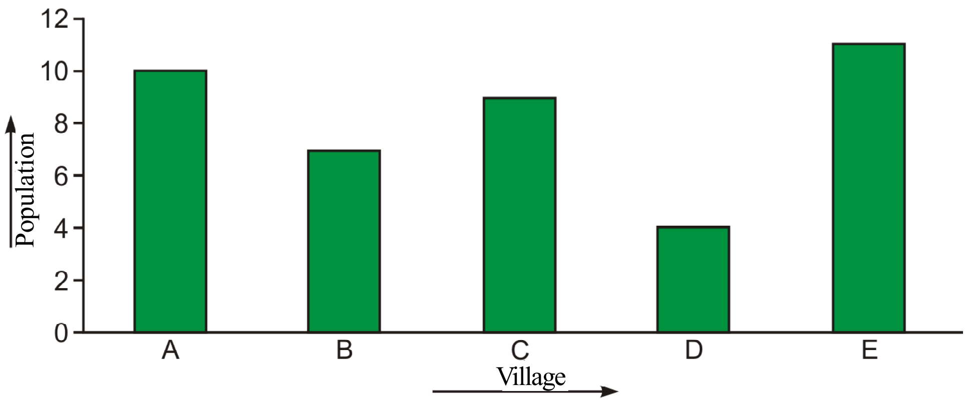


Figure 18.4

Now you can read the bars from the fig. used 18.4 above and you can find the population of a village, maximum population village, village with minimum population. You can compare the population of two villages.

Why do we need an appropriate scale?

(i) This will help us to decide the proportionate height of the Bar.

(ii) This will help us to draw the Bar Graph on paper by reducing the figures proportionately.

(iii) This gives us simple method of interpretation/drawing inference.

(iv) This helps in making the bar in proportion and looking good, neither two small nor two large.

Intext Questions 18.6

Ashok obtained following marks in an examination:

English	70
Hindi	80
Science	65
Social Science	55
Mathematics	85

Show this information by a Bar Graph.

18.10 How to draw bar graph on a graph sheet?

Let us now see how do we draw a bar graph on a graph sheet? Suppose in your neighbouring village, the number of scooters are:

Village		No. of Scooter
A	=	136
B	=	78
C	=	120
D	=	108
E	=	94

You want to represent these data on a graph sheet. Follow the steps given below:

Step 1 : Take a graph sheet

Step 2 : Draw two lines, one horizontal and other vertical perpendicular to each other.

Step 3 : To represent villages take the horizontal line and to represent the number of scooters take the vertical line. Horizontal line to be named as x-axis and the vertical line to be named as y-axis.

Step 4 : As the number of villages is 5, we need to draw 5 bars with equal width and equal space between two adjacent bars. Take a big square as the width of the bar and equal space between two bars. Now draw lines as the width of the bar and other lines at equal distances.

Step 5 : Now as the number of scooters is large, take an appropriate number to represent the number of scooters, take one big square as 20 scooters and one small part equal to 2 scooters. Accordingly get the height of each bar.

Village A has 136 scooters. The length of bar for village A = 136 $\div$ 20 = 6.8

6 big squares and 8 small parts

Similarly

For village B height = 78 $\div$ 20 = 3.9 or 3 big and 9 small parts

For village C height = 120 $\div$ 20 = 6 big square

For village D height = 108 $\div$ 20 = 5.4 or 5 big squares and 4 small parts

94 $\div$ 20 = 4.7 or 4 big squares and 7 small parts

Now draw the bar graph as per the heights of number of villages:

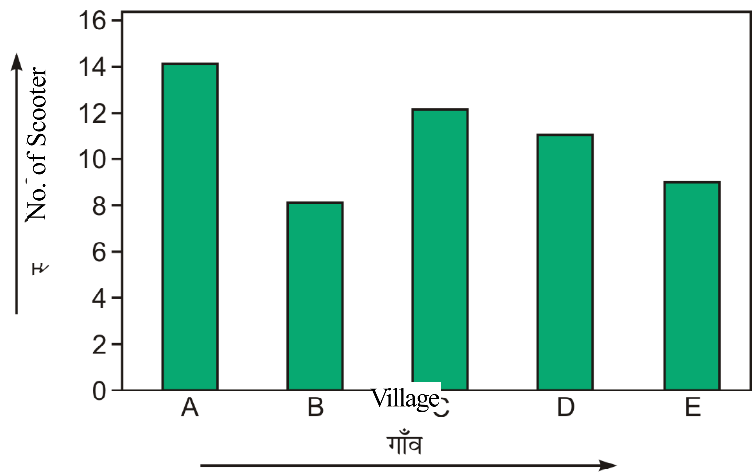


Figure 18.5

Following important points to be kept in mind for drawing bar graph

1. Make it clear on the bar graph that for what the bar is drawn.
2. The method of making the scale for x-axis and y-axis separately in this question on vertical side 2 scooters = 1 small part or 1 big part = 20 scooters.
3. Here x-axis denotes the names of villages and y-axis the number of scooters in a particular village.
4. Each bar has to be named representing what.

Note

Intext Questions 18.7

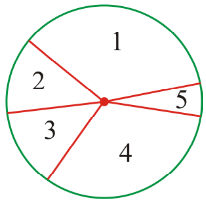
Time taken by the planets moving around the solar system is given below:

Birhspati	–	11.9 yrs
Saturn	–	29.5 yrs
Urenus	–	84 yrs
Neptune	–	165 yrs
Pluto	–	248 yrs

Draw the bar graph from the above data:

18.11 Pie-chart

The number of forests in five states of India is shown by a pie-chart:



If we assume that the states with more number of forests will receive more rain fall then (fig. 18.6 (i))

- Which state has maximum rainfall?
- Which state has least rainfall?

In a parliamentary selection four candidates were in fray. Pie-chart in fig. 18.6 (ii) shows their votes they received. Answer the following questions with the help of pie-chart

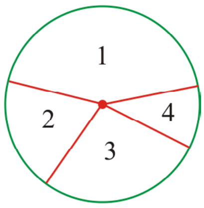


Figure 18.6

- Which candidate got maximum votes?
- Which candidate got least votes?

You know that the sum total measure of all angles formed at the center of the circle is 360°. The largest angle formed at the center is corresponding to the votes received by candidate number 1. Similarly the least angle subtended at the center is corresponding to the number of votes received by candidate number 4.

Let us now understand this with the help of an example:

Example 18.1: For a school in Delhi, the number of students in classes 6 to 10, is shown in the following table

Class	6	7	8	9	10	
No. of students	216	180	150	110	64	Total 720

For drawing a pie-chart for this data, we first find the total number of students in the school. Find the angle for 1 student to be made at the inter $(360 \div 720)^{\circ} = \frac{1^{\circ}}{2} = 0.5^{\circ}$.

Then find the measure of the angle for the number of students in each class

for class 6, the measure of angle $= \frac{360^{\circ}}{720} \times 216 = 108^{\circ}$

for class 7, the measure of angle $= \frac{360^{\circ}}{720} \times 180 = 90^{\circ}$

for class 8, the measure of angle $= \frac{360^{\circ}}{720} \times 150 = 75^{\circ}$

for class 9, the measure of angle $= \frac{360^{\circ}}{720} \times 110 = 55^{\circ}$

for class 10, the measure of angle $= \frac{360^{\circ}}{720} \times 64 = 32^{\circ}$

Now draw a circle of any radius (not too small) and make angles, corresponding to the number of students, using protector and mark the angle with corresponding sector and also write the class to represent by this sector as shown in fig. 18.7

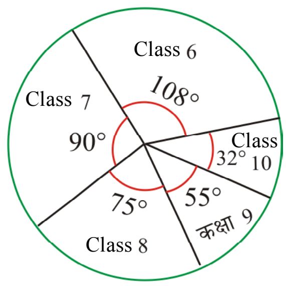


Figure 18.7

Example 18.2: The interest of class VII

Student in various sports (in %) is given below

Name of sport	Cricket	Football	Hockey	Handball	Volley Ball	Total
Interest in sport %	65	15	10	3	7	100

Draw a pie-chart for the above data

Solution:

Name of Sport	Interest in Sport	Angle at the center of circle
Cricket	65	$\frac{65}{100} \times 360^{\circ} = 234^{\circ}$
Football	15	$\frac{15}{100} \times 360^{\circ} = 54^{\circ}$

Hockey	10	$\frac{10}{100} \times 360° = 36°$
Hand Ball	3	$\frac{3}{100} \times 360° = 10.8° (Approx - 11°)$
Volley Ball	7	$\frac{7}{100} \times 360° = 25.2° (Approx. - 25°)$
Total Students	100	Total angle 360°

The pie chart for the above data is shown in figure 18.8 when the data is represented by the sectors of a circle corresponding to the data, this is called "Pie-Chart".

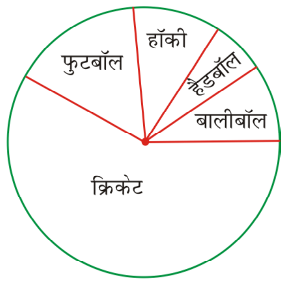


Figure 18.8

Example 18.3: The agriculture produce of a farmer is shown in the pie-chart in fig. 18.9. If the total produce is 720 quintal, then from the pie-chart find the produce for each crpp.

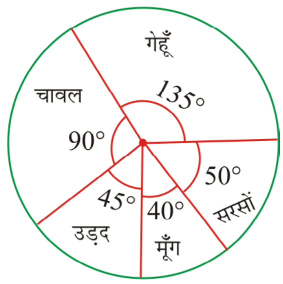


Figure 18.9

Solution:

Total produce - 720 quintal

So, $360° = 720$ quintal

$1° = 2$ quintal

$\therefore$ Angle at the center for wheat produce = 135°

$\therefore$ The wheat produce 135 x 2 = 270 quintal

Rice produce 90 x 2 = 180 quintal

Split black gram produce 45 x 2 = 90 quintal

Split green gram produce 40 x 2 = 80 quintal

Mustard produce 50 x 2 = 100 quintal

Intext Questions 18.8

1. No. of students in a hostal, speaking different languages, is given in the table below - represent the data by a pie-chart.

Language	Hindi	English	Marathi	Tamil	Bangla	Total
No. of Students	40	12	9	7	4	72

2. Monthly income of a family is ₹12000. Monthly expenditure is given in the table below. Draw a pie-chart for these data.

Event	House Rent	Food	Education	Entertainment	Health
Expenditure	₹ 1500	600	1200	1800	1500

Let us Revise

- Data is the numerical observation of a particular group.
- In the group of data each number represents an observation.
- The original data collected & used by you are called primary data.
- The source from where the data is collected directly is primary source.
- The data collected by some other person or official record are secondary data.
- The source from where the secondary data is taken is secondary source.
- The data collected from either from primary/secondary source is ungrouped data.
- Once the data is collected it can be presented in a table/figure/graph/chart.
- The number of times an observation is made is called frequency.
- The data is shown by frequency in the frequency distribution table.
- The difference of maximum value & minimum value of observation is Range.
- We use tally marks to make it easy for recording individual observations.
- Conventionally we use four tables straight and fifth as diagonal to represent group of five.
- Statistics is defined as a collection, presentation, analysis of numerical data and drawing inference in a scientific manner.
- Three steps involved in statistics:
  (i) Collection of data
  (ii) Presentation of data
  (iii) Drawing inference
- Data are represented in tabular or graphical form.
- Bar chart/Bar graph and pie-chart are pictorial representation of data.
- The pictorial form of representation of data is also called pictograph.

Note

- The width of bars and the gap between two bars is same in drawing bar graph.
- All the bars should be treated as a line, so that the height is true representation of data.
- Bar graph can be drawn on a plane/graph paper.
- Bar graph helps in drawing correct and immediate inference.
- We need a correct measure to make the height of each bar proportional to the data given. This helps to draw the graph on a paper using the available space.
- For drawing a pie-chart, we need to find the total of data and for each separately? Find the measure of the angle be drawn at the center.

Exercise

1. Fill in the blanks with correct word:
   (a) The number of times an observation is made is called it's __________.
   (b) In a __________ table data are shown as per their number.
   (c) The difference of maximum and minimum observations is called __________.
   (d) After collection of data next step is _________ in order.
2. What are different sources of data collection?
3. What are the different types of data?
4. What are ungrouped data?
5. Differentiate the primary and secondary data.
6. What does the range of data indicate?
7. What is the range of data if maximum observation is 80 and minimum is 35?
8. If the range of data is 42 and the upper value data is 68, what is the least observation?
9. The least value & range of data are 27 & 35 respectively. Find the maximum value.
10. The height in cm of 15 girls in a class is shown below:

84	92	88	99	105
96	82	100	110	115
84	80	91	101	93

Find out

(i) Height of the shortest girl

(ii) Height of the tallest girl

(iii) Range of data

11. There are 20 families in a village. Below given the number of family members in each family:

5	4	6	3	7	6	4	5	8	4
6	5	5	5	6	4	7	5	9	7

Prepare a frequency distribution table and answer the following

(a) Total population of villages?

(b) How many members in the smallest family?

(c) How many families are with less number?

(d) How many members are there in the largest family?

(e) How many such families with maximum number of member?

(f) What is the number of families with maximum frequency?

(g) Find the range of data?

12. If you want to know that how many boys & girls are attending the school, what will you do? From where you will collect data/ will it be a primary source/secondary source.

13. What is a Bar Graph?

14. Why is it easy to draw inference from bar graph/chart as compared to frequency distribution table?

15. Why do we need a specific measure to present data in bar chart form?

16. In the following bar chart, the income pattern of a family for last 5 years is shown: Read the bar-chart carefully and answer the following questions

(a) In which year was the maximum saving?

(b) In which year was the minimum savings?

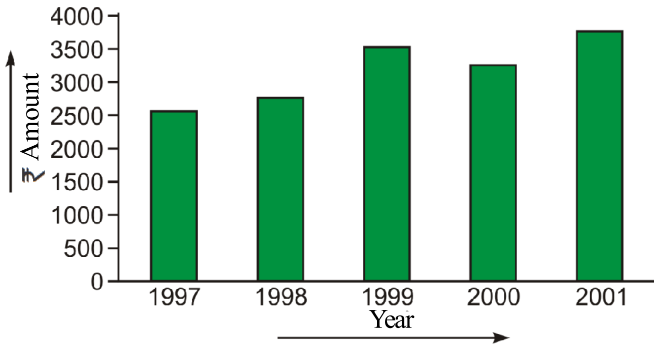


Figure 18.10

(c) What was the amount of savings in the year 1999?

(d) In which year the savings was ₹2500.

17. Read the bar chart 18.11 and answer the following questions:

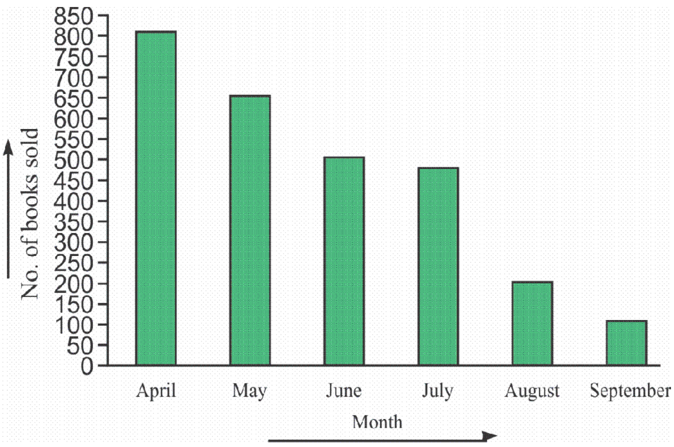


Figure 18.11

(a) In which month, sale was maximum?

(b) In which month, sale was minimum?

(c) How many books were sold in June?

(d) In which month 800 books were sold?

18. The literacy rate in the country is given below (six decades)

Years of census	Literacy %
1951	18 33
1961	28 30
1971	34 45
1981	43 57
1991	52 21
2001	65 38

Draw the bar chart for the above data

19. According to 2001 census, the literacy rate of following states is given. Draw bar-chart for these data.

State	Literacy rate %
Kerala	90.92
Assam	64.28
Andhra Pradesh	61.11
Uttar Pradesh	57.36
Bihar	47.53
West Bengal	69.22

20. Population density of our country is given below

Census year	Population density
1951	117
1961	142
1971	177
1981	216
1991	267
2001	324

Using a graph paper draw the bar graph from the above data.

21. From the below drawn bar-chart answer the followed by

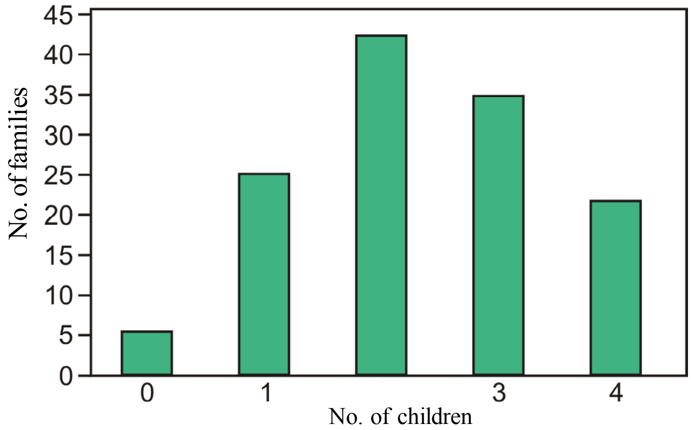


Figure 18.12

(a) How many families have 3 children?

(b) How many families do not have any child?

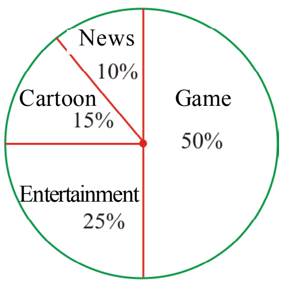
(c) How many families have 2 or less than 2 children?

(d) How many families have more than 3 children?

22. From the pie-chart in fig. 18.13 answer the following questions:

(a) What type of programmes are seen most?

(b) What type of programmes are least seen?



Persons viewing number of different channels on TV

Figure 18.13

Answers

Intext Questions 18.1

(a) Primary (b) Primary (c) Secondary (d) Secondary

(e) Non-Refined

Intext Questions 18.2

Q1. Range 66

Q2,

<table>
<tr><th>Number of Children</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>0</td><td>|</td><td>1</td></tr>
<tr><td>1</td><td>|||</td><td>3</td></tr>
<tr><td>2</td><td><s>||||</s> ||</td><td>7</td></tr>
<tr><td>3</td><td><s>||||</s> |</td><td>6</td></tr>
<tr><td>4</td><td>||||</td><td>4</td></tr>
<tr><td>5</td><td>||</td><td>2</td></tr>
<tr><td>6</td><td>|</td><td>1</td></tr>
</table>

Intext Questions 18.3

2 for four Friends

1 for 8 in Friends

Total 14 Chocoletes to be distributed by Mother.

Intext Questions 18.4

(a) Bar chart shows the number of satellites of our solar systems.

(b) Saturn

(c) Budh & Shukra have no satellite

Note

Intext Questions 18.5

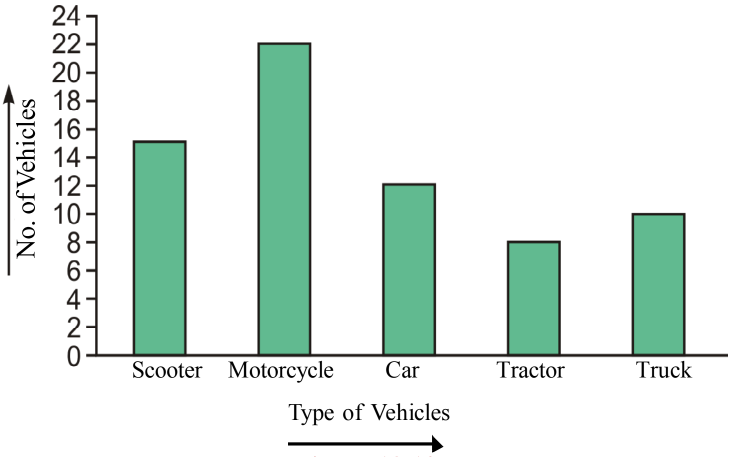


Figure 18.10

Intext Questions 18.6

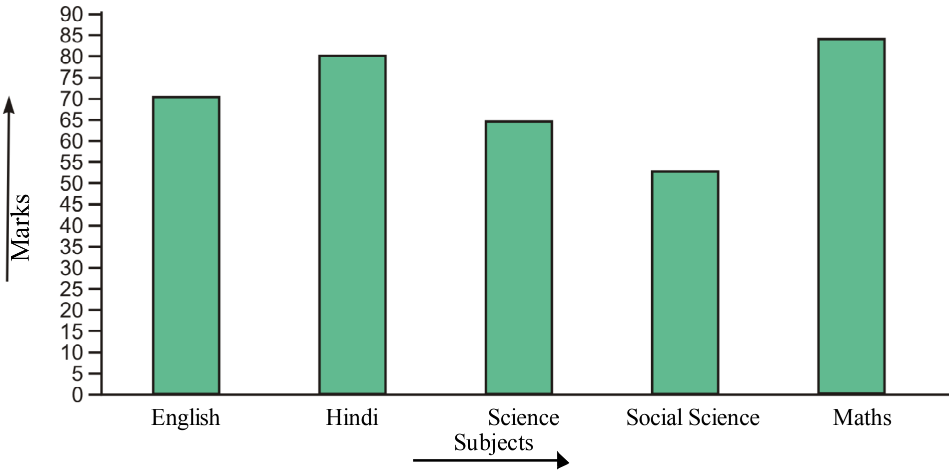


Figure 18.11

Intext Questions 18.7

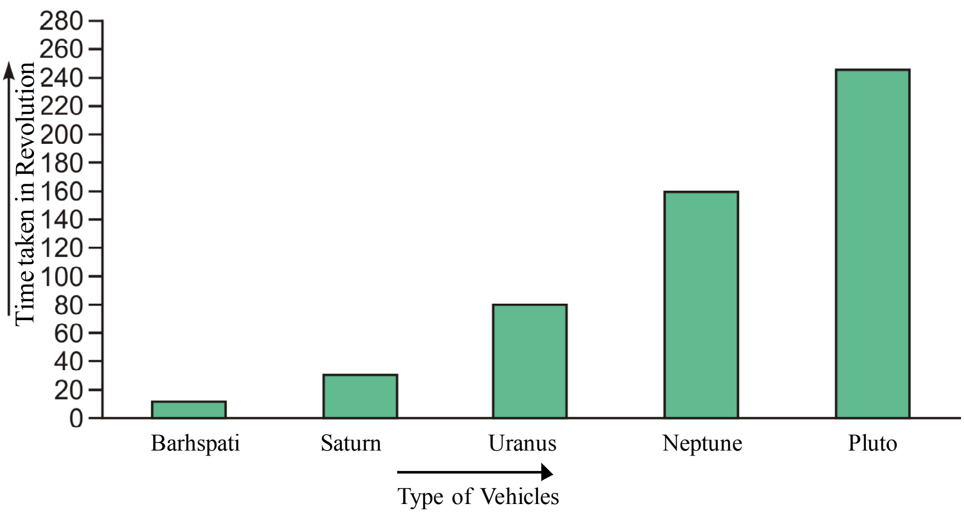


Figure 18.12

Intext Questions 18.8

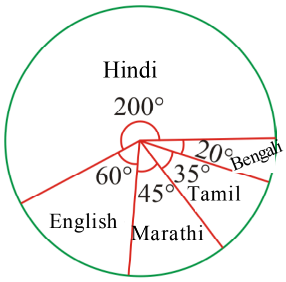
1 

Figure 18.13

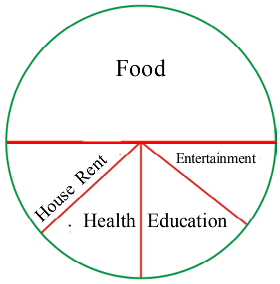
2 

Figure 18.14

Exercise

1. (a) Frequency

   (b) Frequency distribution

   (c) Range

   (d) Presentation

2. Primary source and secondary source

3. Primary data and secondary data

4. The data personally collected through primary/secondary source are ungrouped data.

5.

	Primary Data	Secondary Data
(a)	Data collected for your experiment is primary data	Data collected by other person and used by you is secondary data
(b)	These are from the primary source	collected from second source
(c)	It takes much time	It takes less time
(d)	It is costly	It is not costly

6. The range is the difference between maximum and minimum observation

7. 45

8. 26

9. 62

10. (a) 80 cm

(b) 115 cm

(c) 35 cm

11. (a) 109 (b) 3

(c) 2 (d) 9

(e) 1 (f) 5

(g) 6

12. Either you will collect the data directly from the families or from the school, where the children are studying. Data taken from the families is primary data and the data collected from school is secondary data.
13. Bar chart is the pictorial representation of numerical data, where bars are drawn to represent the data. All the bars of same width and some distance between two bars. Height denotes the frequency.
14. In the bar chart data is represented by bars representing proportionally the values for each bar. Looking at the bars the information is clear as bars are more attractive then the numerical data of frequencies. Looking at the height of the bars you can know about the frequency for that bar. Looking at all the bars, you can easily compare the different informations.
15. Following are the reasons for taking an appropriate scale
    (i) To give each bar a proportionate height, so that in the given space all the bars can be errected.
    (ii) Explaination becomes easy.
    (iii) To make all the bars attractive
16. (a) 2001

    (b) 1997

    (c) 3500

    (d) 1997
17. (a) April

    (b) September

    (c) 500

    (d) April
18.

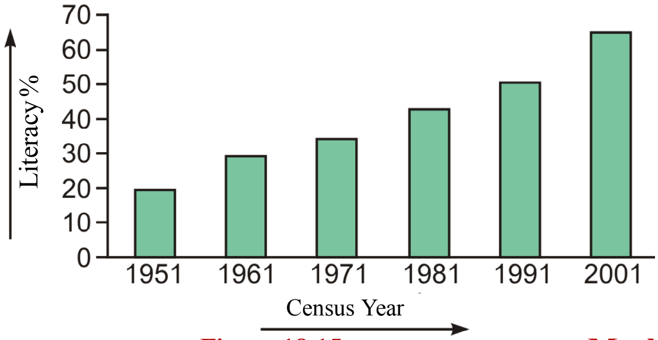


Figure 18.15

Note

19.

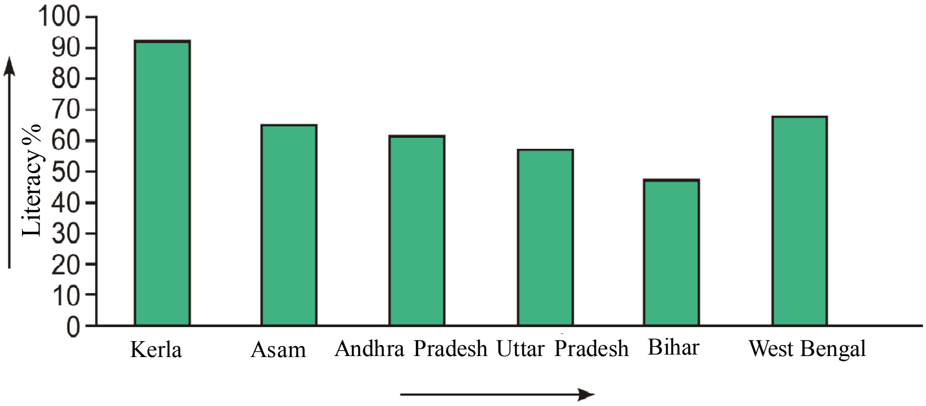


Figure : 18.16

20.

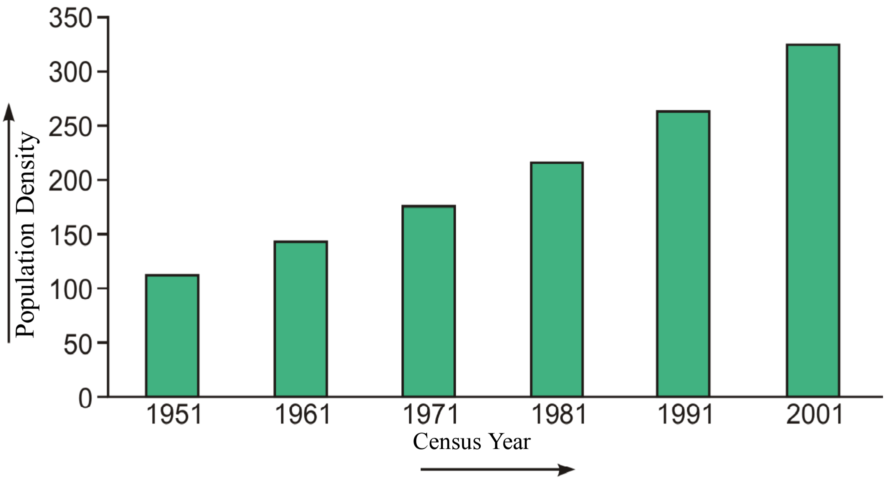


Figure 18.17

21. (a) 35

    (b) 5

    (c) (5+25+42) = 72

    (d) 23

22. (a) Sports

    (b) News

Module - VI

Vedic Mathematics

In India, from the Vedic period, teaching-learning of Mathematics has been followed. Many Indian Mathematicians have significantly contributed in the development of Mathematics.

In this continuity Bharti Krishan Teerathji Maharaj is known for the development of Mathematics. He has written a book on Vedic Mathematics touching new heights in Mathematics. He has explained 16 sutras and 13 subsutras in this book. Book contains 40 lessons. These sutras are explained in the book. He has solved mathematical problem in a very effective manner. The solution of problems through vedic mathematics sutras is very easy and intereasting. People use these sutras and are attracted towords their wonderful solution. With the help of these sutra, the 5-6 line solution can be in one line. Using these sutras the mathematical solutions are tested and verified. Studying the use of these sutras, the creativity of children reaches new height of development. As a result student's interest is developed for learning and understanding mathematical concepts. Vedic sutras have a claimed appreciation of national & international mathematicians because of this in other countries also the vedic mathematics is being followed and appreciated a lot. Using vedic mathematics solutoin of mathematical problems is easy and fast. As a result this becomes interesting & motivating.

19

INTRODUCTION OF VEDIC MATHEMATICS

In ancient education system the solution of Mathematics problems was given very fast in India. But now it is being experienced that learners finding difficulty in solving mathematical problem. Through introduction of Vedic Mathematics, learners have developed interest and facing less problem in solving mathematical problems.

From this lesson, you will learn

- Developing interest in teaching-learning mathematics
- Increase in the confidence level for competitive exams
- To save time by solving problem in shortest possible time
- Developing learner's brain to a level so that he/she could solve their life problems
- Developing reasoning power using vedic mathematics sutras
- Developing self confidence by solving problems using Vedic Mathematics
- Developing speed & accuracy for caculations among learners
- Increasing memory power of learners

19.1 Importance of teaching-learning Vedic Mathematics

- Useful for competitive examinations
- Making modern mathematics interesting
- Increasing numerical calcuations among the learners
- Increasing reasoning power of students
- Solving many problems in less time
- Using Vedic mathematics problem can be solved in only one line

19.2 Vedic Mathematics Sutras and their meaning

Swami Bharti Krishan Teerathji has explained the 16 sutras in the following way:

	Surtas	Meaning
(i)	5 रू 4ग्न 5	One more than the previous
(ii)	रू. 5 रू7 2 5	All from nine and last from 10
(iii)	6 7 52	Vertically and cross-wise
(iv)	4 57 5	Using addition by making '0' at end
(v)	5ग्न 5 28	Equals give answer zero
(vi)	5रू8 5 5ग्न 2'	If one is in ratio, the other one is zero
(vii)	रू श 7 रू 52	Adding and subtracting
(viii)	4य 54य 55 52	By the completion or non-completion
(ix)	रू रू5 52	By calculus
(x)	57 रू2	By the deficiency
(xi)	श 2	Use the average
(xii)	555 रू 2 5	The remainders by the last digit
(xiii)	545' 2' 2	The ultimate & twice the penultimate
(xiv)	' रूरू 4ग्न 5	By one less then the previous one
(xv)	8 5 28	The whole product
(xvi)	85 28	Set of multipliers

19.3 Using double sign digits (Using विनकुलम अंक)

In Vedic Mathematics we use this method many a times. Let us now understand these and their application.

19.3.1 Definition of विनकुलम

When we use positive and negative sign digits together this is called विनकुलम. $1\bar{2}$, 1 is positive and 2 is negative. Using विनकुलम the number operations become easy.

19.4 विनकुलम operation

विनकुलम numbers are used in many Vedic Mathematics Sutras. The use of this method is to convert big numbers in to smaller, but the numbers are so arranged that the number does not change.

Note

Example: 9 =10 -1= $1\bar{1}$, 9 is written as $1\bar{1}$ (one bar one)

In this method we use sutra number 2, which means all from 9 and the last from 10. We can change the following into विनकुलम numbers.

$$8 = 10 - 2 \text{ [here 8 is last]} = 1\bar{2}$$

$$99 = 100 - 1 = 10\bar{1}$$

$$996 = 1000 - 4 = 100\bar{4}$$

$$987 = 1000 - 13 = 10\overline{13}$$

19.5 Addition

In Vedic Mathematics, addition can be done in different ways. Normally, we add ones and tens of two or more numbers. In Vedic Mathematics we can add from left sides. This method is called making a number ending with zero and then add the remaining. In this way learner can add orally.

19.6 Sutra Ending with zero (शून्यांत)

The number at the end of which there is a zero as 10, 100, 1000, 2000, 3000... the addition can be made easy and interesting using this sutra.

Example 19.1: Add 76 + 87

Sol.

```
 7 6
 8 7
_____

_____
```

Step 1: 7 + 8 = 15tens write it 150 for the next step

Step2 : 150 + 6 + 7 = 163

Example 19.2: Add 68 + 53 + 85 + 36

```
 5 3
 8 5
 3 6
_____

_____
```

Step 1 : 6 + 5 +8 + 3 = 22 tens = 220

Step 2 : 220 + 8 + 3 + 5 + 6 = 242

Example 19.3: Add 532 + 674 + 378

Sol.

```
  5 3 2
  6 7 4
  3 7 8
 _______
1 5 8 4
 _______
```

Step 1 : 5 + 6 + 3 = 14 $\Rightarrow$ 140

Step 2 : 140 + 3 + 7 + 7 = 157 $\Rightarrow$ 1570

Step 3 : 1570 + 2 + 4 + 8 = 1584

Example 19.4: Add 632 + 621 + 712 + 821

Sol. 6 3 2
6 2 1

Step 1 : 6 + 6 + 7 + 8 = 27 $\Rightarrow$ 270

Step 2 : 270 + 3 + 2 + 1 + 2 = 278 $\Rightarrow$ 2780

Step 3 : 2780 + 2 + 1 + 2 + 1 = 2786

Example 19.5: Add 937 + 32 + 61 + 635

Sol.

```
  9 3 7
    3 2
    6 1
  6 3 5
 -------
1 6 6 5
 -------
```

Step 1 : 9 + 6 = 15 $\Rightarrow$ 150

Step 2 : 150 + 3 + 3 + 6 + 3 = 165 $\Rightarrow$ 1650

Step 3 : 1650 + 7 + 2 + 1 + 5 = 1665

Intext Questions 19.1

1. 47 + 21 + 63
2. 54 + 72 + 91
3. 65 + 62 + 73
4. 79 + 86 + 14
5. 173 + 241 + 203
6. 776 + 234 + 541
7. 642 + 607 + 242
8. 553 + 345 + 244
9. 643 + 672 + 923
10. 675 + 723 + 644
11. 475 + 67 + 72 + 265
12. 675 + 76 + 34 + 892

19.7 Addition (Sutra-Nikhlam)

Using Sutra-Nikhlam, addition is very easy. The addition of numbers around base/sub base can be done easily. Bases are 10, 100, 1000, ... etc sub-bases 20, 30, 40, 200, 300, ... 2000, 3000, 4000, ... are taken.

Example 19.6 : Add 427 + 99

427 + (100 –1)

= (427 + 100) – 1

= 526 Addition of 10, 100, 1000 number is very easy.

Example 19.7 Add 725 + 597

Sol. 725 + (600 – 3)

= (725 + 600) – 3

= 1325 – 3

= 1322

Example 19.8 : Add 4462 + 2005

Sol. 4462 + (2000 + 5)

= (4462 + 2000) + 5

= 6467

Example 19.9 : Add 7237 + 3999

Sol. 7237 + (4000 – 1)

= (7237 + 4000) – 1

= 11237 – 1

= 11236

Example 19.9 : Add 6546 + 5998 + 7002

Sol. 6546 + (6000 – 2) + (7000 + 2)

= (6546 + 6000 + 7000) – 2 + 2

= 19546

Intext Questions 19.2

1. 67 + 95
2. 72 + 98
3. 65 + 93
4. 665 + 997
5. 720 + 901
6. 925 + 996
7. 1772 + 9005
8. 6725 + 4995
9. 6761 + 1011
10. 7256 + 7999 + 1002
11. 67650 + 998 + 997 + 1005
12. 4970 + 5998 + 6001 + 7997

19.8 Subtraction

For subtraction, Vedic Sutra श्रे 5 is very useful and interesting for the students.

Note

We can subtract from left and this becomes easy and interesting.

Example 19.11: Subtract 28 from 67

Sol. 67 - 28 = 39

Step 1 : $6 - 2 = 4 \Rightarrow 40$

Step 2 : $40 + 7 - 8 = 39$

Example 19.12: Subtract 278 from 624

Sol. 624 - 278 = 346

Step 1 : $6 - 2 = 4 \Rightarrow 40$

Step 2 : $40 + 2 - 7 = 35 \Rightarrow 350$

Step 3 : $350 + 4 - 8 = 346$

Example 19.13 : Subtract 8278 from 3487

Sol. 8278 - 3487 = 4791

Step 1 : $8 - 3 = 5 \Rightarrow 50$

Step 2 : $50 + 2 - 4 = 48 \Rightarrow 480$

Step 3 : $480 + 7 - 8 = 479 \Rightarrow 4790$

Step 4 : $4790 + 8 - 7 = 4791$

Example 19.14 : 6421 from 971

Sol. 6421 – 971 = 5450

Step 1 : $64 - 9 = 55 \Rightarrow 550$

Step 2 : $550 + 2 - 7 = 545 \Rightarrow 5450$

Step 3 : $5450 + 1 - 1 = 5450$

Example 19.15 : Subtract 627 from 72735

Sol. 72735 – 627 = 72108

Step 1 : $727 - 6 = 721 \Rightarrow 7210$

Step 2 : $7210 + 3 - 2 = 7211 \Rightarrow 72110$

Step 3 : $72110 + 5 - 7 = 72108$

Intext Questions 19.3

1. 6470 – 2315
2. 4135 – 1756
3. 6443 – 2172
4. 7485 – 2579
5. 6477 – 3288
6. 2177 – 1288
7. 7667 – 2778
8. 2765 – 1765
9. 6465 – 2578
10. 8875 – 1987
11. 7263 – 2465
12. 9265 – 6227

Note

19.9 Operations using addition and subtraction

The present time is of competition. In the competitive examination mixed operations are often asked to solve. It takes too much time. Using Vedic Mathematics the mixed solution becomes easy and interesting. We can do orally and in one line only using Vedic Mathematics.

Example 19.16 : If we have numbers 65 + 32 + 72 – 93 + 42 – 34

Generally we add positive numbers and then separate negative numbers then we subtract the two results but with the help of Vedic Mathematics we can do it in one line and orally.

Sol.

+ 65
+ 32
+ 72
– 93
+ 42
– 34

84

Step 1 : $6 + 3 + 7 - 9 + 4 - 3 = 8 \Rightarrow 80$

Step 2 : $80 + 5 + 2 + 2 - 3 + 2 - 4 = 84$

Example 19.17 : Solve : 66 + 47 – 76+24 – 54+26

Sol.

+ 66
+ 47
– 76
+ 24
– 54
+ 26

33

Step 1 : $6 + 4 - 7 + 2 - 5 + 2 = 2 \Rightarrow 20$

Step 2 : $20 + 6 + 7 - 6 + 4 - 4 + 6 = 33$

Example 19.18 : Solve : 421 + 512 – 417 + 612 + 723 – 156

Sol.

+ 421
+ 512
– 417
+ 612
+ 723
– 156

1695

Step 1 : $4 + 5 - 4 + 6 + 7 - 1 = 17 \Rightarrow 170$

Step 2 : $170 + 2 + 1 - 1 + 1 + 2 - 5 = 170 \Rightarrow 1700$

Step 3 : $1700 + 1 + 2 - 7 + 2 + 3 - 6 = 1695$

Note

Intext Questions 19.4

1. 437 + 635 – 125
2. 534 – 235 + 432 – 137
3. 567 + 135 – 211 + 145
4. 625 + 137 – 457 + 512
5. 789 – 378 + 512 – 415
6. 882 + 172 – 765 + 121
7. 627 + 672 – 475
8. 997 – 788 + 122 – 234
9. 675 + 321 – 375
10. 887 – 765 + 432 – 317
11. 794 –219 + 425 –317
12. 763 + 411 – 255 – 307

Let us Revise

- Using vinkulam number operations become easy
- Sutra where adding two numbers we get '0' or '00' at the end this makes addition easy. For adding 932+764+378, we first add 932 & 378, gives 1310 then add 764.
- Using sutra शून्यांत, it is easy to subtract from left.
- Calculation with mixed operations.

Exercise

1. Write the name of the person who has written Vedic Maths Book.
2. Write the number of sutras & sub sutras in Vedic Mathematics.
3. Write the objectives of learning Vedic Mathematics.
4. Write four uses of teaching-learning Vedic Mathematics.
5. Write any four sutras of Vedic Mathematics and their Explanation.
6. Define विनकुलम numbers.
7. Which sutra is used in विनकुलम numbers?
8. How विनकुलम operation are useful for us?
9. Convert the following into विनकुलम numbers.

   (i) 97 (ii) 96 (iii) 996

   (iv) 989 (v) 987 (vi) 994

   (vii) 979 (viii) 888 (ix) 999

10. Add the following using sutra शून्यांत:

(i)	67 + 23 + 52	(ii)	172 + 421 + 321
(iii)	462 + 502 + 722	(iv)	822 + 611 + 322
(v)	1421 + 3121 + 1452	(vi)	731 + 514 + 302
(vii)	741 + 517 + 602		

11. Using sutra निखलं add the following:

(i)	522 + 998	(ii)	725 + 997
(iii)	441 + 990	(iv)	627 + 985
(v)	423 + 799	(vi)	627 + 498
(vii)	848 + 397	(viii)	720 + 195

12. Using the sutra to get '0' or '00' at the end, subtract the following:

(i)	721 – 455	(ii)	672 – 344
(iii)	674 – 277	(iv)	872 – 285
(v)	723 – 478	(vi)	811 – 177
(vii)	625 – 256	(viii)	428 – 179

13. Solve following mixed numbers

(i)	247 + 301 – 241	(ii)	47 + 51 – 24 + 52
(iii)	32 + 42 – 22 + 45 – 30	(iv)	241 + 522 – 102
(v)	672 – 172 + 525 – 122	(vi)	422 + 133 – 211
(vii)	4221 + 5112 – 7112	(viii)	5147 – 1241 + 2134

Answers

Intext Questions 19.1

1. 131	2. 217	3. 200	4. 179
5. 617	6. 1551	7. 1491	8. 1142
9. 2238	10. 2042	11. 879	12. 1677

Intext Questions 19.2

1. 162	2. 170	3. 158	4. 1662
5. 1621	6. 1991	7. 10777	8. 11720
9. 7772	10. 16257	11. 70650	12. 24966

Intext Questions 19.3

1. 4155	2. 2379	3. 4271	4. 4906
5. 3189	6. 889	7. 4889	8. 1000
9. 3887	10. 6888	11. 4798	12. 2538

Intext Questions 19.4

1. 947	2. 594	3. 636	4. 817
5. 508	6. 410	7. 824	8. 97
9. 621	10. 237	11. 683	12. 612

Exercise

1. Jagat guru Swami Bhart Krishan teerath ji

2. 16 sutras, 13 sub-sutras

3. (i) Developing interest in teaching learning Mathematics
   (ii) To save time by solving problems in less time
   (iii) To increase interest in the development of mathematics
   (iv) Increasing exactness in calculations for the learners

4. (i) Use for competative exams
   (ii) Useful in making mathematics simple and interesting
   (iii) To increase calculation capabilities of learnear
   (iv) Answer could be done in only one line

(i) 5 रू4ग्र 5 One more them the previous

(ii) रू. 5 स्र्7 2 5 All from 9, last from 10

(iii) 6 7 52 Straight/diagonal or bath

(iv) 4 57 5 Transpose and adjust (Transpose and apply)

Note

7रू 8 2 numbers are those where positive and negative both types of digits are there.

7. In 7रू 8 2 Sutra रू. 5 रूा 2 2 5 is used.

8. 7रू 8 2 is the method to convert large numbers into small numbers, which makes calculation simpler.

9. (i) $10\bar{3}$ (ii) $10\bar{4}$ (iii) $100\bar{4}$

(iv) $100\bar{1}\bar{1}$ (v) $10\bar{1}\bar{3}$ (vi) $100\bar{6}$

(vii) $10\bar{2}\bar{1}$ (viii) $1\bar{1}\bar{1}\bar{1}$ (ix) $100\bar{1}$

10. (i) 142 (ii) 914 (iii) 1686

(iv) 1755 (v) 5994 (vi) 1547

(vii) 1860

(i) 1520 (ii) 1722 (iii) 1431

(iv) 1612 (v) 1222 (vi) 1125

(vii) 1245 (viii) 915

(i) 266 (ii) 328 (iii) 397

(iv) 587 (v) 245 (vi) 634

(vii) 369 (viii) 249

(i) 307 (ii) 126 (iii) 67

(iv) 661 (v) 903 (vi) 344

(vii) 2221 (viii) 6040

20

APPLICATION OF VEDIC MATHEMATICS

In the previous chapter we have been acquainted with sutras. Vedic Mathematics Sutras are not helpful in only solving mathematics problems but also a part of life. Using Vedic Mathematics Sutras are also helpful in making our life stress free.

Arithmetic, Algebra and Geometrical problems are solved with the help of these sutras. In this chapter we shall learn multiplication, squaring, cubes, square root & cube root of numbers.

From this lesson, you will learn

- Multiplication of two numbers
- To find the square of numbers
- To find the cube of numbers
- To find the cube root of numbers

20.1 Multiplication - first method - Sutra एकन्यूनेन पूर्वेण

Meaning of this sutra is, one less than the earlier number. This method is not used for multiplication of all numbers. We can only multiply with numbers where all the digits of multiplier are 9, any digit could be any other number.

Example 20.1: Solve 524×999 In the multiplier all digits are 9 and in the multiplicand 5, 2, & 4 are digits

Left Side : 524 – 1 = 523	Step 1 : Answer will be of two parts left side & right side. Subtract 1 from the number other than '9' digit number
Left Side : 999 – 523 = 476	Step 2 : Subtract the result from 999
∴ 523476	Step 3 : Write the two results together as left & right in order

Note

Example 20.2 : Solve 6251×9999

Sol. Left side number - 1 = 6251-1=6250 (1)

Right side number - The result of step (i) = 9999-6250

= 3749 (ii)

∴ Result will be writing (i) & (ii) as left & right combine

Answer : 62503749

Example 20.3 : Solve 372 × 9999

Sol. Out of the two numbers as shown above

Left number -1 = 372-1=371 (i)

Right number result obtained in (i)

= 9999 – 371= 9628 (ii)

Answer : 3719628

Example 20.4 : Solve 67246 × 9999

Let number -1= 67245 (i)

Right number - result from (i)

= 9999 - 67245 Which is not possible

67245 9999 (ii)

Step 1 : Write the second number along with the result of (i)

∴ 672459999

-67245

672392754

Step 2 : Subtract the result obtained in (i)

∴ ii is obtained by multiplying

The number 67245 by 1000 instead of 9999 say one more time hence subtract 67245

Example 20.5 : Solve 56729 × 999

Answer : 56728999 - 56728 = 56672271

Intext Questions 20.1

1. 4567 × 9999
2. 7250 × 9999
3. 7219 × 9999
4. 5672 × 99999
5. 70421 × 999999
6. 61234 × 999999
7. 6241 ×999
8. 42157 × 9999

Note

9. 64725 × 99999

10. 346721 × 999999

11. 50721 × 999

12. 74252 × 999999

20.2 Multiplication Sutra -

This sutra also helps in multiplication. This is applicable when the sum of ones digits of two numbers is 10 and the remaining digits of two numbers are same. Example 56 × 54. These 6 & 4 make to and the other digits is same

Example 20.6 : Solve 53 × 57

Sol. 53 × 57 — Step 1 : Multiply the ones digits and write them 3 × 7 = 21 (i)

= (5+1)×5/3×7 — Step 2 : Add 1 to the left digit and multiply by the same digit (5+1) × 5 = 30 (ii)

$= 6 \times 5/3 \times 7$

= 3021 — Write the two results as (ii) (i) together say 3021

Example 20.7 : Solve 74×76

Sol. $(7+1) \times 7/4 \times 6$

$= 8 \times 7/4 \times 6$

= 5624

Example 20.8 : Solve 102 × 108

Sol. $(10+1) \times 10/2 \times 8$

$= 11 \times 10/2 \times 8$

= 11016

Example 20.9 : Solve 291 × 299

Sol. $(29+1) \times 29/1 \times 9$

$= 30 \times 29/1 \times 9 = 87009$ [1×9=9. But number be in two digits so we write 1×9=09 on the right side]

Example 20.10 : Solve 992 × 998

Sol. $100 \times 99/2 \times 8$

= 9.900/16

= 990016

Intext Questions 20.2

1. 47 × 43	2. 34 × 36
3. 64 × 66	4. 104 × 106
5. 203 × 207	6. 193 × 197
7. 294 ×296	8. 404 × 406
9. 502 × 508	10. 392 × 398
11. 491 × 499	12. 595 × 595

20.3 Multiplication Sutra (Base-such base)

Multiplication Sutra 5 2 , is very simple and easy, this is used for numbers which are near to base/sub base. 10 or powers of 10 is called a base. Sub-base are those numbers which all the multiples of 10, as 20, 30, ... 200, 300, ... 2000, 3000 ... etc. Use of deviation is also done in this sutra. Deviation is the number obtained by subtracting/adding to number base the number from base/ sub-base. As 7 is deviation in 107 form 100 base), 15 is the deviation in 1015 and 7 is also deviation in 993 (∵ 100-7=993)

Example 20.11 : Solve 104 × 109

Sol. Numbers Deviation

Numbers	Deviation
104	4
109	9
104 + 9	/ 4 × 9

113/36 ⇒ 11336

Step 1 : At first write the deviation

Step 2 : Multiply the two deviation. This becomes the right part

Step 3 : First numbers & deviation of second or second & deviation of first are added, left part

Step 4 : Write left & right part together in order left & right

Note : If the multiplication of deviations is single digit then put '0' on left to make it two digits or two as the case may be to make it equal to the numbers of '0's in the base

1001	1
1009	9
1010	1009 ⇒ 1010009

Here the base is 1000, so use the product in three digits

Example : 20.12 : Solve 102×124

Sol. Numbers Deviations

124 24 [∴ 102+24=126 and 24×02=48 writing together

Note

102	02	left side]
102+24	/24×02 ⇒ 12648	

Example 20.13 : Solve 97×95

Sol. Number	Deviation
97	–3
× 95	–5
(97 – 5)	/ (–3) × (–5)

⇒ 92/15 ⇒ 9215

Step 1 : The number are smaller than the base ∴ so the dividation will be in (-) and brought of both diviation will be positive

Step 2 : Add the deviation of first to the second numbers or vice verses

Step 3 : Write them from left to right without any symbol

Example 20.14 : Solve 985×975

Sol. Number	Deviation
985	–15
× 975	– 25
(985 – 25)	/ (–15) × (–25)

= 960/375 ⇒ 960375

Intext Questions 20.3

1.	106 × 111	2.	107 × 112	3.	103 × 114
4.	106 × 115	5.	107 × 109	6.	95 × 97
7.	98 × 95	8.	92 × 97	9.	98 × 85

20.4 Sutra निखिलम and आनुरूप्येण (Sub-base)

Using निखिलम Sutra, multiplication of numbers can be done which are near to the sub-base. say 20, 30, 40, ... 200, 300, 400... etc.

Sol. Number	Deviation
602	2
× 606	6
6(602 + 6)	/ 2 × 6

= 6(608)/12

= 364812

Step 1. Multiplication of deviation will form right part of answer

Step 2 : Multiplication of sub-base and the sum of first number+deviation of second 6 (602+6) form the left part

Step 3 : Write the two parts from left to right

Example 20.16 : Solve 705×712

Sol.

Number	Deviation
705	5
× 712	12
7(705 + 12)	/ 5 × 12

$= 7(717)/60 = 501960$

Intext Questions 20.4

1.	405 × 408	2.	225 × 203	3.	508 × 512
4.	709 × 706	5.	909 × 911	6.	765 × 701
7.	806 × 809	8.	807 × 812	9.	606 × 615

20.5 Multiplication Sutra - उर्ध्वतिर्यग्भ्याम्

Using this sutra, multiplication is universal or any number can be multiplied by another number

20.5.1 : Multiplication of two digits number

Multiplication will be in three steps. Let us take one example to understand this

Example 20.17 : Solve 43×57

Sol.

4		3
× 5		7
4 × 5	4 × 7 + 5 × 3	3 × 7

Answer : 2451

Step 1 : Multiplication of ones digits, but we write only the one digit, carry over will be added to next (3×7=21) verticle multiplication

Step 2 : Cross multiplication of the digits and add the carry over from previous step

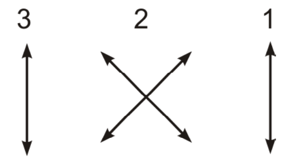
Step 3 : Only write the extreme right digits obtained in step 2 and the rest is carried over and added to the next 4×5=20 (Vertical multiplication)

Step 4 : Add carry over 20+4=24 and write the two results left & right

Application of vedic Mathematics

Example 20.18 : Solve 32×43

Note : Step of multiplicaton



	3	2
	× 4	3
3 × 4	3 × 3 + 4 × 2	2 × 3

Answer = 1376

Example 20.19 : Solve 65×41

Sol.

6	5	
× 4	1	
6 × 4	6 × 1 + 5 × 4	5 × 1

Answer = 2665

Intext Questions 20.5

1. 43 × 52
2. 31 × 63
3. 32 × 55
4. 24 × 36
5. 55 × 62
6. 92 × 93
7. 34 × 43
8. 44 × 65
9. 73 × 46

20.6 Three digits multiplication using & Sutra - उर्ध्वतिर्यग्भ्याम

Using this sutra three digits multiplication can be done easily. Multiplication will be in 5 steps as shown below:

Example 20.20 : Solve 431 × 250

Sol.

		4	3	1
		× 2	5	0
4 × 2	4 × 5 + 2 × 3	4 × 0 + 2 × 1 + 3 × 5	3 × 0 + 5 × 1	1 × 0

Answer = 107750

Step 1: 1 × 0 = 0

Step 2: 3 × 0 + 5 × 1 = 5

Step 3: 4 × 0 + 2 × 1 + 3 × 5 = 17

Step 4: 4 × 5 + 2 × 3 = 26

Step 5: 4 × 2 = 8

Example 20.21 : Solve 509 × 432

Sol.

	5	0	9	
	× 4	3	2	
5 × 4	5 × 3 + 4 × 0	5 × 2 + 4 × 9 + 0 × 3	0 × 2 + 3 × 9	9 × 2

Answer = 219888

Intext Questions 20.6

1. 161 × 432	2. 121 × 922	3. 363 × 432
4. 162 × 454	5. 155 × 335	6. 193 × 412
7. 413 × 305	8. 512 × 205	9. 601 × 712
10. 625 × 441	11. 325 × 433	12. 423 × 812

20.7 To find the square - sutra य

Squaring means multiplying the number by it self. Like multiplication squaring is easy by vedic mathematics. Squaring can be done in one line using sutra यावदूनं

Note

Using sutra यावदूनं , we can find the squares of these numbers which are near to the base say 10, 100, 1000 .... etc. Let us do same examples

Example 20.22 Solve $(107)^2$

Sol. $107 + 7 / 7^2$

$= 11449$

Step 1 : In the right side we square the deviation say $(7)^2 = 49$. Base is 100, hence these will be two digits in right side. if not equal digits to the number of '0's in the base, we make it equal by putting 0, in the left. If the result is more than two digits then take it carryover.

Step 2 : Add deviation to side.

Step 3 : Write them side-by side

Example 20.23 : Solve $(109)^2$

Sol. $109 + 9 / 9^2$

$= 11881$

Example 20.24 : Solve $(113)^2$

Note : $(113)^2$ given us 169, Three digits, hence 69 will be written and 1 is added to the left side to make 127

Sol : $113 + 13 / 13^2$

$= 126\ {}_1 69$ $[13^2) = 169$, as base is 100, with 2 zeros, hence

$= 12769$ There will be two digits third will be taken as carriover $[2\times 2 = 4$ or $= 04]$

Example 20.25 Solve (98)2

Sol. $98\text{-}2/2^2 \Rightarrow 9604$

Note : 98 is less than base 100 $\therefore$ we write it as 98-2 [Hence '–' does not mean subtraction but shows 2 less than 100

$2\times 2 = 4$ but base is 100. so we write as 04

Example 20.26 : Solve (96)2

Sol. $96 - 4 / (4)^2 = 9216$

Example 20.27 : Solve $(89)^2$

Sol. $89 - 11 / 11^2$

$= 89\text{-}11/121$

$= 7921$

Example 20.28 : Solve $(1021)^2$

Sol. $1021 + 21 / (21^2)$

$= 1042441$

Note : Here the base is 1000, hence in the right side we would write the result in 3 digits

Example 20.29 : Solve $(1008)^2$

Sol. $1008 + 8 / (8^2)$

$= 1016064$

Note : $(8)^2$ is 64 only two digits but this is same as 064

Example 20.30 : Solve $(1050)^2$

Sol. $1050 + 50 / 50^2$

$= 1100\ {}_2500 = 1102500$

Example 20.31 : Solve (985)2

Sol. $985 - 15 / 15^2$

$= 970225$

Intext Questions 20.7

1. 105^2	2. 106^2	3. 94^2
4. 97^2	5. 85^2	6. 112^2
7. 1012^2	8. 1015^2	9. 1021^2
10. 975^2	11. 979^2	12. 984^2

20.8 Sutra

This is used to square any number. This can be done in one line.

Example 20.32 : Solve $(42)^2$

Sol. $(42)^2 = 4^2 \;|\; 4 \times 2 \times 2 \;|\; 2^2$

$= 16{+}1 \quad {}_164$

$= 1764$

Step 1 : Square the one's digit $(2)^2=4$

Step 2 : Multiply the two numbers and make it double $(4\times2)\times2 = 16$

Step 3 : Square of the right digit $4^2=16$

Note : Steps 2 & 3 in volves two digits but base is sub-base 40. We write only one digit and second in takes as carryon

Note

Applications of Vedic Mathematics

Exmaple 20.33 : Solve $(64)^2$

Sol. $(64)^2 = 6^2 \mid 6 \times 4 \times 2 \mid 4^2$

$= 40_4\, 9_1\, 6$

Example 20.34 : Solve $(91)^2$

Sol. $(91)^2 = 9^2 \mid 9 \times 1 \times 2 \mid 1^2$

$= 82\ {}_1 81$

Example 20.35 : Solve $(83)^2$

Sol. $(83)^2 = 8^2 \mid 8 \times 3 \times 2 \mid 3^2$

$= 68_4\, 89$

Intext Questions 20.8

1. 31^2	2. 64^2	3. 72^2
4. 62^2	5. 43^2	6. 92^2
7. 84^2	8. 67^2	9. 42^2
10. 54^2	11. 46^2	12. 71^2

20.9 Sutra यावदूनं द्वारा Cube

We take numbers near to the base

Example 20.36 : Solve (98)3

Sol. $(98)^2 = 98 - 2 \times 2 \mid 3 \times (2)^2 \mid (-2)^3$

$= 9412(\overline{08})$

$= 941200 - 08$

$= 941192$

Step 1 : Write the cube of deviation and put '0' as the base is 10

Place = over $0\bar{8}$

Step 2 : Square of the deviation × by 3. $2^2 \times 3 = 12$

Step 3 : Subtract double of the deviation

Example 20.37 : Solve $(105)^3$

Sol. $= 105 + 2 \times 5 \mid 3 \times 5^2 \mid 5^3$

$= 11575\ {}_1 25$

$= 1157625$

Example 20.38 : Solve $(106)^3$

Sol. $(106)^3 = 106 + 2 \times 6 \mid 3 \times 6^2 \mid 6^3$

$= 19_1\ 08_2\ 16$

$= 1191016$

Intext Questions 20.9

1. 104^3
2. 95^3
3. 106^3
4. 99^3
5. 101^3
6. 98^3
7. 97^3
8. 105^3

20.10 Sutra आनुरूप्येण for cube

Example 20.39 : Solve $(41)^3$

Sol. $(41)^3 = 4^3 \mid 3 \times 4^2 \times 1^2 \mid 3 \times 4 \times 1^2 \mid 1^3$

$64 \mid {}_4 8 \mid {}_1 2 \mid 1$

$= 68921$

Step 1 : Right term $1^3 = 1$

Step 2 : Next to right $3 \times$ ten $\times (1)^2$

$3 \times 4 \times 1^2 = 12$

Step 3 : $3 \times (\text{ones})^2 \times (\text{tens})^2$

Step 4 : Cube of digit at ten place

Intext Questions 20.10

1. 53^3
2. 45^3
3. 31^3
4. 42^3
5. 61^3
6. 91^3
7. 31^3
8. 22^3

20.11 : Square root by Sutra - विलोकनम्

Square root of 4 digit number which are perfect squares, can be computed using विलोकनम् Sutra"

20.11.1 : In the table are given ones digit of squares

Digit									
Ones digit of square									

Note

20.11.2 : Ones digit of whole square numbers

Digit at unit place in the number	Digit at unit place in the square root
	or
	or
	or
	or

Example 20.40 : Find the root of 5184

Sol. $\overline{51}\ \overline{84}$

$\sqrt{5184} = 72$

Step 1 : First make pairs from right side

Step 2 : First pairs from left is 51. This is near to 72 also $7^2 < 51$

$\therefore$ Tens digit will be 7 in the square root

Step 3 : Last digit is 4, The last digit of square root will be 2 or 8

To choose about 2 or 8 we shall do following activity. Square of tens digit + tens digit

$7^2 + 7 = 56$

$\therefore$ 51 < 56 Then smaller digit will be taken

Example 20.41 : Find the square root of 7569

Sol. $\sqrt{\overline{75}\ \overline{69}}$

Tens digit is 8 as $8^2 < 75$. In the number ones digit is 9 $\therefore$ last digit of square root will be 3 or 7 $\therefore$ we shall check as $8^2+8 = 72$. Here, 75 > 72 hence bigger digit will be chosen (7)

$\therefore \sqrt{7569} = 87$

Intext Questions 20.11

Find the sqaure root of following

1. 841 2. 361

Note

3. 529 4. 9409

5. 8281 6. 3249

20.12 Cube root of six or less than six digit numbers

Cube root of six or less than six digits can be calculated by sutra विलोकनम

20.12.1 Below given table will help us to find the one's digit or last digit.

Last digit of number	Last digit of cube root

Example 20.42 : Find the cube root of 17576

Sol. $\overline{17}\ \overline{576}$

Step 1 : From right side make groups of 3, on the left may be 1 or 2 or 3.

Step 2 : Last digit of the number is 6. hence the last digit of cube root will also be 6.

Step 3 : The second group is 17, we shall deal in the following way $2^3=8$, $3^3=27$, 17 is in between 8 & 27

$\therefore$ Tens digit will be 2 as $2^3 < 17 < 3^3$

Step 4 : Tens digit is 2, unit's digit is 6 $\therefore$ Cube root is 26

Example 20.43 : Find the cube root of 29791

Sol. $\overline{29}\ \overline{791}$ Last digit is 1 so last digit of cube root is 1

Now $2^3 = 8$, $3^3 = 27$, $4^3 = 64$ $\therefore$ $3^3 < 29 < 64\ (4^3)$

Tens digit is 3 so cube root is 31

Intext Questions 20.12

1. 85184 2. 729 3. 5832

4.	2197	5.	1728	6.	42875
7.	3375	8.	1331	9.	9261

Let us Revise

- The meanig of sutra एकन्यूनेन पूर्वेण is one less than previous
- Multiplication sutra एकाधिकेन and अन्त्योर्दशकेऽपि helps in multiplication when the sum of units digits is 10 and other digits of both numbers are same
- Sutra निखिलम help in multiplication of numbers which are near to the base or sub base
- The cube root of numbers with 6 or less than 6 digits can be calculated by sutra विलोकनम्

Exercise

1. Multiply using sutra एकन्यूनेन

(i) 756×999	(ii) 6545×9999	(iii) 7246×999999
(iv) 6754×999	(v) 8754×99	(vi) 96761×999999

2. Multiply using sutras, एकाधिकेन and अन्त्योर्दशकेऽपि

(i) 42×48	(ii) 292×298	(iii) 394×396
(iv) 992×998	(v) 704×706	(vi) 601×609

3. Multiply using sutra उर्ध्वतिर्यग्भ्याम्

(i) 47×32	(ii) 54×33	(iii) 241×232
(iv) 731×651	(v) 702×721	(vi) 612×723

4. Find square using sutra यावदूनं

(i) 107^2	(ii) 91^2	(iii) 88^2
(iv) 105^2	(v) 988^2	(vi) 977^2

5. Find cube using sutra यावदून

(i) 102^3	(ii) 97^3	(iii) 96^3
(iv) 104^3	(v) 106^3	(vi) 92^3

Answer

Intext Questions 20.1

1. 45665433	2. 72492750	3. 72182781
4. 567194328	5. 70420929579	6. 61233938766
7. 6234759	8. 421527843	9. 6472435275
10. 346720653279	11. 50670279	12. 74251925748

Intext Questions 20.2

1. 2021	2. 1224	3. 4224
4. 11024	5. 42021	6. 38021
7. 87024	8. 164024	9. 255016
10. 156016	11. 245009	12. 354025

Intext Questions 20.3

1. 11766	2. 11984	3. 11742
4. 12190	5. 11663	6. 9215
7. 9310	8. 8924	9. 8330

Intext Questions 20.4

1. 165240	2. 45675	3. 260096
4. 500554	5. 828099	6. 536265
7. 652054	8. 655284	9. 372690

Intext Questions 20.5

1. 2236	2. 1953	3. 1760
4. 864	5. 3410	6. 8556
7. 1462	8. 2860	9. 3358

Intext Questions 20.6

1. 69552	2. 111562	3. 156816
4. 73548	5. 51925	6. 79516
7. 125965	8. 104960	9. 427912
10. 275625	11. 140725	12. 343476

Intext Questions 20.7

1. 11025	2. 11236	3. 8836
4. 9409	5. 7225	6. 12544

Note

7.	1024144	8.	1030225	9.	1042441
10.	950625	11.	958441	12.	968256

Intext Questions 20.8

1.	961	2.	4096	3.	5184
4.	3844	5.	1849	6.	8464
7.	7056	8.	4489	9.	1764
10.	2916	11.	2116	12.	5041

Intext Questions 20.9

1.	1124864	2.	857375	3.	1191016
4.	970299	5.	1030301	6.	941192
7.	912673	8.	1157625		

Intext Questions 20.10

1.	178877	2.	91125	3.	29791
4.	74088	5.	226981	6.	753571
7.	29791	8.	10648		

Intext Questions 20.11

1.	29	2.	19	3.	23
4.	97	5.	91	6.	57

Intext Questions 20.12

1.	44	2.	9	3.	18
4.	13	5.	12	6.	35
7.	15	8.	11	9.	21

Exercise

1.	(i)	755244	(ii)	65443455	(iii)	7245992754
	(iv)	6747246	(v)	866646	(vi)	96760903239
2.	(i)	2016	(ii)	87016	(iii)	156024
	(iv)	99016	(v)	497024	(vi)	366009

3.	(i)	1504	(ii)	1782	(iii)	55912
	(iv)	475881	(v)	506142	(vi)	442476
4.	(i)	11449	(ii)	8281	(iii)	7744
	(iv)	11025	(v)	976144	(vi)	954579
5.	(i)	1061208	(ii)	912673	(iii)	884736
	(iv)	1124864	(v)	1191016	(vi)	778688